Department of Biomedical Engineering

Digital Electronics / Second stage

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Lecture One

Numbering system

A **numbering system** is a mathematical framework for representing and organizing numbers using a specific set of symbols and rules. It defines how numbers are expressed and manipulated within a given base or radix, determining the number of unique symbols representing values. Numbering systems are essential for various fields, including mathematics, engineering, and computer science, as they provide a structured way to handle numerical data.

Common Numbering Systems:

1. **Decimal (Base 10):** The most widely used system, with digits 0–9.
   - **Example**: 123_9 (decimal).
2. **Binary (Base 2):** Used in computing and digital systems, with digits 0 and 1.
   - **Example**: 1010_2 (binary).
3. **Octal (Base 8):** Used in computing, with digits 0 –7.
   - **Example**: 74_8 (octal).
4. **Hexadecimal (Base 16):** Used in computing and programming, with digits 0–9 and letters A–F.
   - Example: $1F3_{16}$ (hexadecimal).

1- BINARY NUMBERS

The binary number system uses only two symbols (0, l). It is said to have a radix of 2 and is commonly called the **base 2** number system. Each binary digit is called a bit.

Counting in binary is illustrated in Fig. 1. The binary number is shown on the right with its decimal equivalent. Notice that the least significant bit (LSB) is the 1s place.

Decimal count	Binary count				
	16s	8s	4s	2s	1s
0					0
1				0	1
2				1	0
3				1	1
4			1	0	0
5			1	0	1
6			1	1	0
7			1	1	1
8		1	0	0	0
9		1	0	0	1
10		1	0	1	0
11		1	0	1	1
12		1	1	0	0
13		1	1	0	1
14		1	1	1	0
15		1	1	1	1
16	1	0	0	0	0
	2^4	2^3	2^2	2^1	2^0
	Power of 2				

Fig. 1 Counting in binary and decimal

Consider the number shown in Fig. 2. This figure shows how to convert the binary 10011 to its decimal equivalent. Note that, for each 1 bit in the binary number, the decimal equivalent for that place value is written below. The decimal numbers are then added (16 + 2 + 1 = 19) to yield the decimal equivalent. Binary 10011 then equals a decimal 19.

Power of 2	2^4	2^3	2^2	2^1	2^0	
Place value	**16s**	**8s**	**4s**	**2s**	**1s**	
Binary	1	0	0	1	1	
Decimal	16	0	0	2	1	= 19

Fig. 2 Binary to decimal conversion

How about converting fractional numbers? Figure 3 illustrates the binary number 1110.101 being converted to its decimal equivalent. The place values are given across the top. The place value of each 1 bit in the binary number is added to form the decimal number. In this problem 8 + 4 + 2 + 0.5 + 0.125 = 14.625 in decimal

Power of 2	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	
Place value	**8s**	**4s**	**2s**	**1s**		**0.5s**	**0.25s**	**0.125s**	
Binary	1	1	1	0	.	1	0	1	
Decimal	8	4	2	0	.	0.5	0	0.125	= 14.625

Converting decimal to binary

The decimal 87 is first divided by 2, leaving 43 with a remainder of 1. The remainder is important and is recorded at the right. It becomes the **LSB** in the binary number. The quotient (43) is then transferred as shown by the arrow and becomes the dividend. The quotients are repeatedly divided by 2 until the quotient becomes 0 with a remainder of 1, as in the last line of Fig. 3. Near the bottom, the figure shows that decimal 87 equals binary 1010111.

LSB

$87_{10} \div 2 = 43$ remainder 1

$43 \div 2 = 21$ remainder 1

$21 \div 2 = 10$ remainder 1

$10 \div 2 = 5$ remainder 0

$5 \div 2 = 2$ remainder 1

$2 \div 2 = 1$ remainder 0

MSB

$1 \div 2 = 0$ remainder 1

1 0 1 0 1 1 1

Fig. 3 Decimal-to-binary conversion

Convert the decimal number 0.375 to a binary number. Figure 5 illustrates one method of performing this task. Note that the decimal number (0.375) is being multiplied by 2. When the product is 1.00, the conversion process is complete. Figure 4 shows a decimal 0.375 being converted into a binary equivalent of 0.011.

$0.375 \times 2 = 0.75$

$0.75 \times 2 = 1.50$

$0.50 \times 2 = 1.00$

$0.375_{10} = .0\,1\,1_2$

Fig. 4 Fractional decimal-to-binary conversions

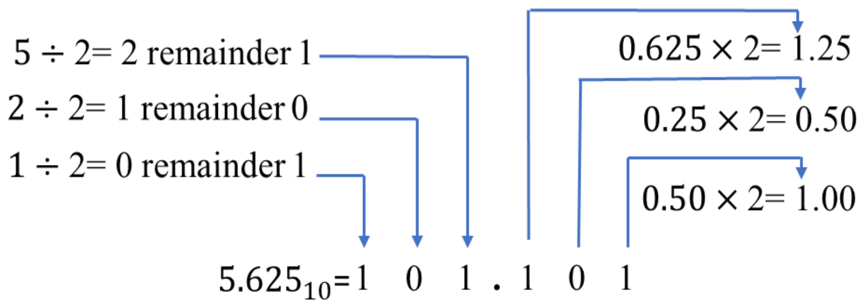

Fig. 5 Decimal-to-binary conversion

Q/ Convert the following binary numbers to their decimal equivalents:

(a) 001100 **(b)** 000011 **(c)** 011100 **(d)** 111100

(e) 101010 **(f)** 111111 **(g)** 100001 **(h)** 111000

Solution:

(a) 001100 = 12 **(b)** 000011= 3 **(c)** 011100 = 28 **(d)** 111100 = 60
(e) 101010, = 42 **(f)** 111111= 63 **(g)** 100001 = 33 **(h)** 111000 = 56

Q/ Convert the binary number 11100.011 to decimal

Solution:

11100.011 = 28.375

Q/ Convert the following decimal numbers to their binary equivalents:

(a) 64 **(b)** 100 **(c)** 111 **(d)** 145 **(e)** 255 **(f)** 500 **(g)** 34.75

Solution

(a) 64 = 1000000 **(b)** 100 = 1100100 **(c)** 111= 1101111 **(d)** 145 = 10010001

(e) 255 = 11111111 **(f)** 500 = 111110100 **(g)** 34.75= 100010.11

Octal-to-Decimal Conversion:

Each digit of the octal number represents a value from 0 to 7. To convert an **octal number** to a **decimal number,** each digit of the octal number is multiplied by 8^n, where n is the position of the digit (counting from right to left, starting from 0). Then, sum all these values.

Example (1): convert $(345)_8$ to its decimal equivalent.

$345_8 = (3\times8^2) + (4\times8^1) + (5\times8^0) = 192+32+5=229_{10}$

Example (2): convert (127_8) to its decimal equivalent

$127^8 = (1\times8^2) +(2\times8^1) +(7\times8^0) = 64+16+7=87_{10}$

Decimal-to-Octal Conversion:

A decimal integer can be converted to octal using the same repeated division method used in the decimal-to-binary conversion but with a division factor of 8 instead of 2. An example is shown below:

Example 1: Convert 229_{10} to octal

$229 \div 8 = 28$ remainder 5

$28 \div 8 = 3$ remainder 4

$3 \div 8 = 0$ remainder 3

$229_{10} = 3\ 4\ 5_8 =$

Example 2: Convert 87_{10} to octal

$87 \div 8 = 10$ remainder 7

$10 \div 8 = 1$ remainder 2

$1 \div 8 = 0$ remainder 1

$87_{10} = 1\ 2\ 7_8 =$

Octal-to-Binary Conversion:

To convert octal to binary, replace each octal digit with its 3-bit binary equivalent

Example 1: Convert 345_8 to binary

Solution

1- Break the octal number into individual digits: 3, 4, 5.

2- Convert each digit to binary: 3 = 011
4 = 100
5 = 101

345_8 = 011100101

Binary-to-Octal Conversion:

To convert binary to octal by grouping the binary digits into groups of three bits starting from the rightmost digit or LSB. If the leftmost group has fewer than three digits, pad it with zeros. Replace each 3-bit binary group with its equivalent octal value.

Example 2: Convert 101011_2 to octal

Solution: $101\ 011_2$
5 3

Example 3: convert 11010110_2 to its octal equivalent

Solution: 011 010 110
3 2 6

Hexadecimal Number

The hexadecimal number system has a radix of 16. It is referred to as the base 16 number system. It uses the symbols 0-9, A, B, C. D. E, and F. Where the character A=10, B=11, C=12, D=13, E=14, F=15 in decimal number.

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
4	1110	16	E
15	1111	17	F

Hexadecimal-to-decimal conversion

Example 1: Convert the hexadecimal number 2B6 to **a** decimal number.

Power of 16	16^2	16^1	16^0
Place value	256	16	1

Hexadecimal number 2 B 6

Decimal number 2 ×256 11×16 6×1

$= 512 + 176 + 6 = \mathbf{694_{10}}$

Example 2: Convert the hexadecimal number A3F.C to **a** decimal number.

Power of 16	16^2	16^1	16^0	16^{-1}
Place value	256	16	1	0.0625

Hexadecimal number A 3 F . C

Decimal number 10×256 3×16 15×1 12×0.0625

$= 2560 + 48 + 15 + 0.75$

$= 2623.75_{10}$

Decimal-to-hexadecimal conversion

Example 1: Convert the decimal number 45 to its hexadecimal equivalent.

$45 \div 16 = 2$ remainder 13

$2 \div 16 = 0$ remainder 2

$\mathbf{45_{10} = 2\ D_{16}}$

Example 2: Convert the decimal number 250.25 to its hexadecimal equivalent

$250 \div 16 = 15$ remainder 10

$15 \div 16 = 0$ remainder 15

$0.25 \times 16 = 4.00$

$250.25_{10} = \text{F A . 4}$

Hexadecimal-to-binary conversion

Each hexadecimal number is converted to its 4-bit binary equivalent then combined to form the binary number.

Example 1: Convert the hexadecimal number **3B9** to its binary equivalent

Solution :

3	B	9_{16}		
↓	↓	↓		
0011	1011	1001	=	001110111001

Example 2: Convert the hexadecimal number **47. FE** to its binary equivalent

Solution :

4	7	.	F	E		
↓	↓		↓	↓		
0100	0111		1111	1110	=	01000111.11111110

Binary-to- hexadecimal conversion

Example 1: Convert the binary number 10010.011011 to its hexadecimal equivalent

Solution:

0001	**0010**	**.**	**0110**	**1100**
↓	↓		↓	↓
1	2	.	6	C

The binary number 10010.011011 then equals $12.6C_{16}$