Al-Mustaqbal University

College of Engineering and Technology

Department of Biomedical Engineering

Stage: Second

Electric Circuits II

2024-2025

Lecture (6): (R.M.S.) VALUE

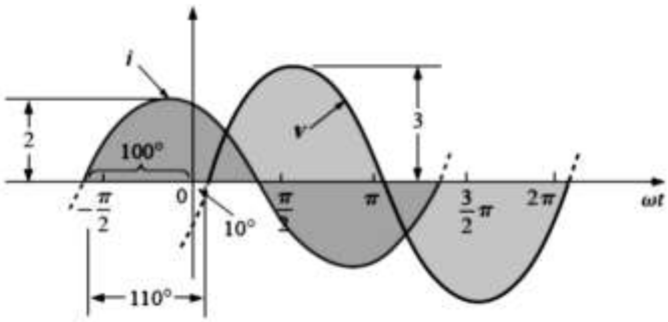

FIG. 8.13 Example 8.7; i leads v by 110°.

d. See Fig. 8.14.

Note

-sin(ωt + 30°) = sin(ωt + 30° - 180°)

= sin(ωt - 150°)

v leads i by 160°, or i lags v by 160°.

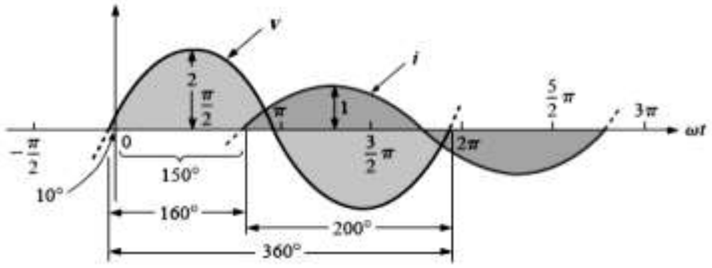

FIG. 8.14 Example 8.7; v leads i by 160°.

e. See Fig. 8.15.

i = -2 cos(ωt - 60°) = 2 cos(ωt - 60° - 180°)

= 2 cos(ωt - 240°)

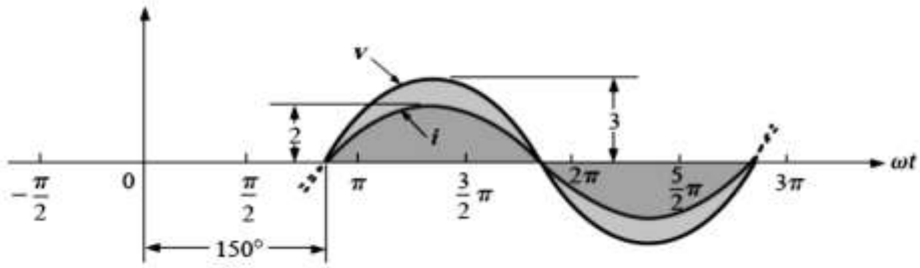

FIG. 8.15 Example 8.7; v and i are in phase.

8.7. EFFECTIVE ROOT-MEAN-SQUARE (R.M.S.) VALUE

The r.m.s. value of an alternating current is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time. It is also known as the effective or virtual value of the alternating current, the former term being used more extensively. For computing the r.m.s. value of symmetrical sinusoidal alternating currents, either mid-ordinate method or analytical method may be used, although for symmetrical but non- sinusoidal waves, the mid- ordinate method would be found more convenient.

8.8. Mid-ordinate Method

In Fig. 8.16 are shown the positive half cycles for both symmetrical sinusoidal and non-sinusoidal alternating currents. Divide time base 't' into **n** equal intervals of time each of duration t/n seconds. Let the average values of instantaneous currents during these intervals be respectively i_1, i_2, i_3,.... i_n (i.e. mid-ordinates in Fig. 8.16).

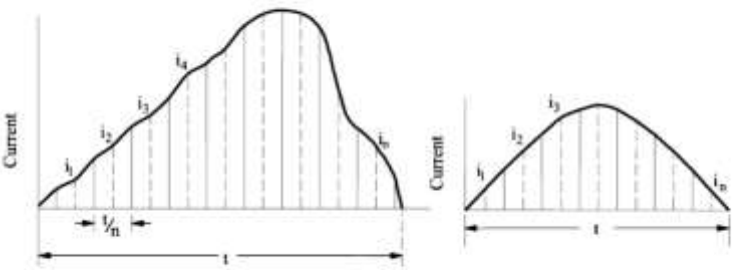

Fig. 8.16

The r.m.s. value of alternating current is

$$I^2 = \frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}$$

$$I = \sqrt{\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}}$$ = square root of the mean of the squares of the instantaneous currents

Similarly, the r.m.s. value of alternating voltage is given by the expression

$$V = \sqrt{\frac{v_1^2 + v_2^2 + \cdots + v_n^2}{n}}$$

8.9. Analytical Method

The general form of r.m.s. value is

$$I^2 = \frac{1}{T}\int_0^T i^2 dt \quad \Rightarrow \quad I = \sqrt{\frac{\int_0^T i^2 dt}{T}} \quad or\ I = \sqrt{\frac{\int_0^{2\pi} i^2 d\theta}{(2\pi - 0)}}$$

The standard form of a sinusoidal alternating current is $i = I_m \sin \omega t = I_m \sin \theta$. The mean of the squares of the instantaneous values of current over one complete cycle is (even the value over half a cycle will do).

$$I=\sqrt{\frac{\int_0^{2\pi} i^2\,d\theta}{(2\pi-0)}}=\sqrt{\frac{I_m^2}{2}}$$

The square root of this value is

$$I=\frac{I_m}{\sqrt{2}}=0.707I_m$$

Hence, we find that for a symmetrical sinusoidal current

r.m.s. value of current = 0.707 × max. value of current

The r.m.s. value of an alternating current is of considerable importance in practice, because the ammeters and voltmeters record the r.m.s. value of alternating current and voltage respectively. In electrical engineering work, *unless indicated otherwise, the values of the given current and voltage are always the r.m.s. values.*

It should be noted that the average heating effect produced during one cycle is

$$=I^2R=\left(\frac{I_m}{\sqrt{2}}\right)^2R=\frac{1}{2}I_m^2R$$

8.10 AVERAGE VALUE

The average value $\mathbf{I_a}$ of an alternating current is expressed *by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time*. In the case of a symmetrical alternating current (i.e. one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only. But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.

(i) Mid-ordinate Method

With reference to Fig. 8.16, $\boldsymbol{I_{av}}=\frac{i_1+i_2+\cdots+i_n}{n}$

This method may be used both for sinusoidal and non-sinusoidal waves, although it is specially convenient for the latter.

(ii) Analytical Method

The general form of average value is

$$\boldsymbol{I_{av}}=\frac{1}{T}\int_0^T i\,dt, \qquad or\ \boldsymbol{I_{av}}=\frac{1}{2\pi}\int_0^{2\pi} i\,d\theta$$

The standard equation of an alternating current is, $i = I_m \sin\theta$