Al-Mustaqbal University

College of Engineering and Technology

Department of Biomedical Engineering

Stage: Second

Electric Circuits II

2024-2025

Lecture (5): Average values

$$I=\sqrt{\frac{\int_0^{2\pi} i^2\,d\theta}{(2\pi-0)}}=\sqrt{\frac{I_m^2}{2}}$$

The square root of this value is

$$I=\frac{I_m}{\sqrt{2}}=0.707I_m$$

Hence, we find that for a symmetrical sinusoidal current

r.m.s. value of current = 0.707 × max. value of current

The r.m.s. value of an alternating current is of considerable importance in practice, because the ammeters and voltmeters record the r.m.s. value of alternating current and voltage respectively. In electrical engineering work, *unless indicated otherwise, the values of the given current and voltage are always the r.m.s. values.*

It should be noted that the average heating effect produced during one cycle is

$$=I^2R=\left(\frac{I_m}{\sqrt{2}}\right)^2R=\frac{1}{2}I_m^2R$$

8.10 AVERAGE VALUE

The average value $\mathbf{I_a}$ of an alternating current is expressed *by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time*. In the case of a symmetrical alternating current (i.e. one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only. But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.

(i) Mid-ordinate Method

With reference to Fig. 8.16, $\boldsymbol{I_{av}}=\frac{i_1+i_2+\cdots+i_n}{n}$

This method may be used both for sinusoidal and non-sinusoidal waves, although it is specially convenient for the latter.

(ii) Analytical Method

The general form of average value is

$$\boldsymbol{I_{av}}=\frac{1}{T}\int_0^T i\,dt, \qquad \text{or } \boldsymbol{I_{av}}=\frac{1}{2\pi}\int_0^{2\pi} i\,d\theta$$

The standard equation of an alternating current is, $i = I_m \sin\theta$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} I_m sin\theta \, d\theta$$

∴ $I_{av} = 0.637\, I_m$

∴ average value of current = 0.637 × maximum value

Note. R.M.S. value is always greater than average value except in the case of a rectangular wave when both are equal.

8.11. Form Factor

It is defined as the ratio, $K_f = \frac{r.m.s.value}{\text{average value}} = \frac{0.707\, I_m}{0.637\, I_m} = 1.11$ (for sinusoidal alternating currents only)

In the case of sinusoidal alternating voltage also, $K_f = 0.707\, E_m/0.637\, E_m = 1.11$

As is clear, the knowledge of form factor will enable the r.m.s. value to be found from the arithmetic mean value and vice-versa.

8.12. Crest or Peak or Amplitude Factor

It is defined as the ratio $K_a = \frac{\text{maximum value}}{r.m.s.value} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$ (for sinusoidal a.c. only)

For sinusoidal alternating voltage also, $K_a = \frac{E_m}{E_m/\sqrt{2}} = 1.414$

Knowledge of this factor is of importance in dielectric insulation testing, because the dielectric stress to which the insulation is subjected, is proportional to the maximum or peak value of the applied voltage. The knowledge is also necessary when measuring iron losses, because the iron loss depends on the value of maximum flux.

Example 8.8: Compute the average and effective values of the square voltage wave shown in Fig. 8.17.

Solution: As seen, for 0 < t < 0.1 i.e. for the time interval 0 to 0.1 second, v = 20 V. Similarly, for 0.1 < t < 0.3, v = 0. Also time-period of the voltage wave is 0.3 second.

∴ $$V_{av} = \frac{1}{T}\int_0^T v dt = \frac{1}{0.3}\int_0^{0.1} 20 dt$$

$$= \frac{1}{0.3}(20 \times 0.1) = 6.6667\, V$$

$$V = \sqrt{\frac{\int_0^T v^2 dt}{T}} = \sqrt{\frac{\int_0^{0.1} 20^2 dt}{0.3}} = \sqrt{\frac{400 \times 0.1}{0.3}} = 11.5\, V$$

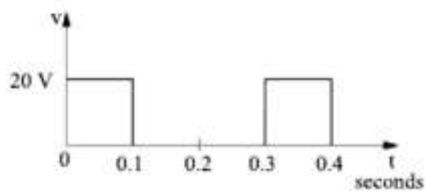

Fig. 8.17

Example 8.9: Calculate the RMS value of the function shown in Fig. 8.18 if it is given that for $0 < t < 0.1$, $y = 10(1 - e^{-100t})$ and $0.1 < t < 0.2$, $y = 10\ e^{-50(t-0.1)}$

Solution:

$$Y^2 = \frac{1}{2}\left\{\int_0^{0.1} y^2 dt + \int_{0.1}^{0.2} y^2 dt\right\}$$

$$= \frac{1}{2}\left\{\int_0^{0.1} 10^2(1 - e^{-100t})^2 dt + \int_{0.1}^{0.2} 10^2\left(e^{-50(t-0.1)}\right)^2 dt\right\}$$

$$= 500 \times 0.095 = 47.5 \therefore Y = \sqrt{47.5} = 6.9$$

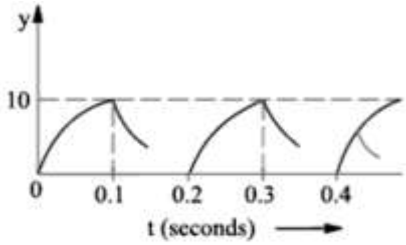

Fig. 8.18

Example 8.10: Determine the r.m.s. and average value of the waveform shown in Fig. 8.19?

Solution: The slope of the curve AB is BC/AC = 10/T.

Next, consider the function y at any time t. It is seen that

y = 10 + (10/T)t

This gives us the equation for the function for one cycle.

$$\boldsymbol{Y_{av}} = \frac{1}{T}\int_0^T \boldsymbol{y\,dt} = \frac{1}{T}\int_0^T \left(\mathbf{10} + \frac{\mathbf{10}t}{T}\right)\boldsymbol{dt} = 15$$

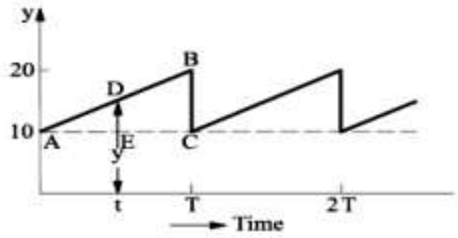

Fig. 8.18

Mean square value $= \frac{1}{T}\int_0^T y^2 dt = \frac{1}{2}\int_0^T \left(10 + \frac{10}{T}t\right)^2 dt = \frac{700}{3}$

or RMS value $= 10\sqrt{7/3} = 15.2$