Al-Mustaqbal University

Department of Biomedical Engineering

Third Stage / 1st Course

“Transport Phenomena for BME”

Dr. A.Najah Saud

amir_Najah@uomus.edu.iq

Chapter 6 *Elementary Fluid Dynamics*

General Applications of Bernoulli's Equation

- *Pitot Tube*

- A Pitot tube measures the velocity of gas passing through a tube. It is also generally used to measure wind and aircraft speeds relative to air. It consists of two tubes, one of which has a wide section through which the gas flows at a speed v, and the other tube has a small section at its end. The two tubes are connected to a liquid manometer. The figure below shows a diagram of this device.

$$\frac{P_1}{\rho} + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2$$

V2=0

$$(P_1\text{-}P_2)=\frac{1}{2}\rho v_1^2$$

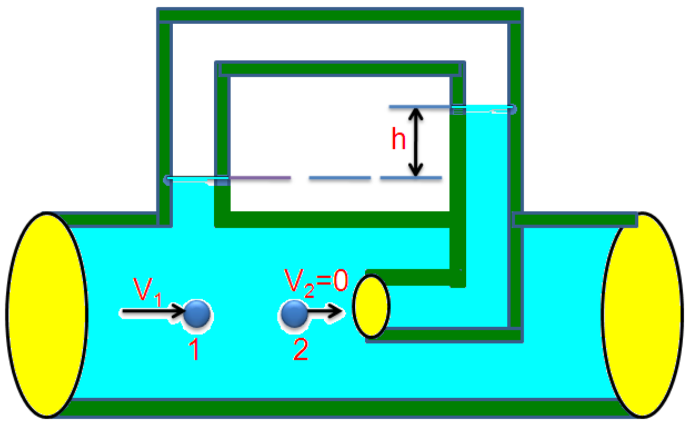
So:-

$$v_1^2 = \frac{2(P_2 - P_1)}{\rho}$$

(p1-p2) is the static and kinetic pressure and equal to

$$(P_2\text{-}P_1)=\rho gh$$

$$v_1 = \sqrt{2gh}$$



General Applications of Bernoulli's Equation

- ***Torricelli's Equation***

The figure shows a vessel containing a liquid flowing from an opening in the vessel and from the surface of the liquid. If we take points 1 and 2,

a- if the tank is open , to calculate the V2

So So

$$P_1 - P_2 = \boldsymbol{\rho} * g * (y_2 - y_1) + (½\, \boldsymbol{\rho}\, (v_2^2 - v_1{}^2)$$

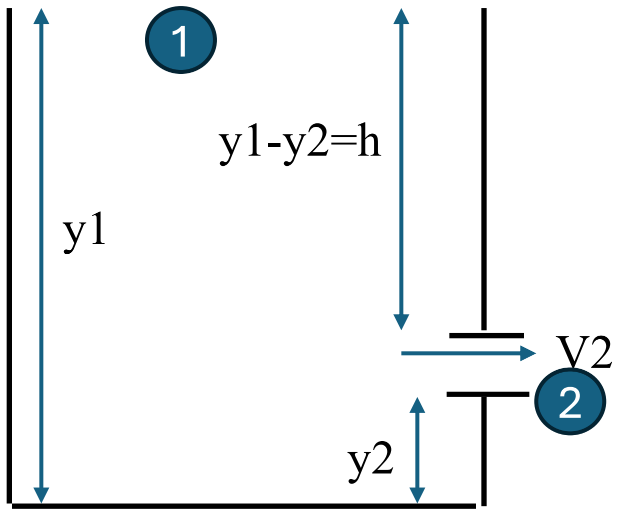
$$½\, \boldsymbol{\rho}\, v_2^2 = \boldsymbol{\rho} * g * h$$

$$v_2 = \sqrt{2}\, g * h$$

Tank open so
$P1 = p_{atm}$, $p2 = p_{atm}$

A1>>A2

A1v1=A2V2
So
As the A increases, the V decreases
so
V1<<V2



A1>>A2

General Applications of Bernoulli's Equation

- ***Torricelli's Equation***

b- if the tank is close , to calculate the V2

So

$P_1\text{-}P_2 = \boldsymbol{\rho}*g*(y_1\text{-}\ y_2) + (½\ \boldsymbol{\rho}\ (v_2^2 - v_1^{\ 2})$

$½\ \boldsymbol{\rho}\ v_2^2 = (p_o - p_{atm}) + \boldsymbol{\rho}*g*h$

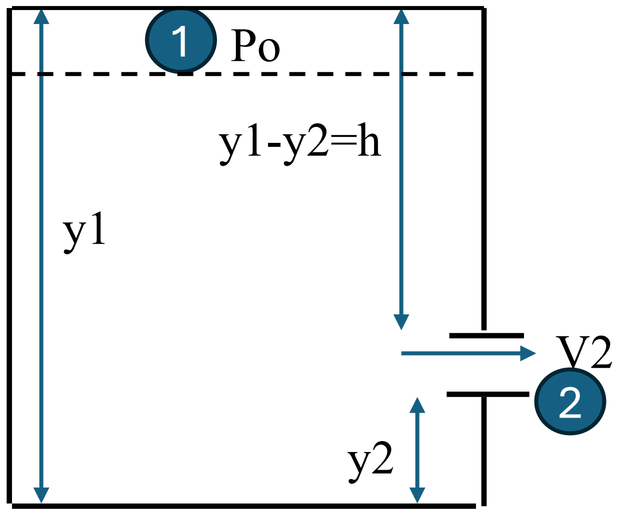
$$v_2^2 = \frac{2(P_o - P_{atm})}{\rho} + 2gh$$

$$V_2 = \sqrt{\frac{2(P_o - P_{atm})}{\rho} + 2gh}$$

Tank open so
$P1 = p_o$, $p2 = p_{atm}$

A1>>A2

A1v1=A2V2
So
As the A increases, the V decreases
so
V1<<V2

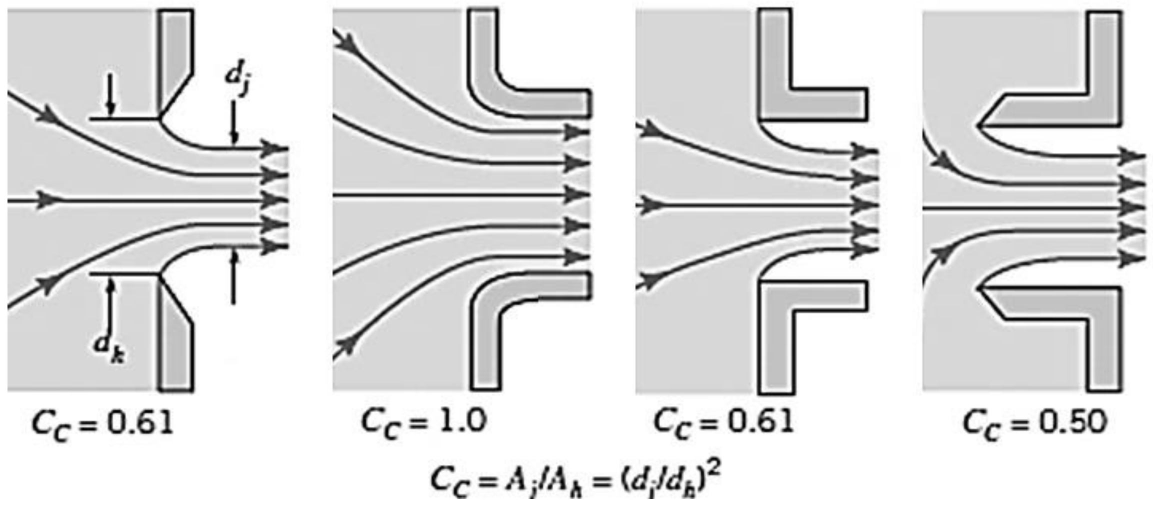


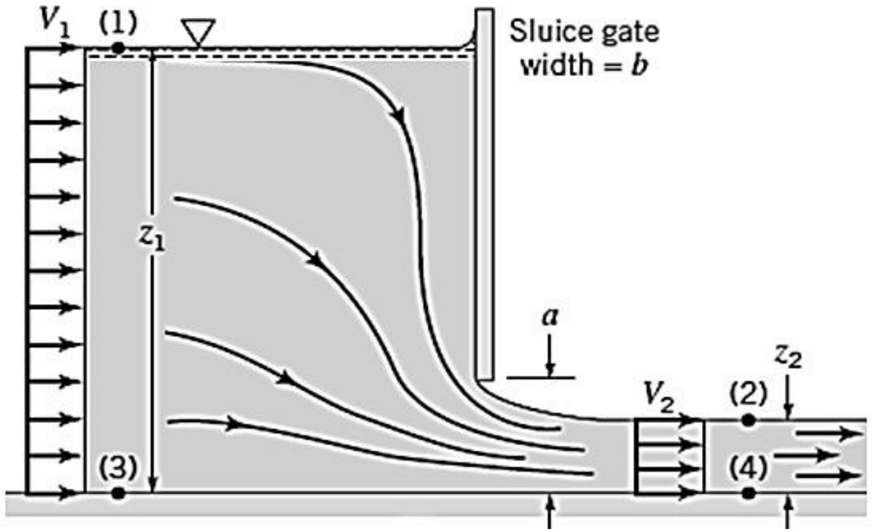
A1>>A2

General Applications of Bernoulli's Equation

Torricelli's Equation

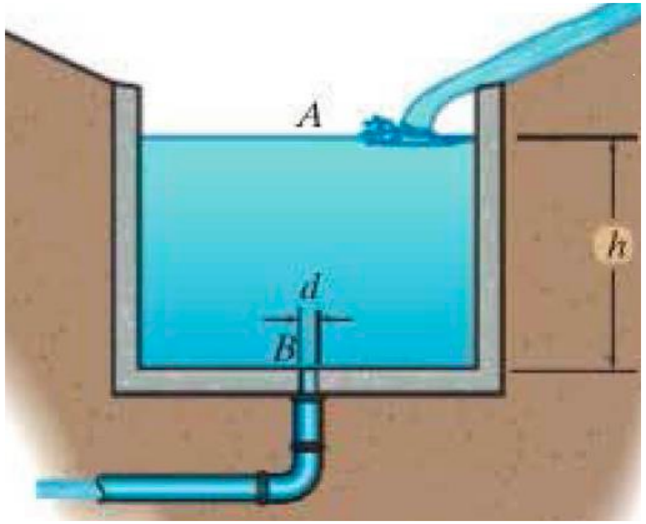
- The fluid cannot turn a sharp 90° corner. A vena contracta results with a contraction coefficient
- $Cc = z2/a$ less than 1.
- Typically Cc is approximately 0.61 over the depth ratio range of $(0 < a/z1 < 0.2)$. For larger values of $a/z1$ the value of Cc increases rapidly.





Problems and solution

Water is discharged through the drain pipe at B from the large basin at 0.03 m^3/s. lf the diameter of the drain pipe is d = 60 mm, determine the pressure at B just inside the drain when the depth of the water is h = 2 m.



$$Q_B = u_B A_B \;\Rightarrow\; u_B = {Q_B}/{A_B} = 0.03/(\pi * \frac{0.06^2}{4}) = 10.61\ m/s$$

The Bernoulli's equation can be applied between A and B

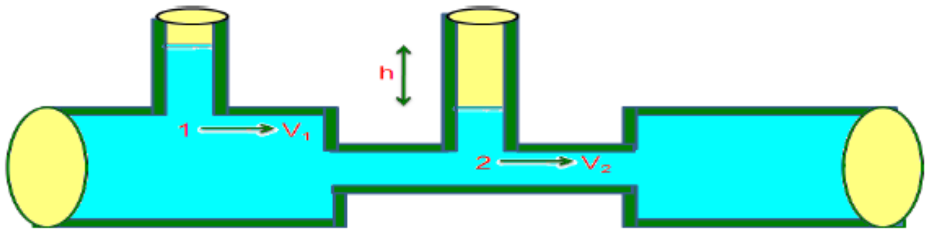
$$\frac{p_A}{\rho} + \frac{u_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{u_B^2}{2} + gz_B \text{ , } but\ p_A = p_{atm}\ and\ assume\ u_A = 0\ (large\ basin)$$

$$\frac{p_A - p_B}{\rho} = \frac{u_B^2}{2} + g(z_B - z_A) \;\Rightarrow\; \frac{p_{atm} - p_{atm} - p_{B,gauge}}{\rho} = \frac{u_B^2}{2} + g(z_B - z_A)$$

$$p_{B,gauge} = \rho\{(-\frac{u_B^2}{2} + g(z_A - z_B)\} = 10^3(-56.3 + 9.81 * 2) = -36666Pa$$
$$= -36.666\ kPa$$

General Applications of Bernoulli’s Equation

- *Venturi Mater*
- A Venturi gauge is a direct application of Bernoulli's equation and consists of a horizontal tube containing a shortening (as shown in the figure below. intersegmental compression. The two tubes should be equal in cross-section. And when applying Bernoulli's equation on segments 1 and 2 we get



$$\frac{P_1}{\rho}+\frac{1}{2}v_1^2+gh_1=\frac{P_2}{\rho}+\frac{1}{2}v_2^2+gh_2$$

$$\frac{1}{2}(v_2^2-v_1^2)=\frac{P_1-P_2}{\rho} \qquad (1)$$

If the difference in liquid level between the two vertical tubes is equal to h, then the pressure difference will be equal to

$$P_1-P_2=\rho g\, h$$

From the continuity equation

$$A_1v_1=A_2v_2$$

$$v_2=A_1v_1/A_2 \qquad (2)$$

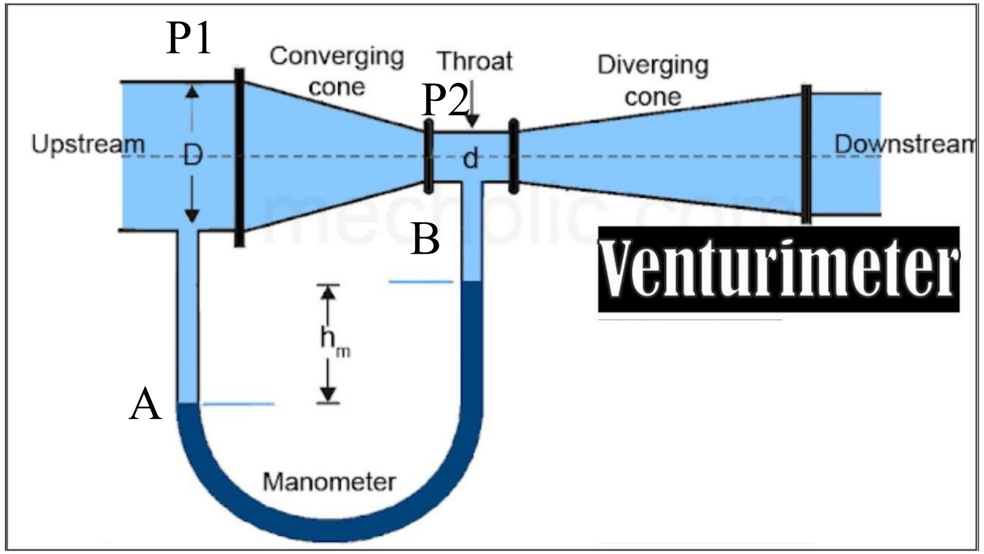
By substituting the equation 2 in 1, we get

$$\frac{1}{2}v_1^2\left(\frac{A_1^2}{A_2^2}-1\right)=\frac{P_1-P_2}{\rho}$$

$$v_1^2=\frac{2(P_1-P_2)}{\rho(A_1^2-A_2^2)}A_2^2$$

$$v_1=A_2\sqrt{\frac{2(P_1-P_2)}{\rho(A_1^2-A_2^2)}} \qquad (3)$$

Since the volume of fluid passing per second is equal to



$$Q = A_1 V_1$$

$$Q = A_1 A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho (A_1^2 - A_2^2)}} \quad (4)$$

How to calculate the P1-P2

$PA = P1 + \boldsymbol{\rho} * g * h$

$PB = P2 + \boldsymbol{\rho}_{Hg} * g * h$

$P1 - P2 = (\boldsymbol{\rho}_{Hg} - \boldsymbol{\rho}_{fluid}) * g * h$

$P1 - p2 = h[(\gamma_{Hg}/\gamma_{fluid}) - 1]$

Determine the pressure head between inlet and throat

Let, $[Z\mathbf{1} + (P\mathbf{1} / \gamma)] - [Z\mathbf{2} + (P\mathbf{2} / \gamma)] = H$

$(P1/\gamma - P2/\gamma) + (Z1 - Z2) = H$

Where $(P1/\gamma - P2/\gamma)$ is the the pressure head between inlet and throat

Problem and solution

- A venturimeter is used to measure liquid flow rate of 7500 litres per minute. The difference in pressure across the venturimeter is equivalent to 8 m of the flowing liquid. The pipe diameter is 19 cm. Calculate the throat diameter of the venturimeter. Assume the coefficient of discharge for the venturimeter as 0.96.

$$Q = C_d \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}},$$

$$A_1 = \frac{\pi}{4} \times 0.19^2 = 0.0284 \text{ m}^2$$

$$\frac{7500 \times 10^{-3}}{60} = \frac{0.96 \times 0.0284\, A_2}{\sqrt{0.0284^2 - A_2^2}} \sqrt{2 \times 9.81 \times 8}\text{, Solving } A_2 = 0.0098 \text{ m}^2$$

Let the diameter be d, $\frac{\pi}{4} \times d^2 = 0.0098$

$$\therefore \quad d = \sqrt{\frac{4 \times 0.0098}{\pi}} = 9.9 \text{ cm}$$

Problem and solution

- A venturi meter of 15 cm inlet diameter and 10 cm throat is laid horizontally in a pipe to measure the flow of oil of 0.9 specific gravity. The reading of a mercury manometer is 20 cm. Calculate the discharge in lit/min?

Solution

For inlet, $A_1 = (\pi\, d_1^2)/4 = (\pi \times 15^2)/4 = 176.7\ cm^2$

For throat, $A_2 = (\pi\, d_2^2)/4 = (\pi \times 10^2)/4 = 78.54\ cm^2$

$H = h_m\,[(\gamma_m / \gamma) - 1] = 20\,[(13.6 / 0.9) - 1] = 282.2$ cm of oil

$$Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{(A_1^2 - A_2^2)}}$$

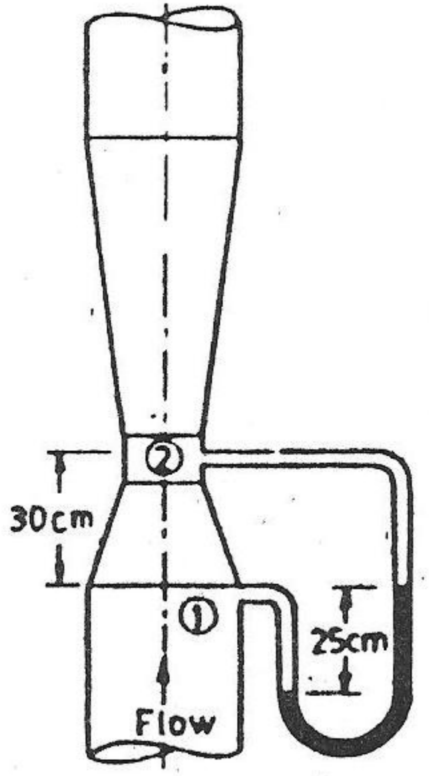
$$Q = \frac{(176.7 x 78.54)\sqrt{2 x 981 x 282.2}}{\sqrt{(176.7)^2 - (78.54)^2}}$$

$Q = 65238.2\ cm^3/sec$

(x 60/1000) Thus, $Q = 3914.3$ lit/min

Problem and solution

- A 30 cm x 15 cm venturi meter is provided to vertical pipe line carrying oil with 0.9 specific gravity. The flow direction is upwards. The difference in elevation between inlet and throat is 30 cm. The reading of a mercury manometer is 25 cm.
- 1- Calculate the discharge?
- 2- Determine the pressure head between inlet and throat?



(1) For inlet, $A_1 = (\pi\, d_1^2)/4 = (\pi \times 30^2)/4 = 706.86\ cm^2$

For throat, $A_2 = (\pi\, d_2^2)/4 = (\pi \times 15^2)/4 = 176.71\ cm^2$

$H = h_m\,[(\gamma_m / \gamma) - 1] = 25\,[(13.6 / 0.9) - 1] = 352.8$ cm of oil

$$Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{(A_1^2 - A_2^2)}}$$

$$Q = \frac{(706.86 x 176.71)\sqrt{2x981x352.8}}{\sqrt{(706.86)^2 - (176.71)^2}}$$

$Q = 151840.6\ cm^3/sec$

(2) $[Z_1 + (P_1 / \gamma)] - [Z_2 + (P_2 / \gamma)] = H$

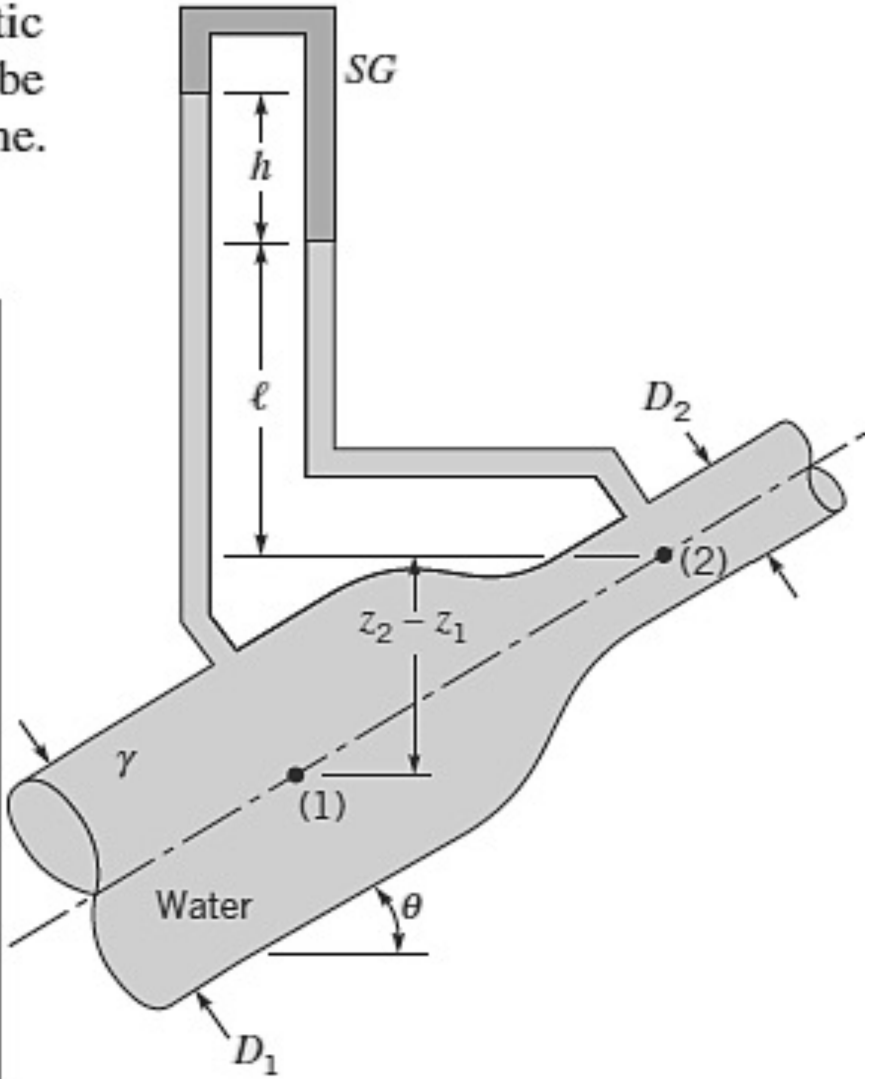
$(P_1 / \gamma) - (P_2 / \gamma) + (Z_1 - Z_2) = 352.8$

$Z_1 - Z_2 = 0 - 30 = -30$

$(P_1 / \gamma) - (P_2 / \gamma) = 352.8 + 30 = 382.8$ cm of oil

For inclined venturi meter

Water flows through a pipe reducer as is shown in Fig. The static pressures at (1) and (2) are measured by the inverted U-tube manometer containing oil of specific gravity, *SG*, less than one. Determine the manometer reading, *h*.



Solution

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

$$Q = A_1 V_1 = A_2 V_2$$

$$p_1 - p_2 = \gamma(z_2 - z_1) + \frac{1}{2}\rho V_2^2[1 - (A_2/A_1)^2] \quad \textbf{(1)}$$

This pressure difference is measured by the manometer and can be determined by using the pressure–depth ideas

$$p_1 - \gamma(z_2 - z_1) - \gamma\ell - \gamma h + SG\,\gamma h + \gamma\ell = p_2$$

$$p_1 - p_2 = \gamma(z_2 - z_1) + (1 - SG)\gamma h \quad \textbf{(2)}$$

$$(1 - SG)\gamma h = \frac{1}{2}\rho V_2^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]$$

$$h = (Q/A_2)^2 \frac{1 - (A_2/A_1)^2}{2g(1 - SG)} \quad \textbf{(Ans)}$$

Power in Fluid Flow

- Power is the *rate* at which work is done or energy in any form is used or supplied. To see the relationship of power to fluid flow, consider Bernoulli's equation:

$$\left(P + \frac{1}{2}\rho v^2 + \rho g h\right)Q = \text{power}.$$

Exp// The left ventricle of a resting adult's heart pumps blood at a flow rate of 83.0 cm3/s, increasing its pressure by 110 mm Hg, its speed from zero to 30.0 cm/s, and its height by 5.00 cm. (All numbers are averaged over the entire heartbeat.) Calculate the total power output of the left ventricle. Note that most of the power is used to increase blood pressure.

Solution

Q= 83 cm3/s = 8.3 $*10^{-5}$

P= 110 mmHg* 1.013 $*10^{5}$ / 760 mmHg = 1.47$*10^{4}$ pa

V = 30 = 0.3 m/s

H= 5 cm = 0.05 m

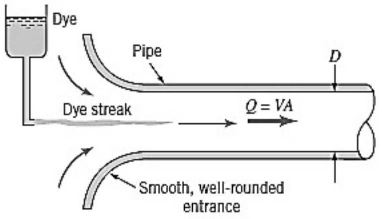
$$\left(P + \frac{1}{2}\rho v^2 + \rho g h\right)Q = \text{power}.$$

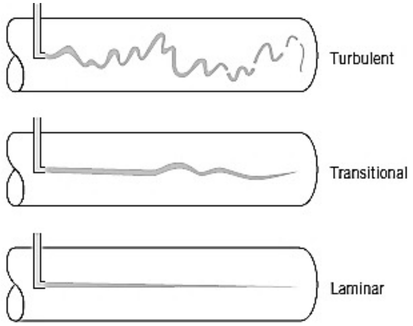
Power = 8.3 $*10^{-5}$ (1.47$*10^{4}$)+ (1/2 * 1050 * $(0.3)^2$+(1050*9.8*0.05)

Power = 1.27 w

Reynolds number

- **Laminar or Turbulent Flow**
- The flow of a fluid in a pipe may be laminar flow or it may be turbulent flow. Reynolds injected dye into a pipe in which water flowed due to gravity. The entrance region of the pipe is depicted in Fig. There are three characteristics, denoted in figure as laminar, transitional, and turbulent flow, respectively





Laminar . when $Re < 2100$

Transient . when $2100 < Re < 4000$

Turbulent . when $Re > 4000$

- For pipe flow the most important dimensionless parameter is the Reynolds number, Re - the ratio of the inertia to viscous effects in the flow. Hence, in the previous paragraph the term flowrate should be replaced by Reynolds number. The Reynolds number is a dimensionless parameter named after Professor Reynolds.

$$R_e = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{F_{\text{Inertial}}}{F_{\text{Viscous}}}$$

$$R_e = \frac{\rho V D}{\mu}$$

Where: ρ = fluid density (kg/m^3)

D = pipe diameter (m)

V = average velocity (m/s)

μ = fluid viscosity (Ns/m^2),

Reynolds number

$$R_e = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{F_{\text{Inertial}}}{F_{\text{Viscous}}}$$

$$R_e = \frac{\rho V D}{\mu}$$

Where: ρ = fluid density (kg/m^3)
D = pipe diameter (m)
V = average velocity (m/s)
μ = fluid viscosity (Ns/m^2),

For laminar flow

The pressure drop,

$$\Delta p = f \frac{\ell}{D} \frac{\rho V^2}{2}$$

The head loss $h_{L,\text{major}} = f \frac{\ell}{D} \frac{V^2}{2g}$

f is termed the friction factor. Thus, the friction factor for laminar fully developed pipe flow is simply

$$f = \frac{64}{Re}$$

We obtain an alternate expression for the friction factor as a dimensionless wall shear stress

$$f = \frac{8\tau_w}{\rho V^2}$$

Reynolds number Problem

- Water at 10°C (ρ= 999.7 kg/m3 and μ= 1.307 ×103 kg/m · s) is flowing steadily in a 0.20 cm-diameter, 15 m-long pipe at an average velocity of 1.2 m/s. Determine (a) The pressure drop, (b) The head loss, and (c) The pumping power requirement to overcome this pressure drop.

Solution

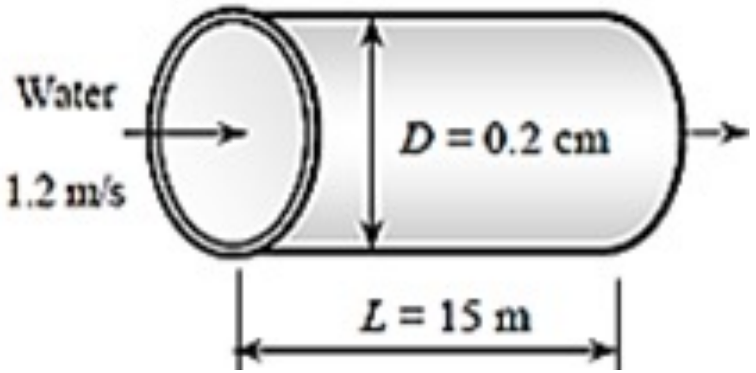
(*a*) First we need to determine the flow regime. The Reynolds number of the flow is

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{(999.7\ \mathrm{kg/m^3})(1.2\ \mathrm{m/s})(2\times10^{-3}\ \mathrm{m})}{1.307\times10^{-3}\ \mathrm{kg/m\cdot s}} = 1836$$

$$f = \frac{64}{\mathrm{Re}} = \frac{64}{1836} = 0.0349$$

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2}$$

$$\Delta P = \Delta P_L = 0.0349\frac{15\ \mathrm{m}}{0.002\ \mathrm{m}}\frac{(999.7\ \mathrm{kg/m^3})(1.2\ \mathrm{m/s})^2}{2}\left(\frac{1\ \mathrm{kN}}{1000\ \mathrm{kg\cdot m/s}}\right)\left(\frac{1\ \mathrm{kPa}}{1\ \mathrm{kN/m^2}}\right) = 188\ \mathrm{kPa}$$



(*b*) The head loss in the pipe is determined from

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g} = 0.0349\frac{15\ \mathrm{m}}{0.002\ \mathrm{m}}\frac{(1.2\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} = 19.2\ \mathrm{m}$$

(*c*) The volume flow rate and the pumping power requirements are

$$Q = VA_c = V(\pi D^2/4) = (1.2\ \mathrm{m/s})[\pi(0.002\ \mathrm{m})^2/4] = 3.77\times10^{-6}\ \mathrm{m^3/s}$$

$$\dot{W}_{pump} = Q\,\Delta P = (3.77\times10^{-6}\ \mathrm{m^3/s})(188\ \mathrm{kPa})\left(\frac{1000\ \mathrm{W}}{1\ \mathrm{kPa\cdot m^3/s}}\right) = 0.710\ \mathrm{W}$$

Therefore, power input in the amount of 0.71 W is needed to overcome the frictional losses in the flow due to viscosity.