Biomedical Engineering Department

Third Stage

Medical Optics

Lecture 3

Lenses and prisms

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Lenses

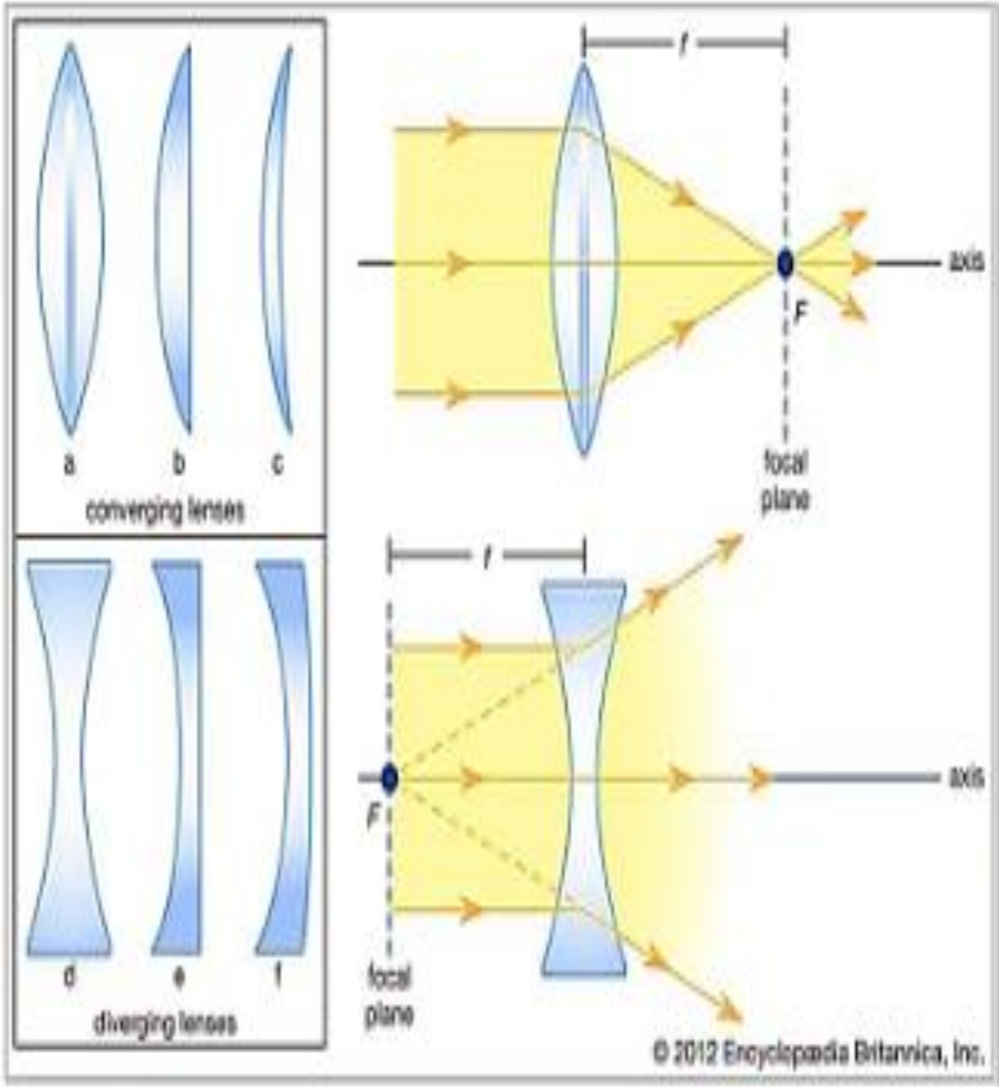
A lens is a piece of glass or other transparent substance that is used to form an image of an object by focusing rays of light from the object. Or a lens is an image forming device. It forms an image by refraction of light at its two bounding surfaces.

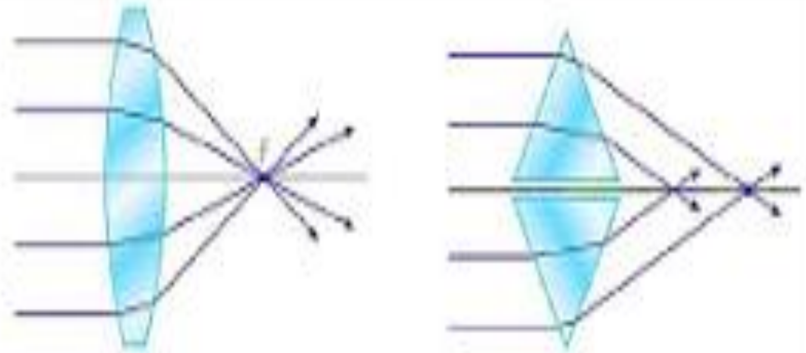
A lens has the valuable property of forming images of objects situated in front of it. Single lenses are used in eyeglasses, contact lenses, pocket magnifiers, projection condensers, viewfinders, and on simple box cameras.

More often several lenses made of different materials are combined together as a compound lens in a tube to permit the correction of aberrations. Compound lenses are used in such instruments as cameras, microscopes, and telescopes.

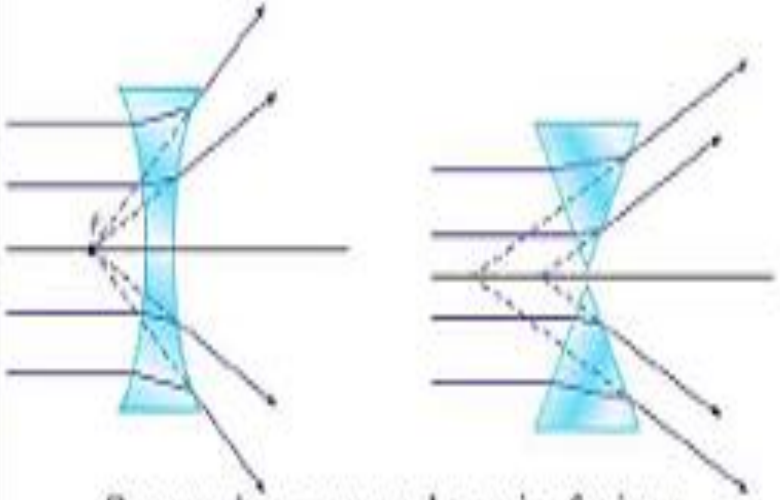
In general, a lens is made of glass and is bounded by two regular (polished) curved surfaces; or by one spherical surface and a plane surface. Spherical surfaces are easy to make. Therefore, most lenses are made of spherical surfaces. Other transparent materials such as quartz, fused silica and plastics are also used in making lenses. A single lens with two refracting surfaces is a simple lens.

Mainly there are two types of lenses: convex lens and concave lens. A convex (bulging) lens is thicker at the center than at the edges while a concave lens is thinner at the center than at the edges. A convex lens is a converging lens since a parallel beam of light, after refraction, converges to a point, F. A concave lens is called a diverging lens since rays coming parallel to the principal axis, after refraction, diverge out and seem to come from a point, F. Examine the following figures.



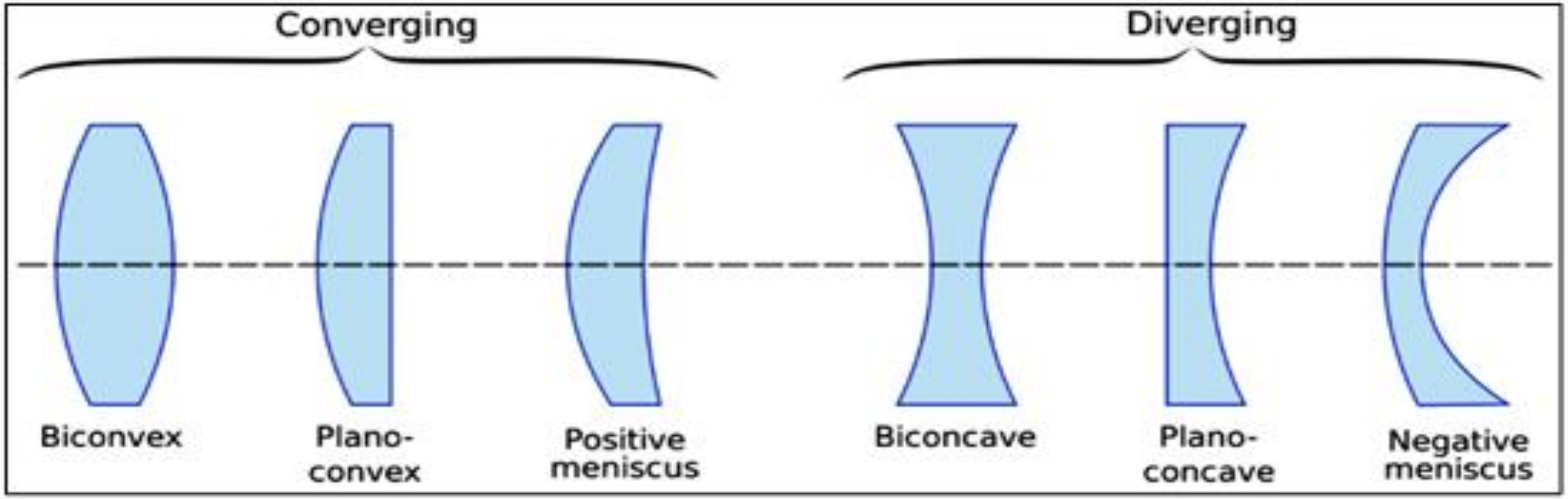


Convex lens compared to pair of prisms



Concave lens compared to pair of prisms

Within the two categories, there is a variety of simple lenses; some of the standard forms are shown in the figure below.

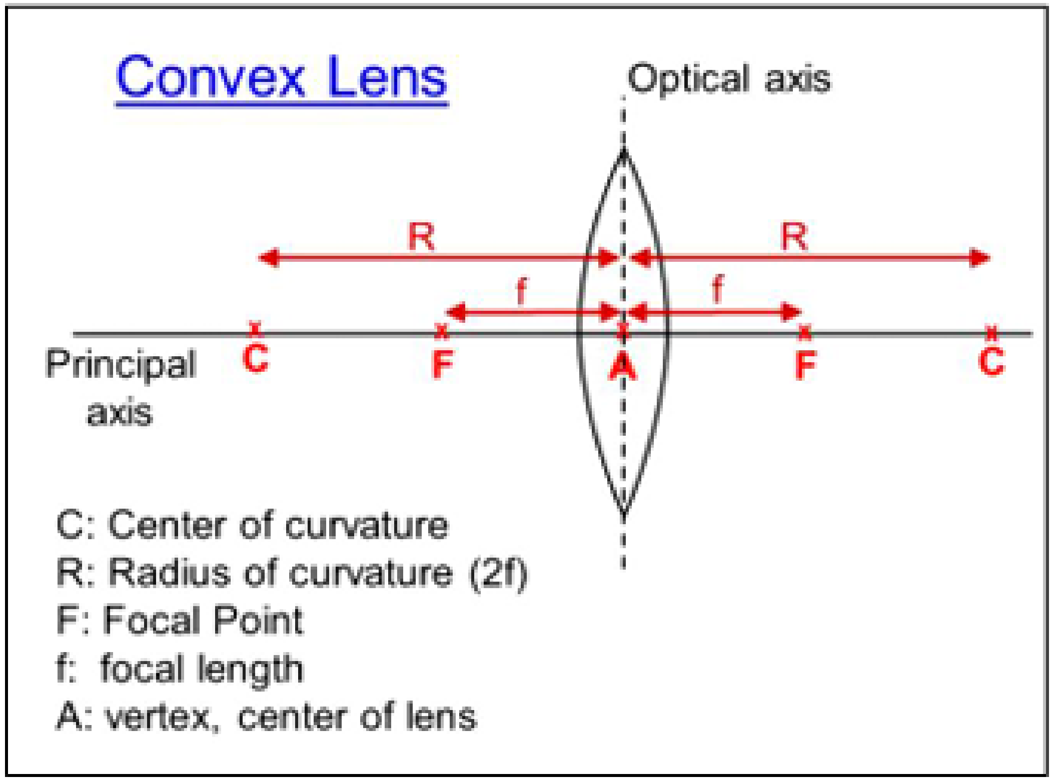


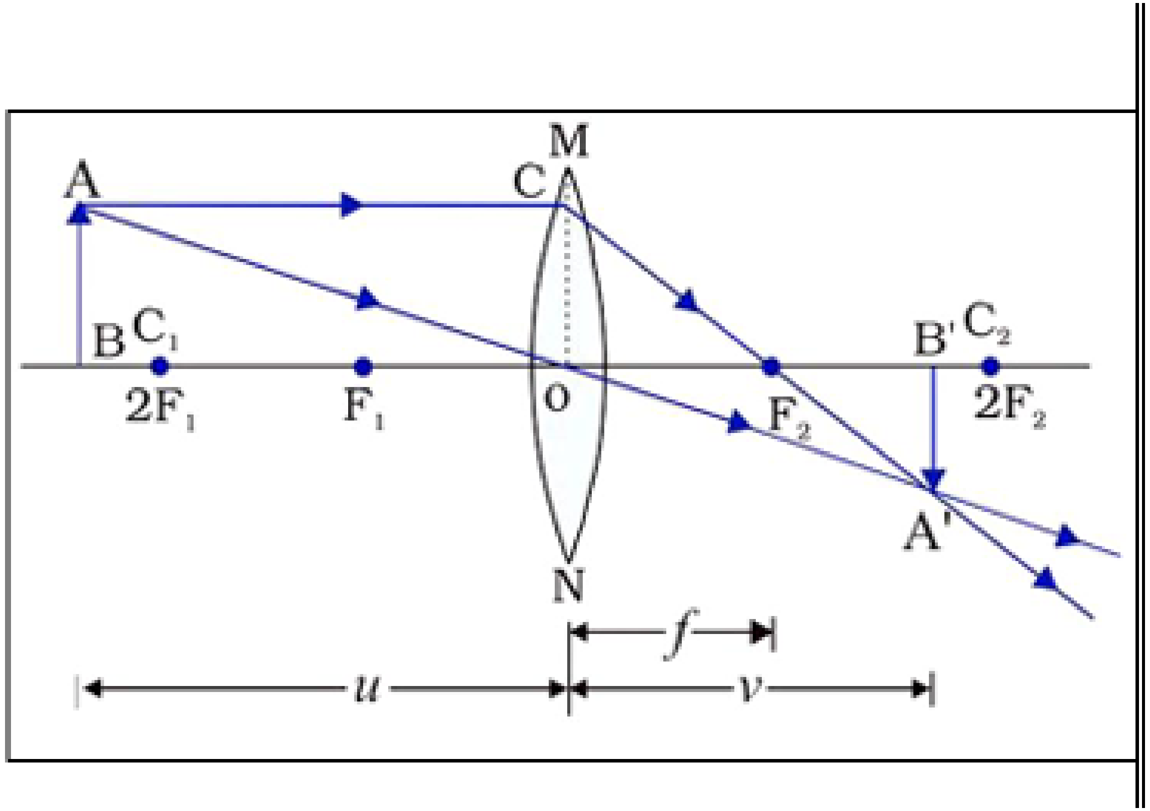
Terminology

- Light can be viewed as beams travelling between points. However, from most light sources, the light radiates outwards as a series of wavefronts. Light from a light source is bent. Wavefronts of light have a property known as curvature which decreases as waves travel further away from its source. It should be noted at this point that light from a source infinitely far away has negligible or zero curvature and can be considered straight. A lens has two curved surfaces, each surface has a curvature. The length of the radius of curvature of the surface is called the radius of curvature, R. The reciprocal of the length of the radius of curvature is known as the curvature C (C = 1/R), Curvature is measured in diopters (D).

- A lens has two centers of curvature and two radii of curvature, one for each refracting surface. The line joining the centers of two curved surfaces is called the principal axis or simply axis of the lens. The points where the principal axis intersects the two refracting surfaces are called the front vertex and the back vertex. The point F to which a set of rays parallel to the principal axis is caused to converge (in case of convex lens) or appear to diverge (in case of concave lens) is the focal point. For every lens, there is a point on the principal axis for which the ray passing through it are not deviated by the lens. Any ray passing through it emerges in a direction parallel to the incident ray. Such a point is called the optical center. When the lens is thin and the radii of curvature of the two refracting surfaces are equal, then the geometric center of the lens becomes the optical center of the lens.

The distance between the focal point F and the optical center of the lens is called the focal length of the lens. The plane perpendicular to the principal axis of lens and passing through its focal point is known as the focal plane. When a point object or a linear object is placed on one side of a convex lens beyond the focal plane, an image is formed on the opposite side, as shown in the figure below. The distance (u) is called the object distance, and the distance (v) is the image distance. The power of a lens is the reciprocal of its focal length.





Conjugate points, Planes and Distances

In figure below, all the rays coming from an object point B are brought to a focus at B'. Similarly, the rays from point A are brought to a focus at A'. A'B' is the image of the linear object AB. It follows from the principle of reversibility of light rays that the object and image positions are interchangeable. Thus, when the image A'B' becomes the object and AB becomes its image. The object and image are therefore conjugate. Any pair of object and image points A and A', are conjugate points. Planes on which the conjugate points lie are conjugate planes and the distances from the vertex to these planes are conjugate distances.

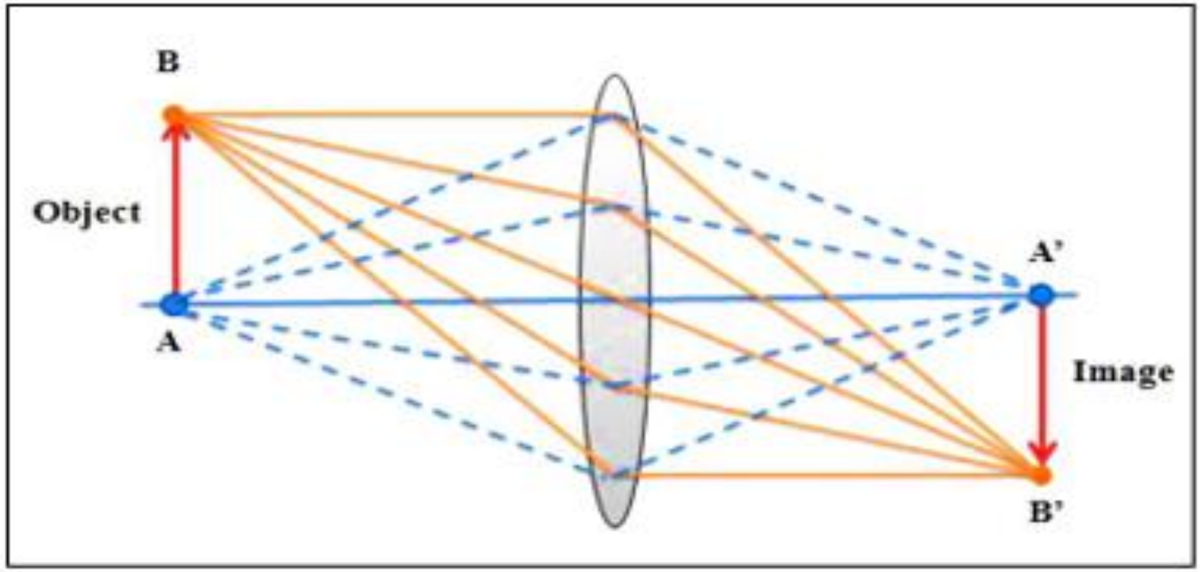
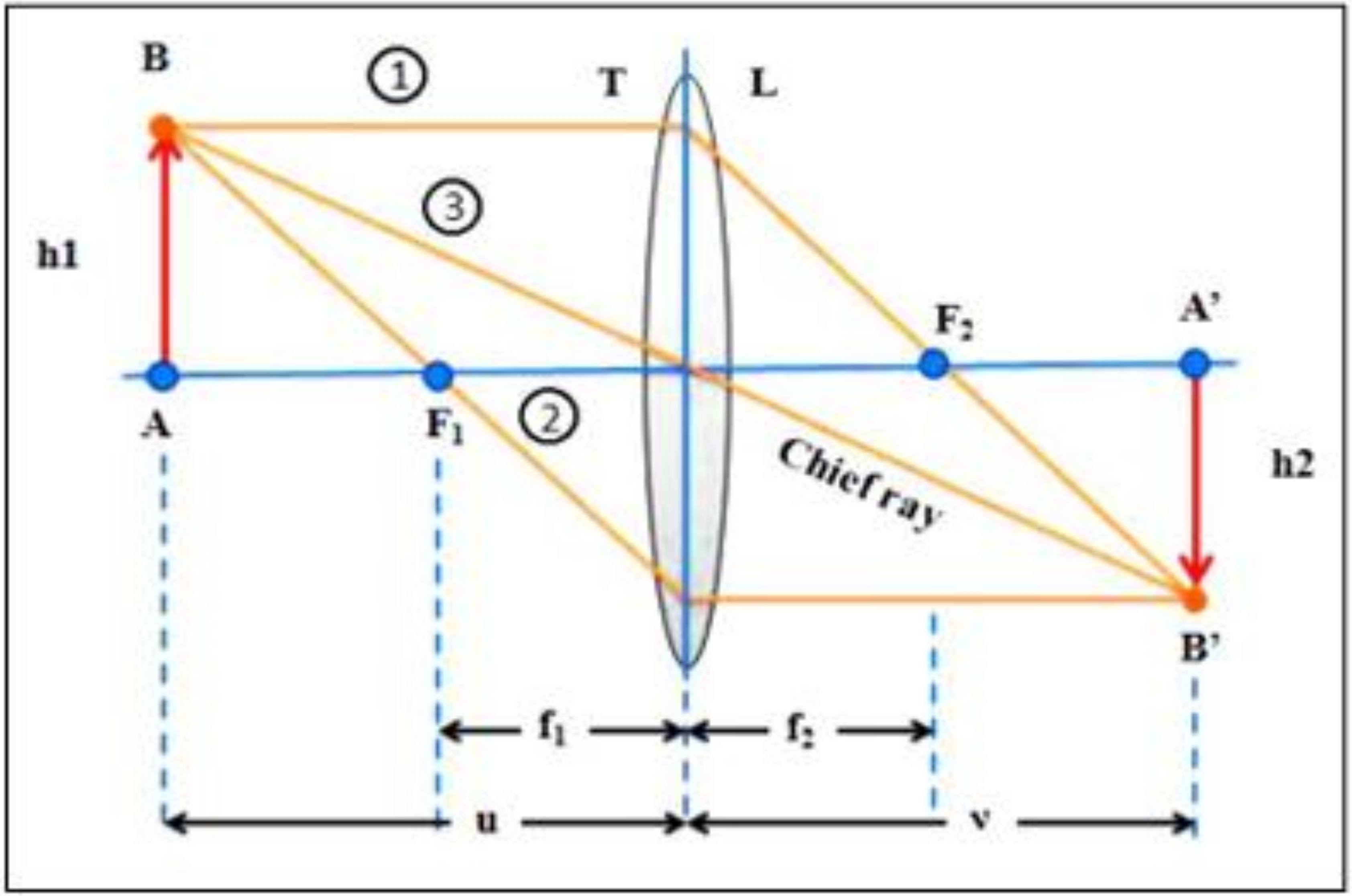


Image Tracing

Graphical ray tracing may be used to determine the position of the image formed by a lens. To find the image, take the help of characteristic rays shown in figure above.

First ray (p-ray) is the parallel to the principal axis, which after refraction, passes through focal point F2. Second ray (f-ray) is the ray that passes through the first focal point F1 of the lens; after refraction, it travels parallel to the principal axis.

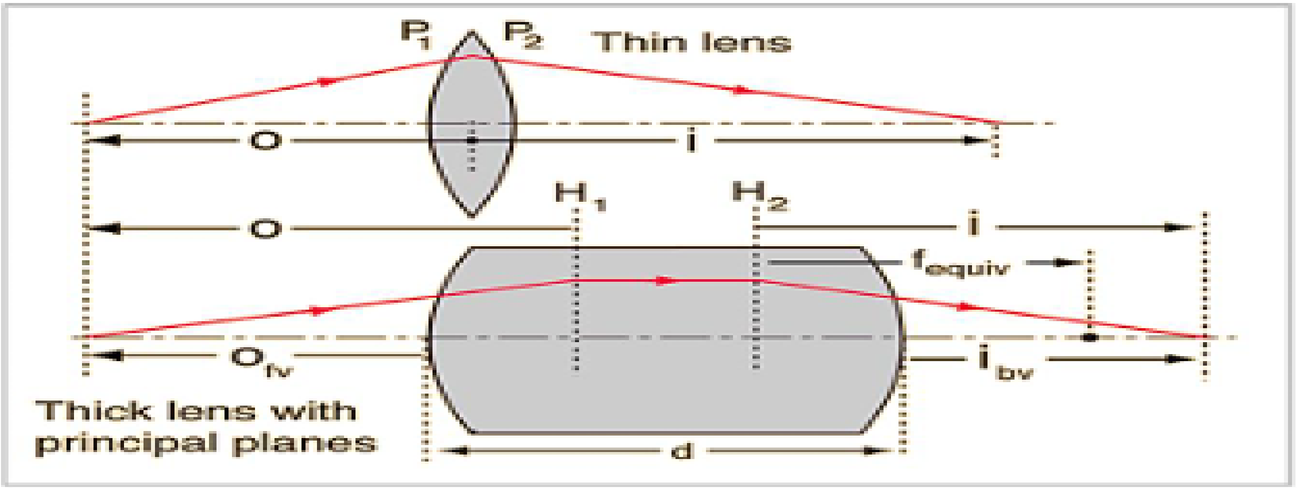
The third ray (c-ray), usually called chief or central ray goes through the optical center of the lens and emerges without deviation. Thus, there are three characteristic rays whose paths are known. Using any two of the three characteristic rays, the image of any object-point or of any extended can be determined.



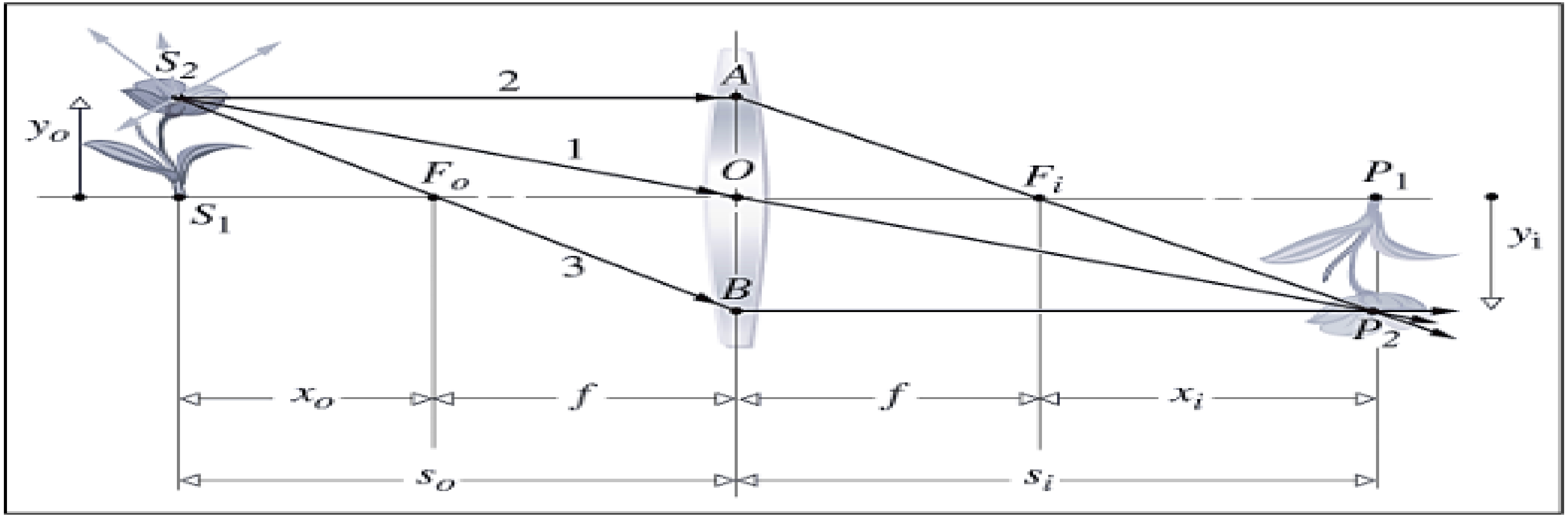
B
①
T
L
③
h1
F₂
A'
A
F₁
②
Chief ray
h2
B'
f₁
f₂
u
v

Thin and Thick Lenses

- The lenses are broadly classified into thin and thick lenses. A lens is said to be thin if the thickness of the lens can be neglected when compared to the lengths of the radii of curvature of its two refracting surfaces, and to the distances of the objects and images from it. No lens is actually a thin lens. Yet many simple lenses commonly used can be treated as equivalent to a thin lens. A thick lens is a physically large lens having two spherical surfaces separated by a distance, which is not negligible in comparison to the radii of curvature of the spherical surfaces.



Thin-Lens Equations



Lenses are used to form a magnified image of an object. Magnification is defined as: Magnification, M = size of the image / size of the object .

$$M = \boldsymbol{yi} \,/\boldsymbol{yo} \;= -\boldsymbol{si} \,/\boldsymbol{so}$$

Where $\boldsymbol{yi}$ and $\boldsymbol{yo}$ are the heights of the image and object respectively, $\boldsymbol{si}$ and $\boldsymbol{so}$ are the distances between the lens and the image and object respectively. The function of a lens is to increase or decrease the curvature of a wavefront. Lenses have a 'power'. This is the curvature which the lens adds to the wavefront. Power is measured in diopters, D (m-1), and is given by the formula:

$$p = \boldsymbol{1} \,/\boldsymbol{f}$$

where $\boldsymbol{f}$ equals the focal length of the lens which is the distance between the lens and the image.

A lens forms an image by refraction of light at its two bounding surfaces. Each surface acts as an image-forming component, and contributes to the final image formed by the lens.

$$1/\boldsymbol{f} = (\boldsymbol{n'} - \boldsymbol{1})(\boldsymbol{1} \,/\boldsymbol{R1} \;- \boldsymbol{1} \,/\boldsymbol{R2}\,)$$

$\boldsymbol{n'} = \boldsymbol{n2}/\,\boldsymbol{n1}\,, \boldsymbol{n2}$ is the refractive index of the lens material, $\boldsymbol{n1}$ is the refractive index of the medium surrounding the lens. The formula relating the curvature of the wavefronts leaving a lens to the curvature of the wavefronts entering it is: The Gaussian' formula for a lens:

$$1/\boldsymbol{f} = \boldsymbol{1} \,/\boldsymbol{si} \;+ \boldsymbol{1} \,/\boldsymbol{so}$$

where ***si*** is the distance between the lens (its center) and the in-focus image formed, ***so*** is the distance between the lens (its center) and the object which the in-focus image is of, and ***f*** is the focal length of the lens. The power of the lens can be substituted in for the reciprocal of ***f***, as they are the same thing.

Example: A biconvex (also called a double convex) thin spherical lens has radii of 100 cm and 20.0 cm. The lens is made of glass with an index of 1.54 and is immersed in air. (a) If an object is placed 70.0 cm in front of the 100-cm surface, locate the resulting image and describe it in detail. (b) Determine the magnification of the image. (c) Draw a ray diagram. Solution: (a) We don't have the focal length, but we do know all the physical parameters

$$1/f = (n' - 1)(1/R1 - 1/R2)$$

$$1/f = (1.54 - 1)(1/100 - 1/20)$$

f=30.86 cm

Now we can find the image. Since so = 70.0 cm, that's greater than $2f$—hence, even before we calculate si, we know that the image will be real, inverted, located between f and $2f$, and minified.

$$1/f = 1/si + 1/so$$

$$1/30.86 = 1/si + 1/70 \quad \rightarrow \quad si = 55.2\ cm$$

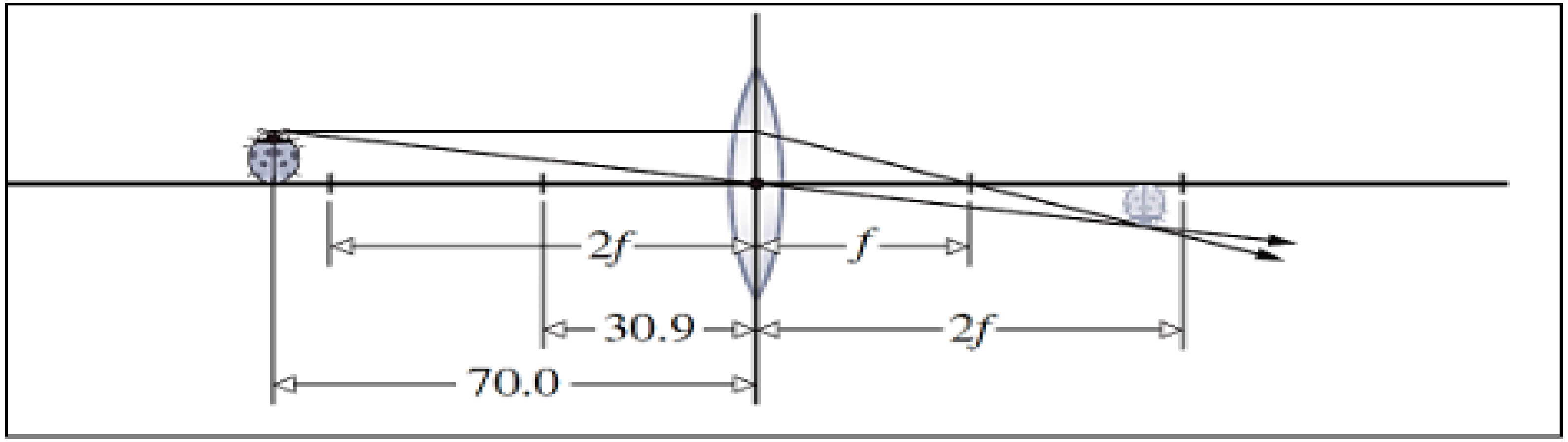
The image is between f and $2f$ on the right of the lens. Note that si=70 cm 0, which means the image is real.

(b) The magnification follows from

$M = -si\ /so\ = -55.2/\ 70\ = -0.788$

and the image is inverted (M< 0) and minified (M<1).

(c)



Example: Both surfaces of an equiconvex thin spherical lens have the same curvature. A 2.0-cm-tall bug is on the central axis 100 cm from the front face of the lens. The image of the bug formed on a wall is 4.0 cm tall. Given that the glass of the lens has an index of 1.50, find the radii of curvature of the surfaces.

Solution:

We have yo = 2.0 cm, so = 100 cm, R1 = R2, $|yi|$ = 4.0 cm, and n'= 1.50. We also know that the image is real, so it must be inverted and therefore yi = -4.0 cm.

$$M = yi\ /yo\ = -si/\ so\ = -4/\ 2\ = -2$$

$$-2\ so = -si\ \rightarrow\ si = 200\ cm$$

$$1/\ f = 1/\ si\ + 1\ /so\ = \ 1\ /100 + \ 1/\ 200\ \rightarrow\ f = 66.67\ cm$$

$$1\ /f = (n' - 1)(1\ /R1\ - \ 1\ /-R2\ )$$

$$1\ /f = (1.5 - 1)(1\ /R1\ - \ 1/-R2\ ) = \frac{1}{2}\frac{2}{R}$$

$$R = 66.67\ cm$$

Location of The Image

A convex lens produces a real or virtual image depending on the location of the object. A concave lens always produces virtual images of real objects. The figure below illustrates the behavior of the convex lens pictorially. As the object approaches the lens, it is seen that the real image moves away from it.

1. When the object is very far away, the image is just to the right of the focal plane. The image is real, inverted, and smaller in size than the object ($M < 1$).

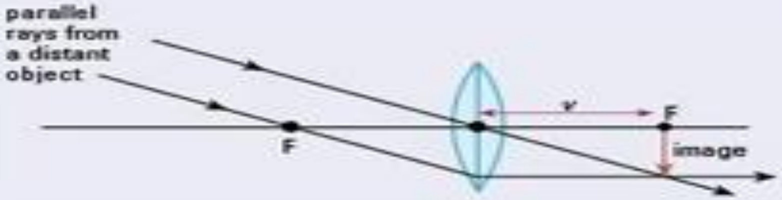
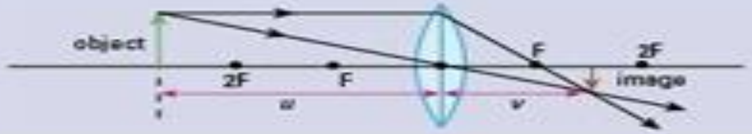
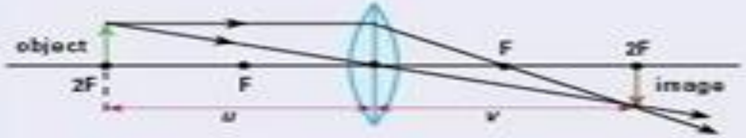
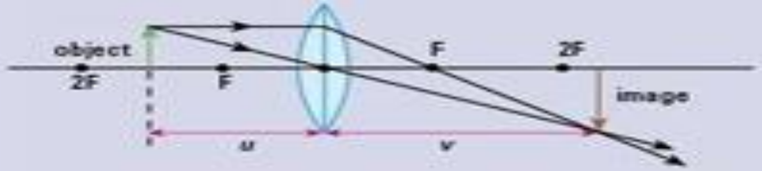
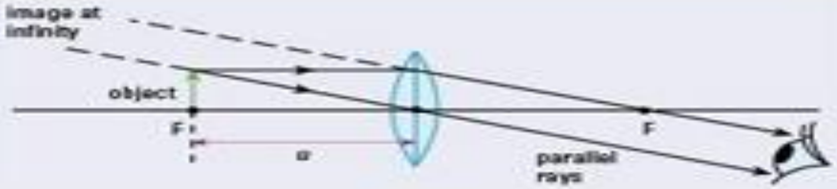
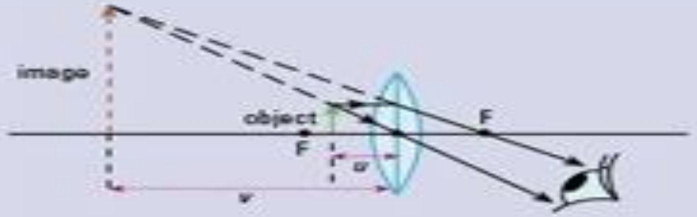
2. As the object approaches the lens, the image moves away to the right of the focal plane getting larger and larger. The image is real and inverted.

3. When the object is at 2F, the image is real, inverted and of the same size as the object ($M = 1$).

4. When the object is in between 2F and F, the image is enlarged ($M > 1$), real and inverted.

5. When the object is precisely at F, there is no image as the emerging rays are parallel and in effect the image is at infinity.

6. When the object closer in than one F, the image appears. It is virtual, erect and enlarged ($M > 1$).

Object distance (u)	Ray diagram	Type of image	Image distance (v)	Uses
$u = \infty$		- inverted - real - diminished	$v = f$ - opposite side of the lens	- object lens of a telescope
$u > 2f$		- inverted - real - diminished	$f < v < 2f$ - opposite side of the lens	- camera - eye
$u = 2f$		- inverted - real - same size	$v = 2f$ - opposite side of the lens	- photocopier making same-sized copy
$f < u < 2f$		- inverted - real - magnified	$v > 2f$ - opposite side of the lens	- projector - photograph enlarger
$u = f$		- upright - virtual - magnified	- image at infinity - same side of the lens	- to produce a parallel beam of light, e.g. a spotlight
$u < f$		- upright - virtual - magnified	- image is behind the object - same side of the lens	- magnifying glass