Al-Mustaqbal University
College of Engineering and Technology
Department of Biomedical Engineering
Stage: Second
Electric Circuits II
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Lecture (3): Mix connection

9.4 IMPEDANCE AND ADMITTANCE

In the preceding section, we obtained the voltage-current relations for the three passive elements as

$$\mathbf{V = RI,\ V = j\omega L\, I,\ V = I / j\omega C} \tag{9.24}$$

These equations may be written in terms of the ratio of the phasor voltage to the phasor current as

$$\mathbf{V/I = R,\ V/I = j\omega L,\ V/I = 1/ j\omega C} \tag{9.25}$$

From these three expressions, we obtain Ohm's law in phasor form for any type of element as

$$\mathbf{Z = V/I \text{ or } V = ZI} \tag{9.26}$$

where Z is a frequency-dependent quantity known as impedance, measured in ohms.

The impedance Z of a circuit is the ratio of the phasor voltage V to the phasor current I, measured in ohms (Ω).

The impedance represents the opposition which the circuit exhibits to the flow of sinusoidal current. Although the impedance is the ratio of two phasors, it is not a phasor, because it does not correspond to a sinusoidally varying quantity. The impedances of resistors, inductors, and capacitors can be readily obtained from Eq. (9.25). we notice that $Z_L = j\omega L$ and $Z_C = - j/\omega C$. Consider two extreme cases of angular frequency. When $\omega = 0$ (i.e., for dc sources), $Z_L = 0$ and $Z_C \rightarrow \infty$, confirming what we already know— that the inductor acts like a short circuit, while the capacitor acts like an open circuit. When $\omega \rightarrow \infty$ (i.e., for high frequencies), $Z_L \rightarrow \infty$ and $Z_C = 0$, indicating that the inductor is an open circuit to high frequencies, while the capacitor is a short circuit.

As a complex quantity, the impedance may be expressed in rectangular form as

$$\mathbf{Z = R + jX} \tag{9.27}$$

where R = Re Z is the resistance and X = Im Z is the reactance. The reactance X may be positive or negative. The impedance, resistance, and reactance are all measured in ohms. The impedance may also be expressed in polar form as

$$\mathbf{Z = |Z| \angle\theta} \tag{9.28}$$

Comparing Eqs. (9.27) and (9.28), we infer that

$$\mathbf{Z = R + jX = |Z| \angle\theta} \tag{9.29}$$

Where $\mathbf{|Z| = \sqrt{R^2 + X^2}\ ,\ \theta = \tan^{-1}\frac{X}{R}}$ (9.30)

and

$\mathbf{R = |Z|\cos\theta, \quad X = |Z|\sin\theta}$ (9.31)

It is sometimes convenient to work with the reciprocal of impedance, known as admittance.

The admittance Y is the reciprocal of impedance, measured in siemens (S).

The admittance Y of an element (or a circuit) is the ratio of the phasor current through it to the phasor voltage across it, or

$\mathbf{Y = 1/Z = I/V}$ (9.32)

The admittances of resistors, inductors, and capacitors can be obtained from Eq. (9.39). As a complex quantity, we may write Y as

$\mathbf{Y = G + jB}$ (9.33)

where G = Re Y is called the conductance and B = Im Y is called the susceptance. Admittance, conductance, and susceptance are all expressed in the unit of siemens (or mhos). From Eqs. (9.27) and (9.33),

$\mathbf{G + jB = \frac{1}{R + jX}}$ (9.34)

the real and imaginary parts gives

$\mathbf{G = \frac{R}{R^2 + X^2}\ ,\ B = \frac{-X}{R^2 + X^2}}$ (9.35)

showing that $G \neq 1/R$ as it is in resistive circuits. Of course, if X = 0, then G = 1/R.

Example 9.4: Find v(t) and i(t) in the circuit shown in Fig. 9.9.

Solution: From the voltage source 10 cos 4t , ω = 4, Vs = 10 ∠0 ◦ V The impedance is

$$Z = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5\ \Omega$$

Hence the current

$$I = \frac{Vs}{Z} = \frac{10\angle 0^\circ}{5 - j\,2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2}$$

$$= 1.6 + j0.8 = 1.789\angle 26.57^\circ\ \text{A} \qquad (9.4.1)$$

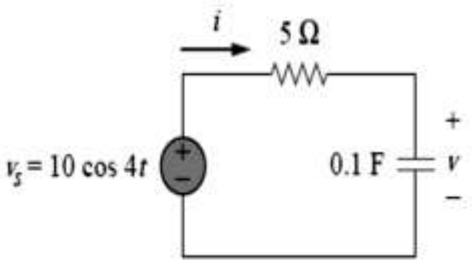


Fig. 9.9

The voltage across the capacitor is

$$V = I\,Z C = \frac{I}{j\omega C} = \frac{1.789\angle 26.57^\circ}{j4 \times 0.1}$$

$$= \frac{1.789\angle 26.57^\circ}{0.4\angle\mathbf{90}^\circ} = 4.47\angle -63.43^\circ\ \text{V} \qquad (9.4.2)$$

Converting I and V in Eqs. (9.9.1) and (9.9.2) to the time domain, we get

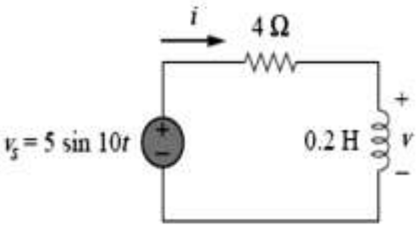
$$i(t) = 1.789 \cos(4t + 26.57^\circ)\ \text{A}$$

$$v(t) = 4.47 \cos(4t - 63.43^\circ)\ \text{V}$$

Notice that i(t) leads v(t) by 90° as expected.

Practice problem 9.4: Refer to Figure below. Determine v(t) and i(t)

Answer: 2.236 sin(10t + 63.43°) V, 1 .118 sin(10t − 26.57°) A.



9.5 KIRCHHOFF'S LAWS IN THE FREQUENCY DOMAIN

We cannot do circuit analysis in the frequency domain without Kirchhoff's current and voltage laws. Therefore, we need to express them in the frequency domain.

For KVL, let $\mathbf{v_1, v_2, \ldots, v_n}$ be the voltages around a closed loop. Then

$$\mathbf{v_1 + v_2 + \cdots + v_n = 0} \qquad (9.36)$$

In the sinusoidal steady state, each voltage may be written in cosine form, so that Eq. (9.36) becomes

$$\mathbf{V_{m1} \cos(\omega t + \theta_1) + V_{m2} \cos(\omega t + \theta_2) + \cdots + V_{mn} \cos(\omega t + \theta_n) = 0} \qquad (9.37)$$

This can be written as

$$\mathbf{Re(V_{m1}\, e^{j\theta 1}\, e^{j\omega t}) + Re(V_{m2}\, e^{j\theta 2}\, e^{j\omega t}) + \cdots + Re(V_{mn}\, e^{j\theta n}\, e^{j\omega t}) = 0}$$

or

$$\mathbf{Re[(V_{m1}\, e^{j\theta 1} + V_{m2}\, e^{j\theta 2} + \cdots + V_{mn}\, e^{j\theta n})\, e^{j\omega t}] = 0} \qquad (9.37)$$

If we let $V_k = V_{mk}\, e^{j\theta k}$, then

$$\mathbf{Re[(V_1 + V_2 + \cdots + V_n)\, e^{j\omega t}] = 0} \qquad (9.38)$$

Since $e^{j\omega t} \neq 0$,

$$\mathbf{V_1 + V_2 + \cdots + V_n = 0} \qquad (9.39)$$

indicating that Kirchhoff's voltage law holds for phasors.

By following a similar procedure, we can show that Kirchhoff's current law holds for phasors. If we let $i_1, i_2, \ldots, i_n$ be the current leaving or entering a closed surface in a network at time t , then