Department of Biomedical Engineering
Digital Electronics / Second stage
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Lecture Two

Arithmetic Operations and Codes

1- Binary Arithmetic

Binary arithmetic is essential in all digital computers and other digital systems. To understand digital systems, you must know the basics of binary addition, subtraction, multiplication, and division.

a- Binary Addition

The four basic rules of addition binary digits are as follows:

0 + 0 = 0	Sum of 0 with a carry of 0
0 + 1 = 1	Sum of 1 with a carry of 0
1 + 0 = 1	Sum of 1 with a carry of 0
1 + 1 = 10	Sum of 0 with a carry of 1

Example 1

Add the following binary numbers:

(a) 11 + 11   (b) 100 + 10   (c) 111 + 11   (d) 110 + 100

Solution

(a) $\begin{array}{r} 11 \\ +11 \\ \hline 110 \end{array}$ $\begin{array}{r} 3 \\ +3 \\ \hline 6 \end{array}$ **(b)** $\begin{array}{r} 100 \\ +10 \\ \hline 110 \end{array}$ $\begin{array}{r} 4 \\ +2 \\ \hline 6 \end{array}$ **(c)** $\begin{array}{r} 111 \\ +\ 11 \\ \hline 1010 \end{array}$ $\begin{array}{r} 7 \\ +3 \\ \hline 10 \end{array}$ **(d)** $\begin{array}{r} 110 \\ +100 \\ \hline 1010 \end{array}$ $\begin{array}{r} 6 \\ +4 \\ \hline 10 \end{array}$

b- Binary Subtraction

The four basic rules of Subtraction of binary digits are as follows:

0 - 0 = 0
1 - 1 = 0
1 - 0 = 1
10 – 1 = 1 0 – 1 with a borrow of 1

Example (1)

Perform the following subtraction

(a) 11 – 01 (b) 11 – 10

Solution

(a)

$$\begin{array}{r} 11 \\ -\ 01 \\ \hline 10 \end{array} \qquad \begin{array}{r} 3 \\ -\ 1 \\ \hline 2 \end{array}$$

(b)

$$\begin{array}{r} 11 \\ -\ 10 \\ \hline 01 \end{array} \qquad \begin{array}{r} 3 \\ -\ 2 \\ \hline 1 \end{array}$$

Example (2)

Subtract 011 from 101

Solution

$$\begin{array}{r} 101 \\ -\ 011 \\ \hline 010 \end{array} \qquad \begin{array}{r} 5 \\ -\ 3 \\ \hline 2 \end{array}$$

c- Binary Multiplication

The four basic rules of multiplying bits are as follows:

0 × 0 = 0
0 × 1 = 0
1 × 0 = 0
1 × 1 = 1

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Example (1)

Perform the following binary multiplications

(a) 11 × 11 (b) 101 × 111

Solution

(a)

```
   11        3
 × 11      × 3
 ----      ---
   11        9
 + 11
 ----
 1001
```

(b)

```
                 111        7
               × 101      × 5
               -----      ---
Partial   {      111       35
products  {     000
          {  + 111
             ------
             100011
```

d- Binary Division

Example (1)

Perform the following binary Division

(a) 110 ÷ 11 (b) 110 ÷ 10 (c) 1001011 ÷ 11

Solution

```
        10       2                  11       3
(a) 11)110     3)6      (b) 10)110     2)6
        11       6                 10       6
        ---      -                 ---      -
        000      0                  10      0
                                    10
                                    ---
                                    00
```

(c)

```
       1 1 0 0 1
    -------------
1 1 ) 1 0 0 1 0 1 1
    -   1 1
      -----
          1 1
        - 1 1
        -----
          0 0 0
        -     0
          -----
              0 1
            -   0
            -----
                1 1
              - 1 1
              -----
                  0
```

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Complements of Binary Numbers

The 1’s complement and the 2’s complement of a binary number are important because they permit negative number representation. The method of 2’s complement arithmetic is commonly used in computers to handle negative numbers.

Finding the 1’s Complement

The 1’s complement of binary number is found by changing all 1s to 0s and all 0s to 1’s as illustrated below:

10110010
↓↓↓↓↓↓↓↓
01001101

Finding the second complement

The 2’s complement of a binary number is found by adding 1 to the LSB of the 1’s complement.

2’s complement = (1’s complement) + 1

Find the 2’s complement of 10110010

Solution

10110010	Binary number
↓	
01001101	1’s complement
+ 1	**add 1**
01001110	**2’s complement**

Note:

The left most bit in a signed binary number is the sign bit, which tells you whether the number is positive or negative.

A 0 sign bit indicates a positive number, and a 1 sign bit indicate a negative number.

Hexadecimal Addition

When adding two hexadecimal numbers, use the following rules:

1- In any given column of an addition problem, think of the two hexadecimal digits in terms of their decimal value. For instance, $5_{16} = 5_{10}$ and $C_{16} = 12_{10}$.

2- If the sum of these two digits is 15_{10} or less, bring down the corresponding hexadecimal digit.

3- If the sum of these two digits is greater than 15_{10}, bring down the amount of the sum that exceeds 16_{10} and carry a 1 to the next column.

Example

Add the following hexadecimal numbers:

(a) $23_{16} + 16_{16}$ (b) $58_{16} + 22_{16}$ (c) $2B_{16} + 84_{16}$ (d) $DF_{16} + AC_{16}$

Solution

(a) $\begin{array}{r} 23_{16} \\ +\,16_{16} \\ \hline \mathbf{39_{16}} \end{array}$ right column: $3_{16} + 6_{16} = 3_{10} + 6_{10} = 9_{10} = 9_{16}$
left column: $2_{16} + 1_{16} = 2_{10} + 1_{10} = 3_{10} = 3_{16}$

(b) $\begin{array}{r} 58_{16} \\ +\,22_{16} \\ \hline \mathbf{7A_{16}} \end{array}$ right column: $8_{16} + 2_{16} = 8_{10} + 2_{10} = 10_{10} = A_{16}$
left column: $5_{16} + 2_{16} = 5_{10} + 2_{10} = 7_{10} = 7_{16}$

(c) $\begin{array}{r} 2B_{16} \\ +\,84_{16} \\ \hline \mathbf{AF_{16}} \end{array}$ right column: $B_{16} + 4_{16} = 11_{10} + 4_{10} = 15_{10} = F_{16}$
left column: $2_{16} + 8_{16} = 2_{10} + 8_{10} = 10_{10} = A_{16}$

(d) $\begin{array}{r} DF_{16} \\ +\,AC_{16} \\ \hline \mathbf{18B_{16}} \end{array}$ right column: $F_{16} + C_{16} = 15_{10} + 12_{10} = 27_{10}$
$27_{10} - 16_{10} = 11_{10} = B_{16}$ with a 1 carry
left column: $D_{16} + A_{16} + 1_{16} = 13_{10} + 10_{10} + 1_{10} = 24_{10}$
$24_{10} - 16_{10} = 8_{10} = 8_{16}$ with a 1 carry

Hexadecimal Subtraction

There are several methods to subtract hexadecimal, one of the most common methods is converting the hexadecimal number to binary. Take the 2's complement of the binary number. Convert the result to hexadecimal.

Example

Subtract the following hexadecimal numbers:

(a) $84_{16} - 2A_{16}$ (b) $C3_{16} - 0B_{16}$

Solution

(a) 2A = 00101010 Binary number

↓

11010101 1's complement

\+ 1 **add 1**

11010110 = D6 2's complement

84
+ D6
~~1~~5A Drop carry

The difference is $5A_{16}$

(b) 0B = 00001011 Binary number

11110100 1's complement

\+ **1** **add 1**

11110101 = F5 2's complement

C3
+ F5 **add**
~~1~~B8 Drop carry

The difference is $B8_{16}$

Binary Coded Decimal (BCD)

Binary coded decimal (BCD) is a way to express each of the decimal digits with a binary code. There are only ten code group in the BCD system, so it is very easy to convert between decimal and BCD. Because we like to read and write in decimal, the BCD code provides an excellent interface to a binary system.

The 8421 BCD code

The 8421 code is a type of BCD code. Binary coded decimal means that each decimal digit, 0 through 9, is represented by a binary code of four bits. The designation 8421 indicates the binary weight of the four bits (2^3, 2^2, 2^1, 2^0)

Decimal digit	0	1	2	3	4	5	6	7	8	9
BCD	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

Invalid codes

In the 8421 code, only ten of (0000 through 1111) are used. The six code combinations that are not used are 1010, 1011, 1100, 1101, 1110, and 1111 which are invalid in the 8421 BCD code.

Example: Convert each of the following decimal numbers to BCD

(a) 35 (b) 98 (c) 170 (d) 2469

Solution

(a) 3 5 → **00110101**

(b) 9 8 → **10011000**

(c) 1 7 0 → **000101110000**

(d) 2 4 6 9 → **0010010001101001**

Example 2

Convert each of the following BCD codes to decimal

(a) 10000110 (b) 001101010001 (c) 1001010001110000

Solution

(a) 1000 0110 → 8 6

(b) 0011 0101 0001 → 3 5 1

(c) 1001 0100 0111 0000 → 9 4 7 0

BCD addition

BCD is a numerical code and can be used in arithmetic operations.
The steps of adding two BCD numbers:

1- Add the two BCD numbers using the role of binary.
2- If a 4-bit sum is equal to or less than 9, it is a valid BCD number.
3- If a 4-bit sum is greater than 9, or if a carry-out of the 4-bit group is generated, it is an invalid result. Add 6 (0110) to the 4-bit sum to skip the six invalid states and return the code to 8421.

Example1

Add the following BCD numbers:

(a) 0011 + 0100 (b) 00100011 + 00010101

(c) 10000110 + 00010011 (d) 01001010000 + 010000010111

Solution

(a)

0011	3
+ 0100	+ 4
0111	7

(b)

0010	0011	23
+ 0001	0101	+ 15
0011	**1000**	38

(c)

1000	0110	86
+ 0001	0011	+ 13
1001	**1001**	**99**

(d)

0100	0101	0000	450
+ 0100	0001	0111	+ 417
1000	**0110**	**0111**	**867**

Example:

Add the following BCD numbers:

(a) 1001 + 0100 **(b)** 1001 + 1001

(c) 00010110 + 00010101 **(d)** 01100111 + 01010011

Solution

The decimal number additions are shown for comparison.

(a)

	1001		9
	+ 0100		+4
	1101	Invalid BCD number (>9)	13
	+ 0110	Add 6	
0001	**0011**	Valid BCD number	
↓ 1	↓ 3		

(b)

	1001		9
	+ 1001		+ 9
1	0010	Invalid because of carry	18
	+ 0110	Add 6	
0001	**1000**	Valid BCD number	
↓ 1	↓ 8		

(c)

0001	0110		16
+ 0001	0101		+ 15
0010	1011	Right group is invalid (>9), left group is valid.	31
	+ 0110	Add 6 to invalid code. Add carry, 0001, to next group.	
0011	**0001**	Valid BCD number	
↓ 3	↓ 1		

(d)

	0110	0111		67
	+ 0101	0011		+ 53
	1011	1010	Both groups are invalid (>9)	120
	+ 0110	+ 0110	Add 6 to both groups	
0001	**0010**	**0000**	Valid BCD number	
↓ 1	↓ 2	↓ 0		

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Gray code

Binary to Gray code conversion

There are two steps in conversion from binary to gray code

1- The most significant bit (left-most) in the Gray code is the same as the corresponding MSB in the binary.

2- Going from left to right, add each adjacent pair of binary code bits to get the next Gray code bit. Discard the carries.

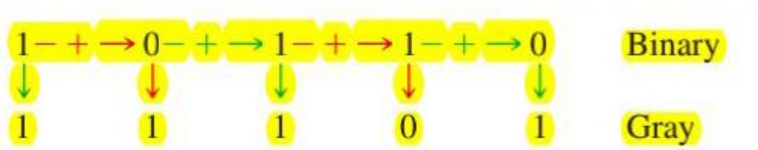
Example 1

(a) Convert the binary number 10110 to Gray code

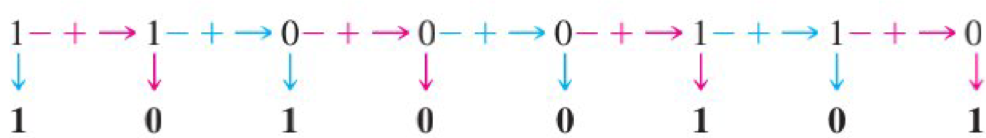
(b) Convert the binary number 11000110 to Gray code

Solution

(a)



(b)



Gray to binary code conversion

There are two steps in conversion from gray code to binary

1- The most significant bit (left-most) in the binary code is the same as the corresponding bit in the Gray code.

2- Add each binary code bit generated to the Gray code bit in the next adjacent position. Discard carries.

Example

(a) Convert the Gray code 11011 to binary

(b) Convert the Gray code 10101111 to binary

Solution

(a)

1	1	0	1	1	Gray
↓	+ ↗ ↓	+ ↗ ↓	+ ↗ ↓	+ ↗ ↓	
1	0	0	1	0	Binary

(b) Gray code to binary:

1	0	1	0	1	1	1	1
↓	+ ↗ ↓	+ ↗ ↓	+ ↗ ↓	+ ↗ ↓	+ ↗ ↓	+ ↗ ↓	+ ↗ ↓
1	**1**	**0**	**0**	**1**	**0**	**1**	**0**