Lec2_C2 Spatial Filtering _Part1

refers to image operators that change the gray value at any pixel (x,y) depending on the pixel values in a square neighborhood centered at (x,y) using a fixed integer matrix of the same size. The integer matrix is called a *filter*, *mask*, *kernel* or a *window*.

The mechanism of spatial filtering, shown below, consists simply of moving the filter mask from pixel to pixel in an image. At each pixel (x,y), the response of the filter at that pixel is calculated using a predefined relationship (linear or nonlinear).

The main difference between linear and non linear spatial filter

Linear filtering is the filtering method in which the value of output pixel is linear combinations of the neighbouring input pixels. it can be done with convolution. For examples, mean/average filters or Gaussian filtering.

A **non-linear filtering** is one that cannot be done with convolution or Fourier multiplication. A sliding median filter is a simple example of a non-linear filter.

In Linear Spatial filtering the spatial mask in convolution process is used. In Non-Linear, the ordering mechanism is used to produce output pixel ie to replace the centre pixel

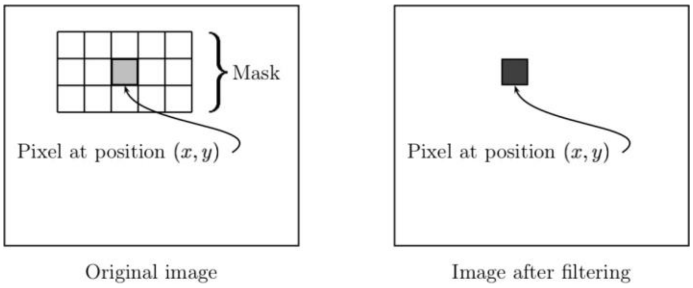


Figure (1), using spatial mask on an image

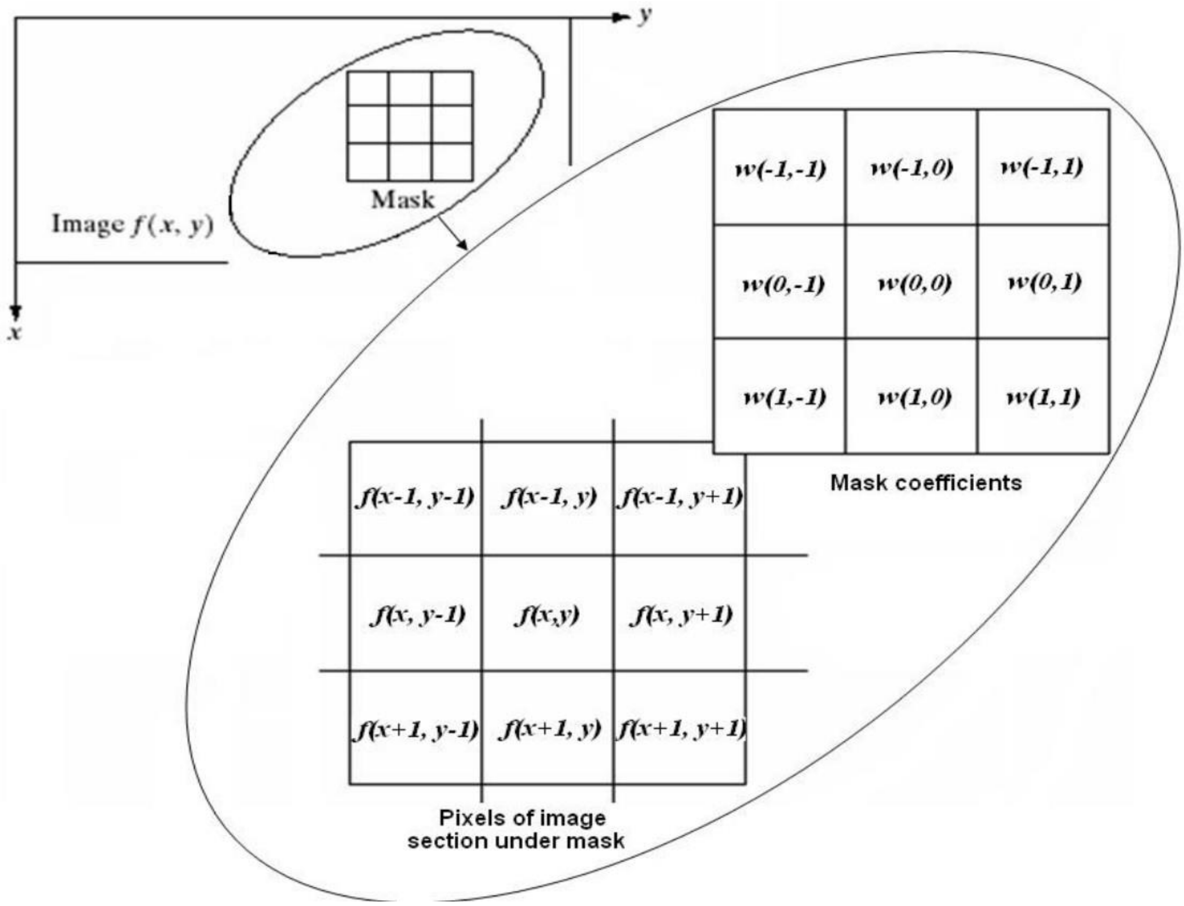


Figure (2), spatial filtering of the image

Note:

The size of mask must be odd (i.e. 3×3, 5×5, etc.) to ensure it has a center. The smallest meaningful size is 3×3.

Linear Spatial Filtering (Convolution)

The process consists of moving the filter mask from pixel to pixel in an image. At each pixel (x,y), the response is given by a sum of products of the filter coefficients and the corresponding image pixels in the area spanned by the filter mask.

For the 3×3 mask shown in the previous figure, the result (or response), R, of linear filtering is:

$$R = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \cdots$$
$$+w(0,0)f(x,y) + \cdots + w(1,0)f(x+1,y) + w(1,1)f(x+1,y+1)$$

In general, linear filtering of an image f of size $M\times N$ with a filter mask of size $m\times n$ is given by the expression:

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)f(x+s,y+t)$$

where a = (m - 1)/2 and b = (n - 1)/2. To generate a complete filtered image this equation must be applied for x = 0,1, 2,..., M-1 and y = 0,1, 2,..., N-1.

Example:

Use the following 3×3mask to perform the convolution process on the shaded pixels in the 5×5 image below. Write the filtered image.

0	1/6	0
1/6	1/3	1/6
0	1/6	0

3×3 mask

30	40	50	70	90
40	50	80	60	100
35	255	70	0	120
30	45	80	100	130
40	50	90	125	140

5×5 image

Solution:

$$0 \times 30 + \frac{1}{6} \times 40 + 0 \times 50 + \frac{1}{6} \times 40 + \frac{1}{3} \times 50 + \frac{1}{6} \times 80 + 0 \times 35 + \frac{1}{6} \times 255 + 0 \times 70 = 85$$

$$0 \times 40 + \frac{1}{6} \times 50 + 0 \times 70 + \frac{1}{6} \times 50 + \frac{1}{3} \times 80 + \frac{1}{6} \times 60 + 0 \times 255 + \frac{1}{6} \times 70 + 0 \times 0 = 65$$

$$0 \times 50 + \frac{1}{6} \times 70 + 0 \times 90 + \frac{1}{6} \times 80 + \frac{1}{3} \times 60 + \frac{1}{6} \times 100 + 0 \times 70 + \frac{1}{6} \times 0 + 0 \times 120 =$$

$$0 \times 40 + \frac{1}{6} \times 50 + 0 \times 80 + \frac{1}{6} \times 35 + \frac{1}{3} \times 255 + \frac{1}{6} \times 70 + 0 \times 30 + \frac{1}{6} \times 45 + 0 \times 80 = 118$$

and so on …

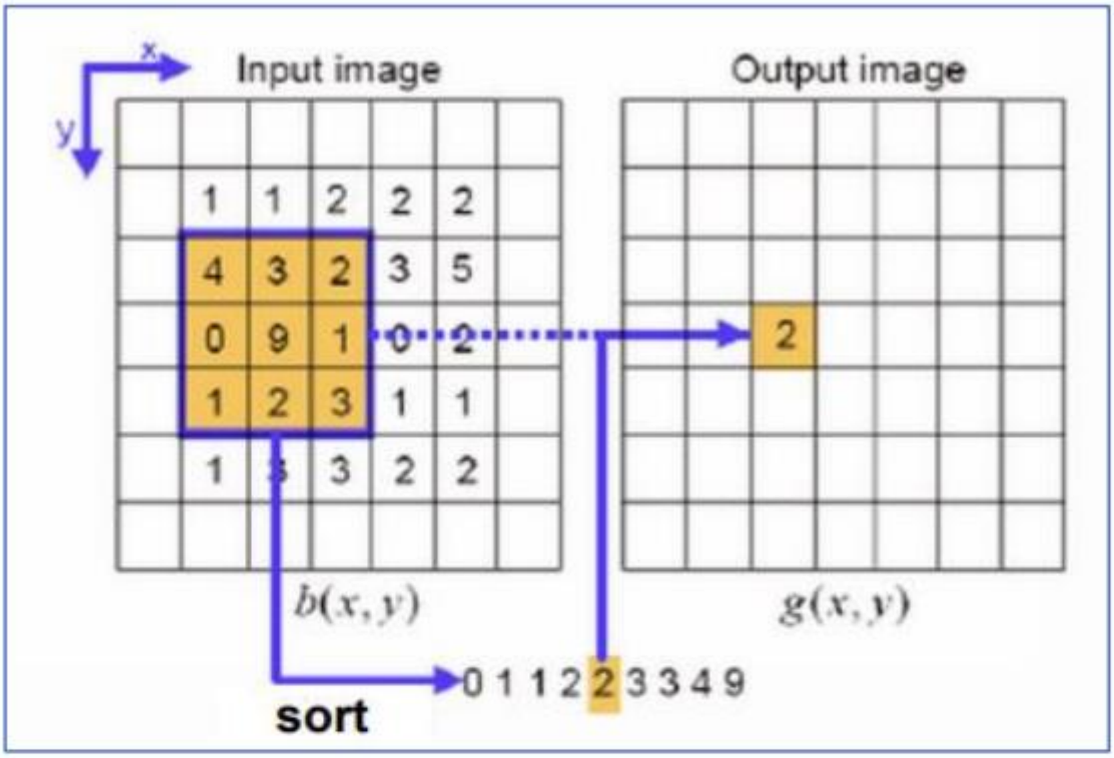
Filtered image =

30	40	50	70	90
40	**85**	**65**	**61**	100
35	**118**	**92**	**58**	120
30	**84**	**77**	**89**	130
40	50	90	125	140

Nonlinear Spatial Filtering

The operation also consists of moving the filter mask from pixel to pixel in an image. The filtering operation is based conditionally on the values of the pixels in the neighborhood, and they do not explicitly use coefficients in the sum-of-products manner.

For example, noise reduction can be achieved effectively with a nonlinear filter whose basic function is to compute the median gray-level value in the neighborhood in which the filter is located. Computation of the median is a nonlinear operation.



Spatial Filters

Spatial filters can be classified by effect into:

1. **Smoothing Spatial Filters:** also called lowpass filters. They include:
    1.1 Averaging linear filters
    1.2 Order-statistics nonlinear filters.
2. **Sharpening Spatial Filters:** also called highpass filters. For example, the Laplacian linear filter.

Smoothing Spatial Filters

are used for blurring and for noise reduction. Blurring is used in preprocessing steps to:

- remove small details from an image prior to (large) object extraction
- bridge small gaps in lines or curves.

Noise reduction can be accomplished by blurring with a linear filter and also by nonlinear filtering.

Averaging linear filters

The response of averaging filter is simply the average of the pixels contained in the neighborhood of the filter mask.

The output of averaging filters is a smoothed image with reduced "sharp" transitions in gray levels.

Noise and edges consist of sharp transitions in gray levels. Thus smoothing filters are used for noise reduction; however, they have the undesirable side effect that they blur edges.

The figure below shows two 3×3 averaging filters.

$$\frac{1}{9}\times\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array}\qquad\qquad \frac{1}{16}\times\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline\end{array}$$

Standard average filter   Weighted average filter

Note:

Weighted average filter has different coefficients to give more importance (weight) to some pixels at the expense of others. The idea behind that is to reduce blurring in the smoothing process.

Averaging linear filtering of an image f of size $M\times N$ with a filter mask of size $m\times n$ is given by the expression:

To generate a complete filtered image this equation must be applied for $x = 0,1, 2,..., M\text{-}1$ and $y = 0,1, 2,..., N\text{-}1$.

Figure below shows an example of applying the standard averaging filter.

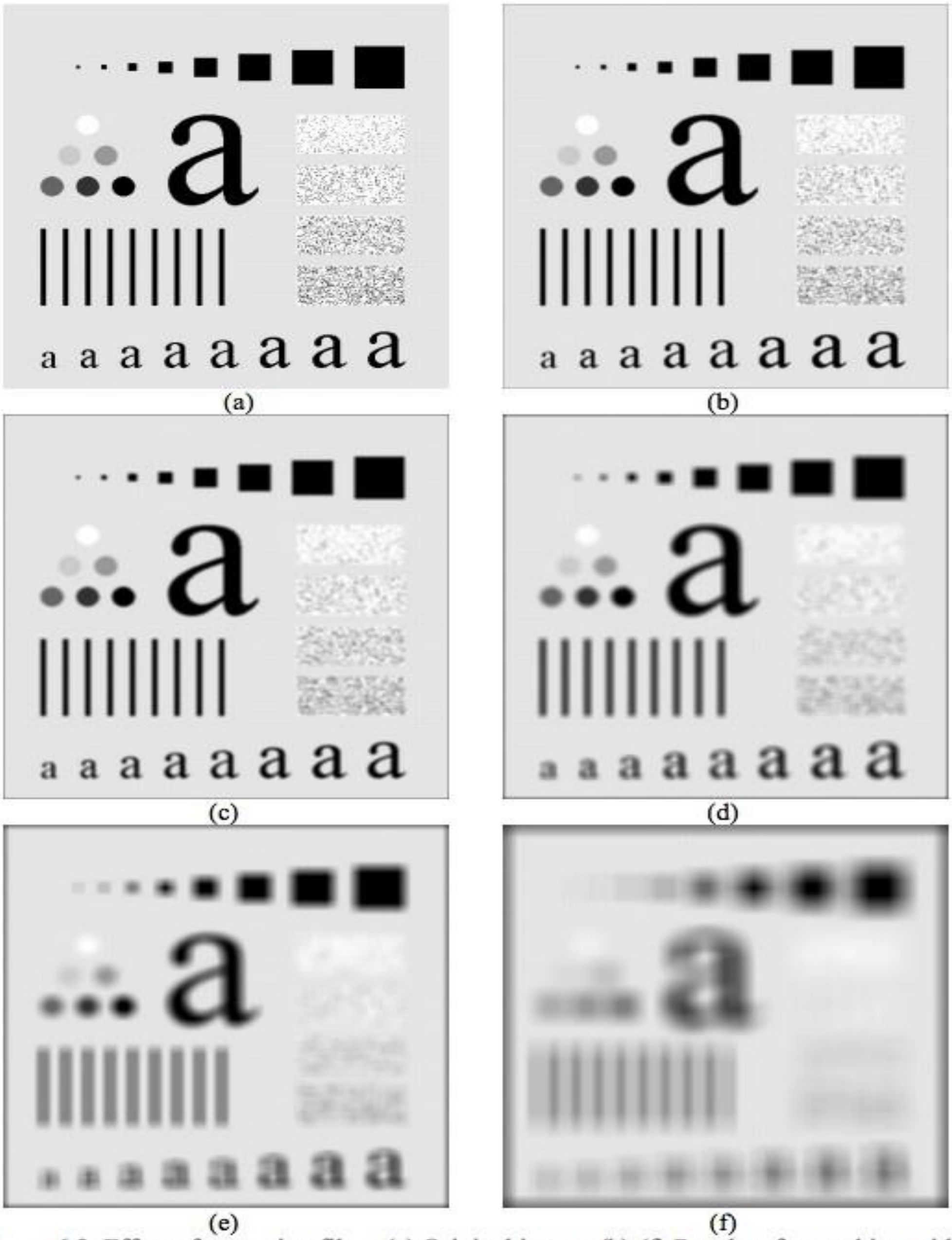


Figure 6.2 Effect of averaging filter. (a) Original image. (b)-(f) Results of smoothing with square averaging filter masks of sizes n = 3,5,9,15, and 35, respectively.

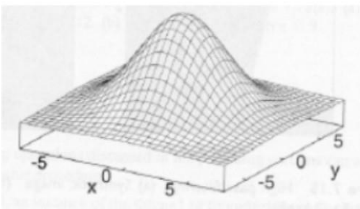
As shown in the figure, the effects of averaging linear filter are:

1. Blurring which is increased whenever the mask size increases.
2. Blending (removing) small objects with the background. The size of the mask establishes the relative size of the blended objects.
3. Black border because of padding the borders of the original image.
4. Reduced image quality.

Smoothing filters: Gaussian

- The weights are Gaussian samples:

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2+y^2}{2\sigma^2}}$$



7 × 7 Gaussian mask

1	1	2	2	2	1	1
1	2	2	4	2	2	1
2	2	4	8	4	2	2
2	4	8	16	8	4	2
2	2	4	8	4	2	2
1	2	2	4	2	2	1
1	1	2	2	2	1	1

$\sigma = 1.4$

mask size is a function of **σ**:

height = *width* = 5σ (subtends 98.76% of the area)

Smoothing filters: Gaussian (cont'd)

- **σ** controls the amount of smoothing
- As **σ** increases, more samples must be obtained to represent the Gaussian function accurately.

$\sigma = 3$

15 × 15 Gaussian mask

2	2	3	4	5	5	6	6	6	5	5	4	3	2	2
2	3	4	5	7	7	8	8	8	7	7	5	4	3	2
3	4	6	7	9	10	10	11	10	10	9	7	6	4	3
4	5	7	9	10	12	13	13	13	12	10	9	7	5	4
5	7	9	11	13	14	15	16	15	14	13	11	9	7	5
5	7	10	12	14	16	17	18	17	16	14	12	10	7	5
6	8	10	13	15	17	19	19	19	17	15	13	10	8	6
6	8	11	13	16	18	19	20	19	18	16	13	11	8	6
6	8	10	13	15	17	19	19	19	17	15	13	10	8	6
5	7	10	12	14	16	17	18	17	16	14	12	10	7	5
5	7	9	11	13	14	15	16	15	14	13	11	9	7	5
4	5	7	9	10	12	13	13	13	12	10	9	7	5	4
3	4	6	7	9	10	10	11	10	10	9	7	6	4	3
2	3	4	5	7	7	8	8	8	7	7	5	4	3	2
2	2	3	4	5	5	6	6	6	5	5	4	3	2	2