Al-Mustaqbal University
College of Engineering and Technology
Department of Biomedical Engineering

Stage: Second

Electric Circuits II

2024-2025

Lecture (2): A.C connection circuit

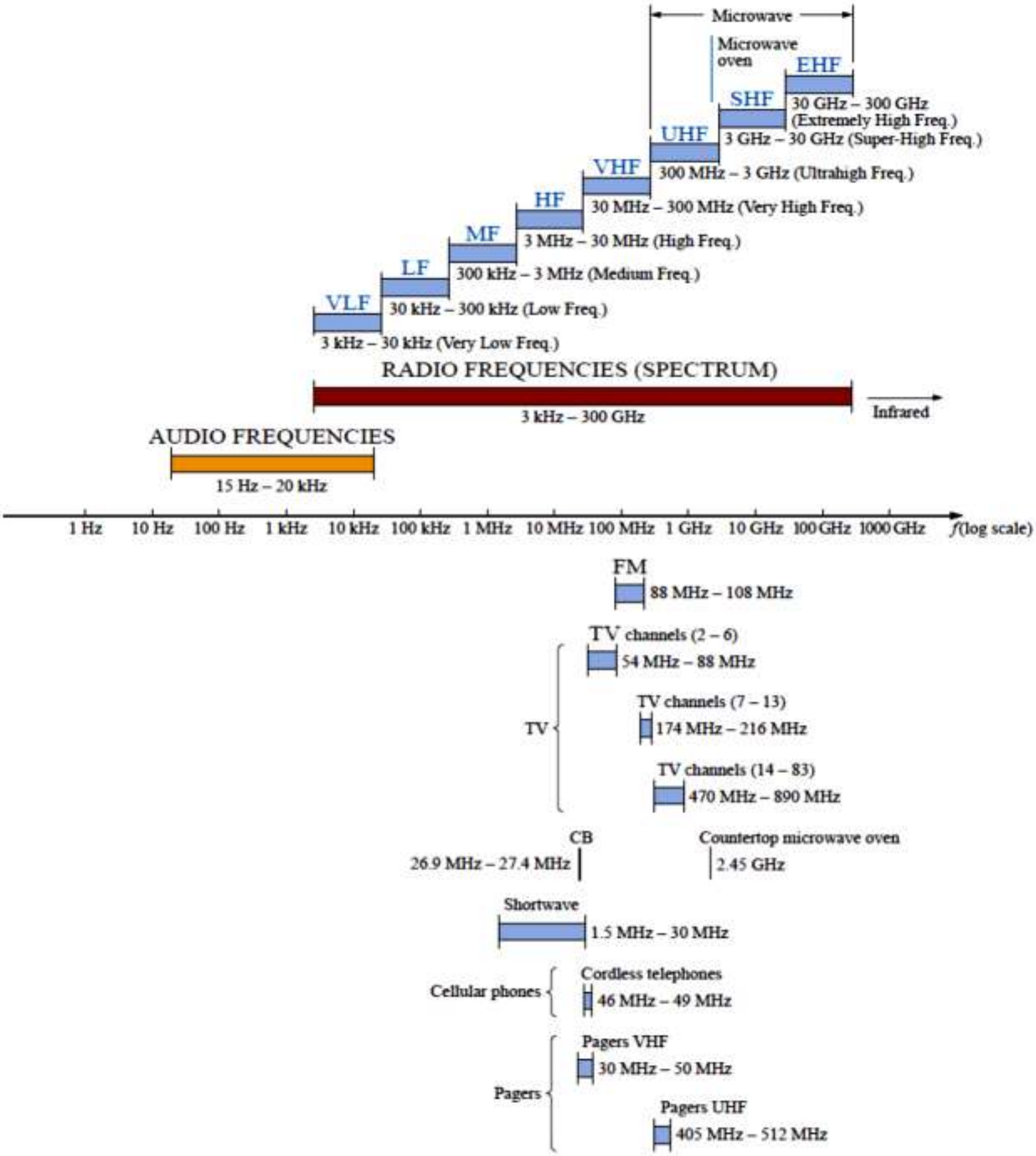


FIG. 8.5 Areas of application for specific frequency bands.

Example 8.1: Find the period of a periodic waveform with a frequency of

a. 60 Hz. b. 1000 Hz.

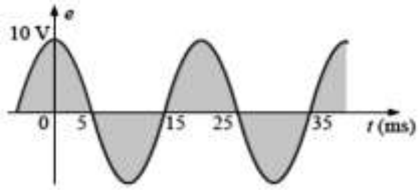
Solutions: a. $\mathbf{T} = \frac{1}{f} = \frac{1}{6Hz} = 0.01667s \text{ or } 16.67\ ms$

b. $\mathbf{T} = \frac{1}{f} = \frac{1}{1000Hz} = 10^{-3}s = 1\ ms$

Example 8.2: Determine the frequency of the waveform of Figure below.

Solution: From the figure, T = (25 ms - 5 ms) = 20 ms, and

$$\mathbf{f = \frac{1}{T} = \frac{1}{20{\times}10^{-3}s} = 50}\ \boldsymbol{Hz}$$



Defined Polarities and Direction

In the following analysis, we will find it necessary to establish a set of polarities for the sinusoidal ac voltage and a direction for the sinusoidal ac current. In each case, the polarity and current direction will be for an instant of time in the positive portion of the sinusoidal waveform. This is shown in Fig. 8.6 with the symbols for the sinusoidal ac voltage and current.

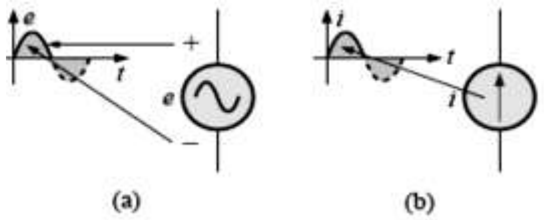


FIG. 8.6 (a) Sinusoidal ac voltage sources, (b) sinusoidal current sources.

8.3 THE SINE WAVE

The terms defined in the previous section can be applied to any type of periodic waveform, whether smooth or discontinuous. The sinusoidal waveform is of particular importance, however, since it lends itself readily to the mathematics and the physical phenomena associated with electric circuits. Consider the power of the following statement: ***The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements.***

In other words, if the voltage across (or current through) a resistor, coil, or capacitor is sinusoidal in nature, the resulting current (or voltage, respectively) for each will also have sinusoidal characteristics, as shown in Fig. 8.7. If a square wave or a triangular wave were applied, such would not be the case.

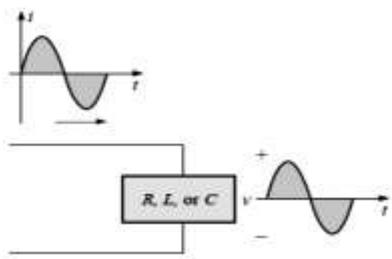


FIG. 8.7

The unit of measurement for the angle axis of is the degree. A second unit of measurement frequently used is the radian (rad). It is defined by a quadrant of a circle where the distance subtended on the circumference equals the radius of the circle.

$$2\pi \text{ rad} = 360° \quad (8.4)$$

With $1 \text{ rad} = 57.296° \cong 57.3°$ (8.5)

The quantity π is the ratio of the circumference of a circle to its diameter.

For 180° and 360°, the two units of measurement are related. The conversion equations between the two are the following:

$$\text{Radians} = \left(\frac{\pi}{180°}\right) \times (\text{degrees}) \quad (8.6)$$

$$\text{Degrees} = \left(\frac{180°}{\pi}\right) \times (\text{radians}) \quad (8.7)$$

The velocity with which the radius vector rotates about the center, called the angular velocity, can be determined from the following equation:

$$\textbf{Angular velocity} = \frac{distance\ (degrees\ or\ radians)}{time\ (seconds)} \quad (8.8)$$

Substituting into Eq. (8.8) and assigning the Greek letter omega (ω) to the angular velocity, we have

$$\omega = \alpha/t \quad \Longrightarrow \alpha = \omega t \quad (8.9)$$

Since q is typically provided in radians per second, the angle α obtained using Eq. (8.9) is usually in radians.

The angular velocity of the rotating radius vector is

$$\boldsymbol{\omega = 2\pi f} \quad (8.10)$$

Example 8.3: Determine the angular velocity of a sine wave having a frequency of 60 Hz.

Solution: $\omega = 2\pi f = (2\pi)(60 \text{ Hz}) = 377 \text{ rad/s}$

Example 8.4: Given ω = 200 rad/s, determine how long it will take the sinusoidal waveform to pass through an angle of 90°.

Solution: Eq. (8.9): α = ω t, and t = α /ω However, a must be substituted as π/2 (= 90°) since qis in radians per second: $t = \alpha / \omega = \frac{\pi/2}{200}$ **= 7.85 ms**

Example 8.5: Find the angle through which a sinusoidal waveform of 60 Hz will pass in a period of 5 ms.

Solution: Eq. (8.9): $\alpha = \omega t$, or $\alpha = 2\pi f t = (2\pi)(60 \text{ Hz})(5\times10^{-3} \text{ s}) = 1.885 \text{ rad} = 108°$

8.4 GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

The basic mathematical format for the sinusoidal waveform is.

$$\mathbf{A_m \sin \alpha} \qquad (8.11)$$

where $\mathbf{A_m}$ is the peak value of the waveform and $\boldsymbol{\alpha}$ is the unit of measure for the horizontal axis, as shown in Fig. 8.8

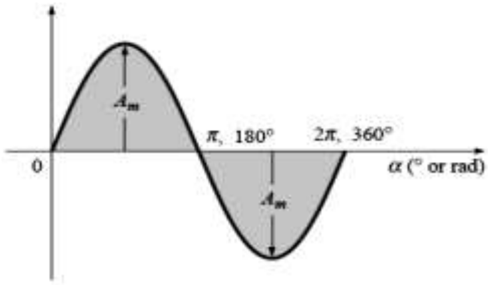


FIG. 8.8 Basic sinusoidal function.

Due to Eq. (8.9), the general format of a sine wave can also be written

$$\mathbf{A_m \sin \omega t} \qquad (8.12)$$

For electrical quantities such as current and voltage, the general format is

$$\mathbf{i = I_m \sin \omega t = I_m \sin \alpha}$$

$$\mathbf{e = E_m \sin \omega t = E_m \sin \alpha}$$

where the capital letters with the subscript m represent the amplitude, and the lowercase letters i and e represent the instantaneous value of current or voltage, respectively, at any time t.

Example 8.6: Given $e = 5 \sin \alpha$, determine e at $\alpha = 40°$ and $\alpha = 0.8\pi$.

Solution: For $\alpha = 40°$

$$e = 5 \sin 40° = 5(0.6428) = 3.214 \text{ V}$$

For $\alpha = 0.8\pi$,

$$a (°) = (180/\pi)(0.8\pi) = 144°$$

and $$e = 5 \sin 144° = 5(0.5878) = 2.939 \text{ V}$$

8.5 PHASE RELATIONS

Thus far, we have considered only sine waves that have maxima at $\pi/2$ and $3\pi/2$, with a zero value at 0, π, and 2π. If the waveform is shifted to the right or left of 0°,the expression becomes

$$A_m \sin(\omega t \pm \theta) \quad (8.13)$$

where θ is the angle in degrees or radians that the waveform has been shifted.

If the waveform passes through the horizontal axis with a positive- going (increasing with time) slope before 0°, as shown in Fig. 8.9, the expression is

$$A_m \sin(\omega t + \theta) \quad (8.14)$$

At $\omega t = \alpha = 0°$, the magnitude is determined by $A_m \sin \theta$.

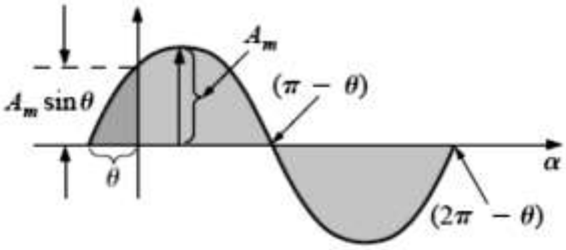


FIG. 8.9 Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope before 0°.

If the wave- form passes through the horizontal axis with a positive-going slope after 0°, as shown in Fig. 8.10, the expression is

$$A_m \sin(\omega t - \theta) \quad (8.15)$$

at $\omega t = \alpha = 0°$, the magnitude is $A_m \sin(-\theta)$, which, by a trigonometric identity, is $-A_m \sin \theta$.

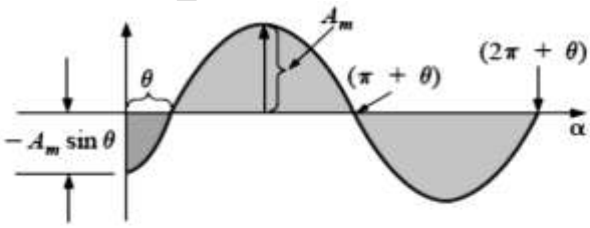


FIG. 8.10 Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope after 0°.

The geometric relationship between various forms of the sine and cosine functions can be listed below:

$\cos \alpha = \sin(\alpha + 90°)$, $\quad \sin \alpha = \cos(\alpha - 90°)$

$-\sin \alpha = \sin(\alpha \pm 180°)$, $\quad -\cos \alpha = \sin(\alpha \pm 270°) = \sin(\alpha - 90°)$ etc.

$\sin(-\alpha) = -\sin \alpha$, $\quad \cos(-\alpha) = \cos \alpha$

<u>Example 8.7:</u> What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30°)$ **d.** $i = -\sin(\omega t + 30°)$

i = 5 sin(ωt + 70°) **v = 2 sin(ωt + 10°)**

b. i = 15 sin(ωt + 60°) **e. i = -2 cos(ωt - 60°)**

v = 10 sin(ωt - 20°) **v = 3 sin(ωt - 150°)**

c. i = 2 cos(ωt + 10°)

v = 3 sin(ωt - 10°)

Solutions:

a. See Fig. 8.11.

i leads v by 40°, or v lags i by 40°.

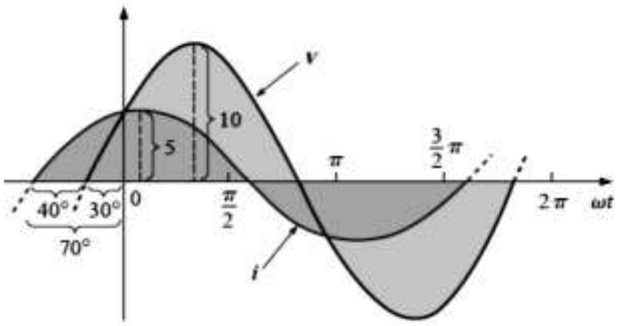


FIG. 8.11 Example 8.7; i leads v by 40°.

b. See Fig. 8.12.

i leads v by 80°, or v lags i by 80°.

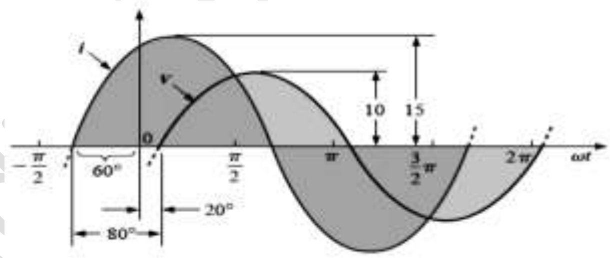


FIG. 8.12 Example 8.7; i leads v by 80°.

c. See Fig. 8.13.

i = 2 cos(ωt + 10°) = 2 sin(ωt + 10° + 90°)

= 2 sin(ωt + 100°)

i leads v by 110°, or v lags i by 110°.