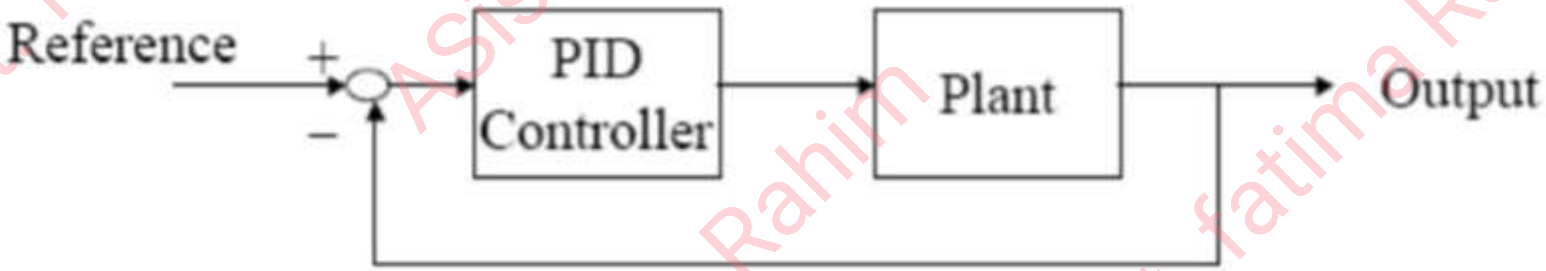
Introduction

- PID Stands for
  - P → Proportional
  - I → Integral
  - D → Derivative



PID Tuning

- The transfer function of PID controller is given as

$$\frac{C_{pid}(s)}{E(s)} = K_p + K_i \frac{1}{s} + K_d\, s$$

integral gain

derivative gain

- It can be simplified as

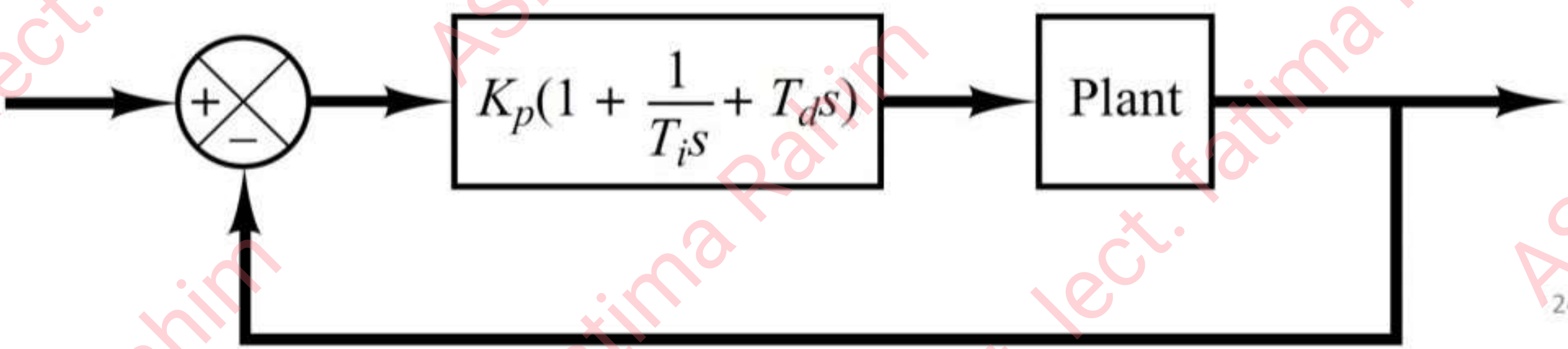
$$\frac{C_{pid}(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s} + T_d\, s\right)$$

- Where

$$T_i = \frac{K_p}{K_i} \qquad T_d = \frac{K_d}{K_p}$$

integral time constant

derivative time constant

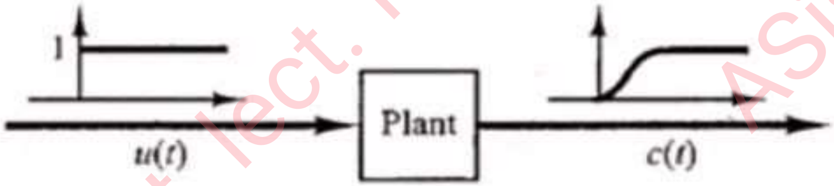


- The process of selecting the controller parameters (Kp, Ti and Td) to meet given performance specifications is known as controller tuning
- Ziegler and Nichols suggested rules for tuning PID controllers experimentally.
- the Ziegler–Nichols tuning rules give an educated guess for the parameter values and provide a starting point for fine tuning, rather than giving the final settings for Kp, Ti and Td in a single shot.

There are two methods called Ziegler–Nichols tuning rules:

- First method (open loop Method)
- Second method (Closed Loop Method)
- **Ziegler–Nichols First method**

1. **Obtain experimentally the response of the plant to *a unit-step input* (open loop).**



2. **For practical: The unit-step response curve may look S-shaped**
3. **For the Math. Model: If the plant involves neither integrator (*s*) nor dominant complex-conjugate poles**

$$G(s) = \frac{K\, e^{-sL}}{Ts+1}$$

L: Delay Time

T: Time Constant

K: System Gain

$$a = \frac{K\,L}{T}$$

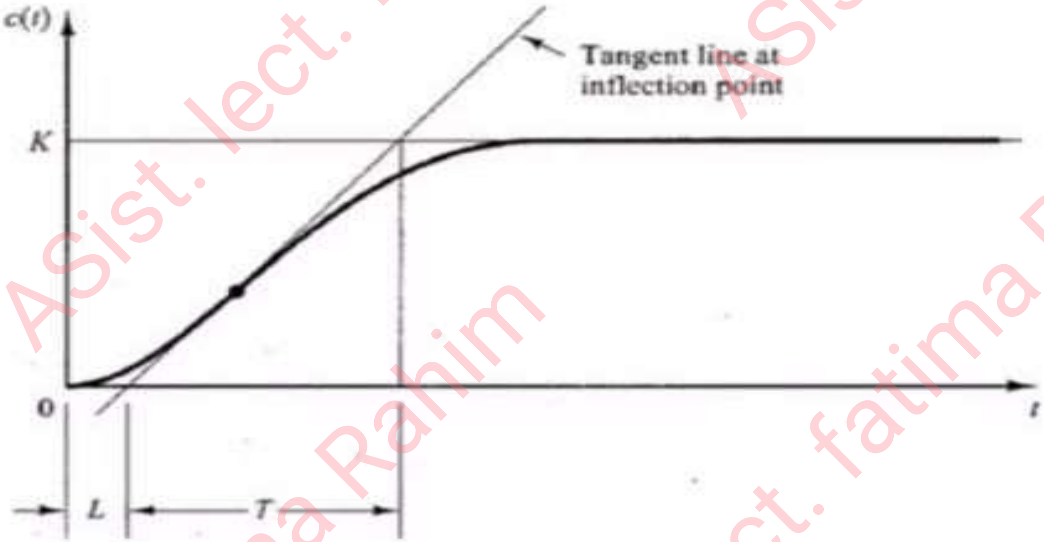


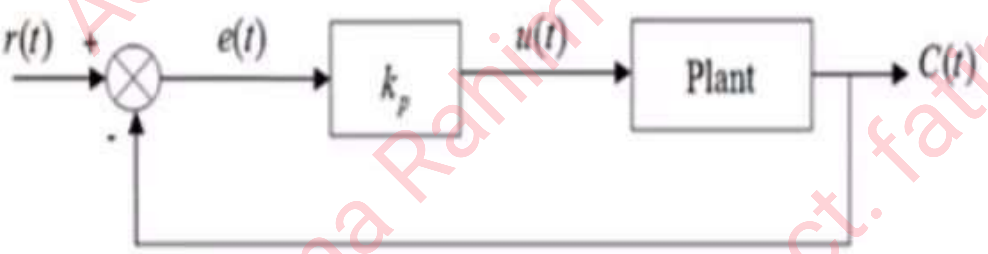
Table-1

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

2. Second method

1. Using the proportional control action only, we first set $T_i = \infty$ ($K_i = 0$) and $T_d = 0$ ($K_d = 0$).
2. For practical: Increase K_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations with period P_{cr}.

- Note: If the output does not exhibit sustained oscillations for whatever value may take K_p , then this method does not apply.



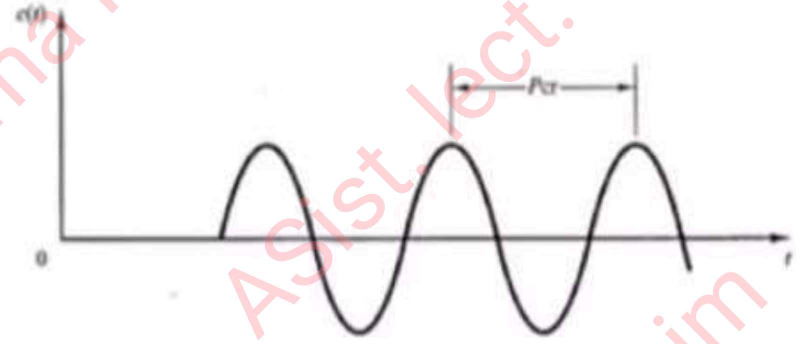
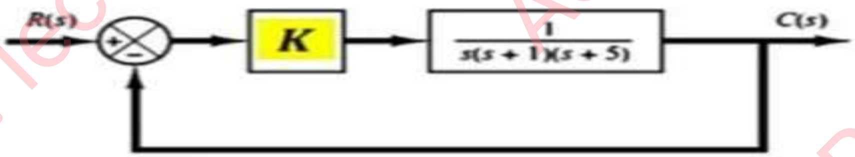


Table-1

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

Example 1



$$G(s) = \frac{K}{s(s+1)(s+2)}$$

$$\frac{C(s)}{R(s)} = \frac{K}{s(s+1)(s+2)+K}$$

$$s^3 + 6s^2 + 5s + K = 0$$

- **Routh table:**

s^3	1	5
s^2	6	K
s^1	$\frac{30-K}{6}$	
s^0	K	

- $\frac{30-K}{6} > 0 \Rightarrow 30 - K > 0 \Rightarrow K < 30$
- $K > 0$

❖ **The range of K for stability** $0 < K < 30$

$$K_{cr} = 30$$

❖ **To obtain** ω_{cr} $\quad 6s^2 + K = 0 \quad 6s^2 + 30 = 0$

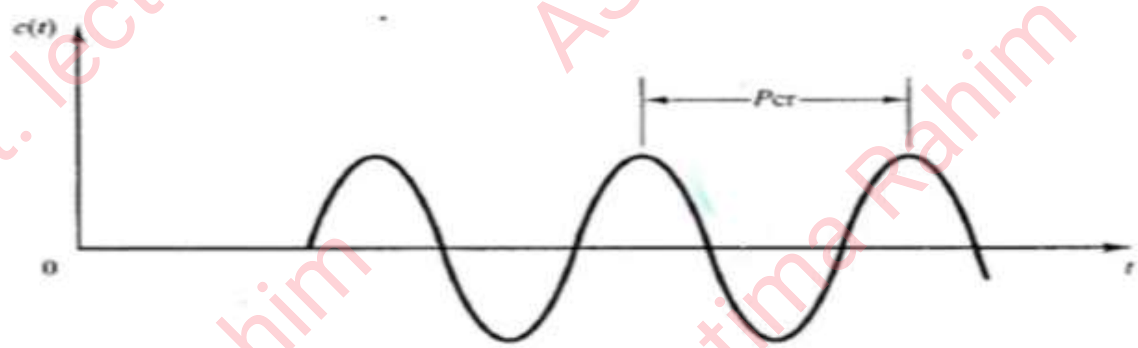
$$6s^2 + 30 = 0$$

$$6s^2 = -30$$

$$s^2 = -5$$

$$s = \mp\sqrt{5}j$$

$$\omega_{cr} = \sqrt{5}$$



$$P_{cr} = \frac{2\pi}{\omega_{cr}} = \frac{2\pi}{\sqrt{5}} = 2.8099$$

PID-Controller

$$K_p = 0.6\,K_{cr} = 18$$
$$T_i = 0.5P_{cr} = 1.4$$
$$T_d = 0.125P_{cr} = 0.35 = 6.3$$

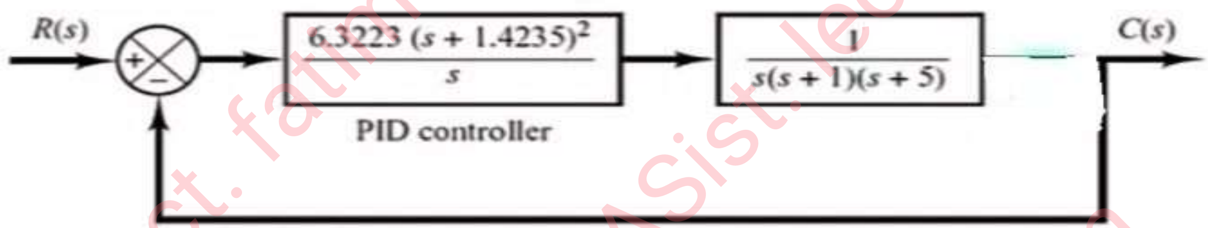
Type of Controller	k_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

PI-Controller

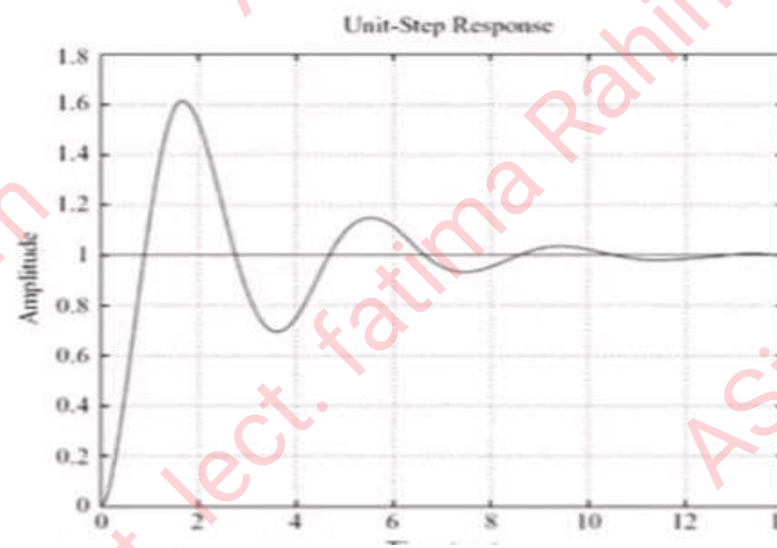
$$K_p = 0.45\,K_{cr}$$
$$T_i = \frac{1}{1.2}P_{cr}$$

P-Controller

$$K_p = 0.5\,K_{cr}$$

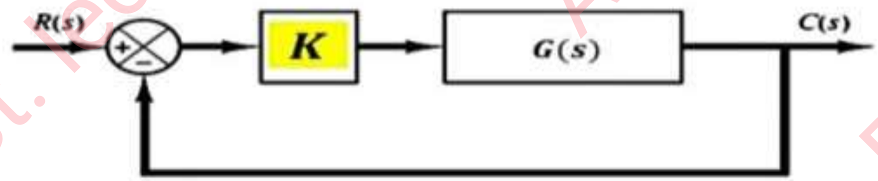


$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$
$$= 18\left(1 + \frac{1}{1.405s} + 0.35124s\right)$$
$$= \frac{6.3223(s + 1.4235)^2}{s}$$
$$\frac{C(s)}{R(s)} = \frac{6.3223s^2 + 18s + 12.811}{s^4 + 6s^3 + 11.3223s^2 + 18s + 12.811}$$



Example 2

$$G(s) = \frac{Ks(s+2)}{(s^2-4s+8)(s+3)}$$



$$\frac{C(s)}{R(s)} = \frac{Ks(s+2)}{(s^2-4s+8)(s+3)+Ks(s+2)}$$

$$= \frac{Ks(s+2)}{s^3+(K-1)s^2+(2K-4)s+24}$$

$$s^3+(K-1)s^2+(2K-4)s+24 = 0$$

- Routh table:

s^3	1	$5K-4$
s^2	$K-1$	24
s^1	$\frac{2K^2-6K-20}{K-1}$	
s^0	24	

- $K-1>0 \quad K>1$
- $\frac{2K^2-6K-20}{K-1} > 0$

$$K^2-3K-10>0$$

$$(K+2)(K-5)>0$$

$$K>5 \qquad K>-2$$

❖ The range of K for stability $K>5$

$$K_{cr} = 5$$

❖ To obtain ω_{cr} $\quad (K-1)s^2+24=0 \quad 4s^2+24=0$

$$K_{cr} = 5 \qquad P_{cr} = 2.567$$

Type of Controller	k_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

PID-Controller

$$K_p = 0.6\,K_{cr} = 3$$
$$T_i = 0.5P_{cr} = 1.2825$$
$$T_d = 0.125P_{cr} = 0.35 = 0.3206$$

PI-Controller

$$K_p = 0.45\,K_{cr}$$
$$T_i = \frac{1}{1.2}P_{cr}$$

P-Controller

$$K_p = 0.5\,K_{cr}$$

Example 3

PID Controller Transfer Function:
Determine the PID parameters K_p, T_i, and T_d according to the formulas shown in the table of Ziegler & Nichols.

Remember: $L = 0.2$ sec and $T = 0.8$ sec.

From the Ziegler & Nichols table, we have for the PID controller:

$$K_p = 1.2\frac{T}{L} = 1.2 \cdot \frac{0.8}{0.2} = 4.8$$
$$T_i = 2L = 2 \cdot 0.2 = 0.4 \text{ sec}$$
$$T_d = 0.5L = 0.5 \cdot 0.2 = 0.1 \text{ sec}$$

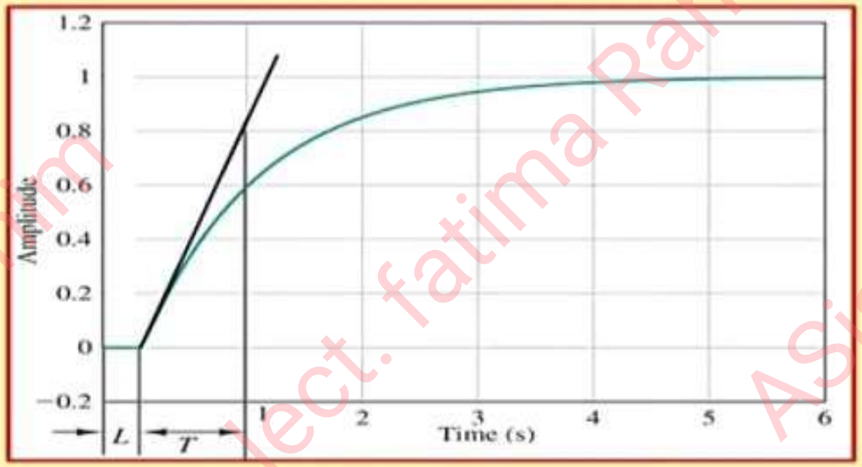
The transfer function for the PID controller becomes:

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$G_c(s) = 4.8\left(1 + \frac{1}{0.4s} + 0.1s\right)$$

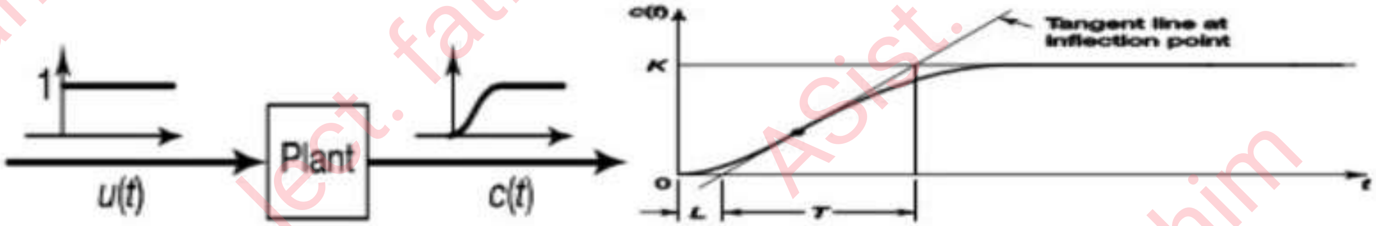
$$\boxed{G_c(s) = 0.48\frac{(s+5)^2}{s}}$$

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$



Example 4

The **Ziegler-Nichols open-loop tuning method** allows you to use the delay time L and the time constant to calculate the gain, integral time and derivative time.



Based on this method, we have the following transfer function of a PI controller:

$$G(s) = 100\left(1 + \frac{2}{0.5\,s}\right)$$

1) Use the table below to find the values of the delay time and time constant.

Table 10–1 Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

2. Use the same table to find the transfer function for a P, PI and PID controller:

$$G_{PID}(s) = \frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Sol. Given

Type of controller	K_P	T_i	T_d
P	T/L	∞	0
PI	$0.9\,T/L$	$L/0.3$	0
PID	$1.2\,T/L$	$2L$	$0.5L$

Transfer function of a PID controller is:

$$\left[G_{PID}(s) = \frac{U(s)}{E(s)} = K_P\left(1 + \frac{1}{T_i s} + T_d s\right) \right] \quad — (1)$$

Based on this method, we have the following transfer function of a PID controller

$$\left[G(PID) = 25\left(4 + \frac{1}{x \cdot s} + 2s\right) \right] \quad — (2)$$

$$G(PID) = 25 \times 4\left(1 + \frac{1}{4 \times s} + \frac{2}{4}s\right)$$

$$\left[G(PID) = 100\left(1 + \frac{1}{4 \times s} + \frac{1}{2}s\right) \right] \quad — (3)$$

compare equation (1) and (3)

$$\left.\begin{array}{l} K_p = 100 \\ T_i = 4\times \\ T_d = \frac{1}{2} \end{array}\right\} \text{—— (4)}$$

(1) From the table for PID controller

$T_d = 0.5L$

$T_d = \frac{1}{2}$ (from equation (4))

$\frac{1}{2} = 0.5L$

$[L = 1]$ —

$K_p = 1.2\ T/L$

$K_p = 100$ (from equation (4))

$100 = 1.2\ T/L$

$T = \frac{100 \times L}{1.2} \Rightarrow \frac{100 \times 1}{1.2}$

$[T = 83.33]$

(2) for P controller

$K_p = T/L$ (from table)

$K_p = 83.33/1$

$[K_p = 83.33]$

$[T_i = \infty]$

$[T_d = 0]$

put all values in equation (1)

$G_p(s) = 83.33\left(1 + \frac{1}{\infty \times s} + 0 \times s\right)$

$[G_p(s) = 83.33]$ Transfer function of the P controller.

for PI controller

$K_p = 0.9T/L$

$K_p = 0.9 \times 83.33/L$

$[K_p = 74.997]$

$[T_i = L/0.3 \Rightarrow \frac{1}{0.3} \Rightarrow 3.33]$

$[T_d = 0]$

put all values in equation ①

$$G_{PI}(s) = 74.997\left(1 + \frac{1}{3.33s} + 0 \times s\right)$$

$$\left[G_{PI}(s) = 74.997\left(1 + \frac{1}{3.33s}\right)\right] \text{—}$$

↳ Transfer function for PI controller.

(3) from equation ④

$T_i = 4X$

from table value of T_i for PID controller is

$T_i = 2L$

from the above two equations

$$4X = 2L$$

$$4X = 2 \times 1$$

$$[X = 0.5] \text{—}$$

Example 5

For the control system shown in fig. (1), find the transfer function of the PID controller $G_c(s)$ by using a Ziegler-Nichols tuning rule.

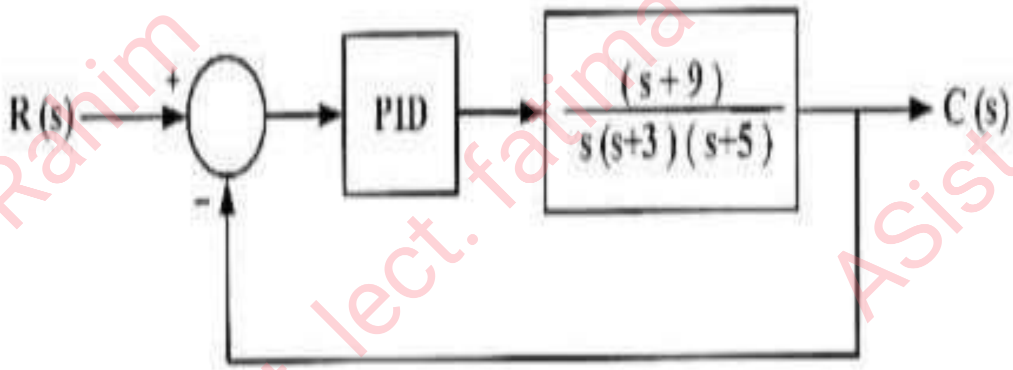


Fig. (1)

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$$G_c(s) = K_P\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$T_d = 0 \quad T_i = \infty \quad \simeq \frac{1}{\infty} = 0$$

$$G_c(s) = K_P\left(1 + \frac{1}{\infty s} + 0 s\right)$$

$$G_c = K_P$$

$$\frac{C(s)}{R(s)} = \frac{G_c(s)\cdot G(s)}{1 + G_c\cdot G_s}$$

$$1 + G_c\cdot G_s$$

$$1 + G_c\cdot G_s$$

$$1 + K_P\cdot\frac{(s+9)}{s(s+3)(s+5)} = 0$$

$$\frac{s(s+3)(s+5) + K_P(s+9)}{s(s+3)(s+5)} = 0$$

$$s(s+3)(s+5) + K_P(s+9) = 0$$

$$s^3 + 8s^2 + 15s + K_P s + 9K_P = 0$$

$$s^3 + 8s\,(15 + K_P)s + 9K_P$$

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Routh

$s^3 \quad 1 \quad 15 + K_p$

$s^2 \quad 8 \quad 9K_p$

$s^1 \quad \frac{8(15+K_p) - 9K_p}{8}$

$s^0 \quad 9K_p$

$9K_p > 0 \Rightarrow K_p > 0$

$\frac{120 + 8K_p - 9K_p}{8} > 0$

$120 - K_p > 0$

$K_p = 120$

$\because K_p = K_{cr} = 120$

$s^3 + 8s^2 + (15+120)s + 9(120) = 0$

$s^3 + 8s^2 + 135s + 1080 = 0$

$\because s = j\omega$

$(j\omega)^3 + 8(j\omega)^2 + 135(j\omega) + 1080 = 0$

$j^3\omega^3 + 8j^2\omega^2 + 135 j\omega + 1080 = 0$

$(-8\omega^2 + 1080) + j(-\omega^3 + 135\omega) = 0$

$-8\omega^2 = 1080 \quad \div -8$

$\omega^2 = 135 \Rightarrow \omega = \sqrt{135} = 11.6$

$\therefore P_{cr} = \frac{2\pi}{\omega} = \frac{2\pi}{11.6} = \frac{2 \times 3.14}{11.6} = 0.54$

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$$\therefore K_p = 0.6\,K_{cr} = 0.6 * 120 = 72$$

$$\therefore T_i = 0.5\,P_{cr} = 0.5 * 0.54 = 0.27$$

$$\therefore T_d = 0.125\,P_{cr} = 0.125 * 0.54 = 0.07$$

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + \frac{1}{T_d s}\right)$$

$$= 72\left(1 + \frac{1}{0.27 s} + 0.07 s\right)$$

التاريخ: الموضوع:

Example :- A unit step has resulted in following open loop response. Determine turning parameters of PID controller and unite transfer function of PID controller ?

Solution :-

from step response

$K=2$, $D=10$, $T=80$

for PID controller

$$K_c = \frac{1.2\,T}{K D} = \frac{1.2(80)}{2 \times 10} = 4.8$$

$$T_I = 2D = 20$$

$$T_D = 0.5\,D = 5$$

$$G(s) = K_c\left[1 + \frac{1}{T_I s} + T_D s\right]$$

$$= 4.8\left[1 + \frac{1}{20s} + 5s\right]$$

for K_P, K_I, K_D form

$$K_P = K_c = 4.8$$

$$K_I = \frac{K_c}{K_I} = \frac{4.8}{20} = 0.24$$

$$K_D = K_c T_D = 24 \Rightarrow G(s) = 4.8 + \frac{0.24}{s} + 24s$$