example 2

For the system with open loop transfer function given below. sketch the root Locus and predict the stability.

$$G(s)\,H(s) = \frac{K}{s(s^2+2s+2)}$$

step $\boxed{1}$ poles and zeros

$$G(s)\,H(s) = 0$$

$$\frac{K}{s(s^2+2s+2)} = 0$$

Poles/

$$\left.\begin{aligned} & s = 0 \\ \text{or } & s^2+2s+2 = 0 \\ & s = -1+J \\ & s = -1-J \end{aligned}\right\} \quad \text{no. of poles} = 3$$

Zeros/ No. of zeros = 0

step $\boxed{2}$ No. of Asymptotes

= No. of poles - No. of zeros

= 3 - 0 = 3

step $\boxed{3}$ Angle of Asymptotes

$$\theta = \frac{2x+1}{\text{No. of poles} - \text{No. of Zeros}} * 180^\circ$$

$x = 0, 1, 2$

$$\theta_1 = \frac{1}{3} * 180^\circ = 60^\circ$$

$$\theta_2 = \frac{3}{3} * 180^\circ = 180^\circ$$

$$\theta_3 = \frac{5}{3} * 180^\circ = 300^\circ$$

step $\boxed{4}$ Angle of Asymptote

$$= \frac{\sum \text{real part of poles} - \sum \text{real part of zeros}}{\text{No. of poles} - \text{No. of Zeros}}$$

$$= \frac{(-1-1+0) - 0}{3-0}$$

$$= \frac{-2}{3} = -0.67$$

Step $\boxed{5}$ Break away point

From charastestic eq.

$$1 + G(s)\,H(s) = 0$$

$$1 + \frac{K}{s(s^2+2s+2)} = 0$$

$$\frac{s(s^2+2s+2)+K}{s(s^2+2s+2)} = 0$$

$$s^3+2s^2+2s+K = 0$$

$$K = -s^3-2s^2-2s$$

$$\frac{dK}{ds} = -3s^2-4s-2 = 0\ )$$

$$3s^2-4s-2 = 0$$

$$s = -0.67+0.47\,j$$

$$s = -0.67-0.47\,j$$

since the point is complex conjegate, its not on root locas plots step $\boxed{6}$ intersection with imaginary axis

From $s^3 + 2s^2 + 2s + K = 0$

s^3	1	2
s^2	2	K
s	$\frac{4-K}{2}$	0
s^0	K	

$$\frac{4-K}{2} = 0 \Rightarrow K = 4$$

Auxilary eq.

$$2s^2 + K = 0$$

$$s^2 = \frac{-4}{2} = -2$$

$$s = \pm\sqrt{-2}$$

$$= \pm\sqrt{2}\,J$$

$$= \pm 1.414\,J$$

step $\boxed{7}$ Angle of Departure.

$$\theta_d = 180^\circ - \left(\sum \theta_p + \sum \theta_z\right)$$

$\theta_d = 180 - (135 + 90)$

$= -45$ for $(-1+j)$

$\theta_d = 45$ for $(-1-j)$ (symetric with reals)