

LECTURE SIX

ACTIVE FILTER

6.1 BASIC FILTER RESPONSES

Filters are usually categorized by the manner in which the output voltage varies with the frequency of the input voltage. The categories of active filters are *low-pass, high-pass, band-pass, and band-stop*.

A popular application uses *op-amps* to build active filter circuits. A filter circuit can be constructed using passive components: *resistors and capacitors*. An active filter additionally uses an amplifier to provide voltage amplification and signal isolation or buffering.

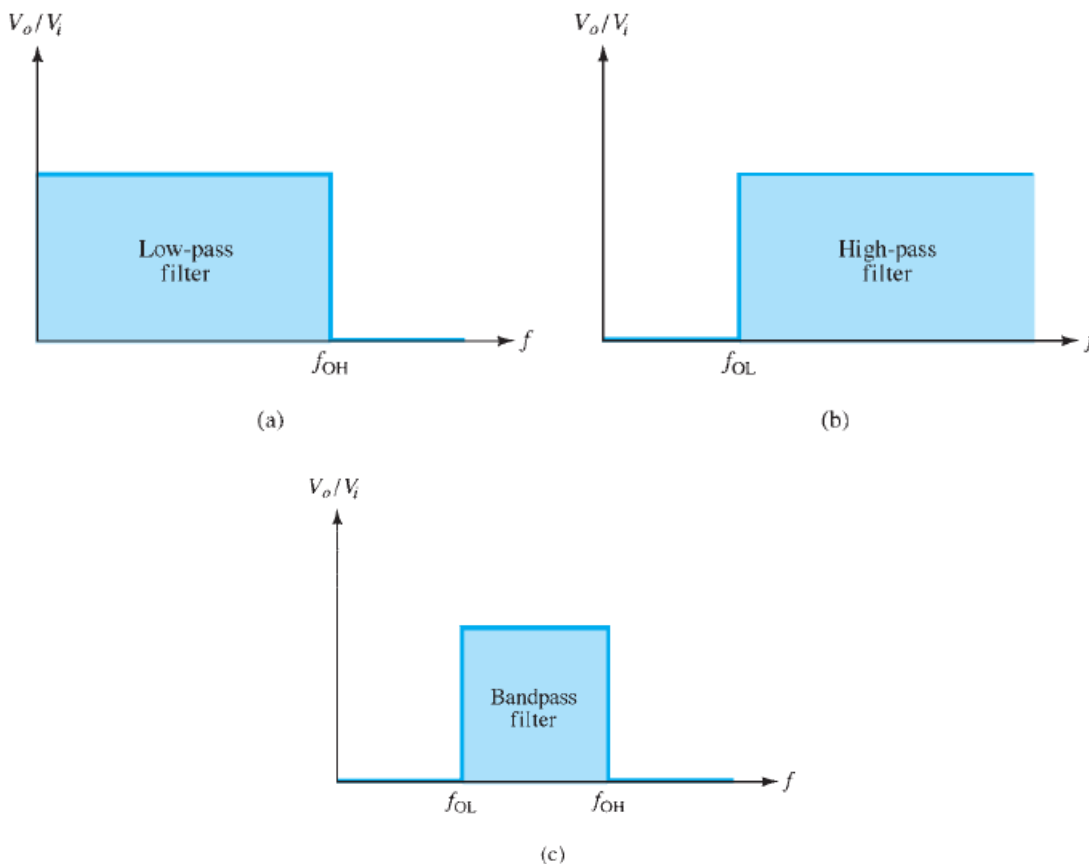


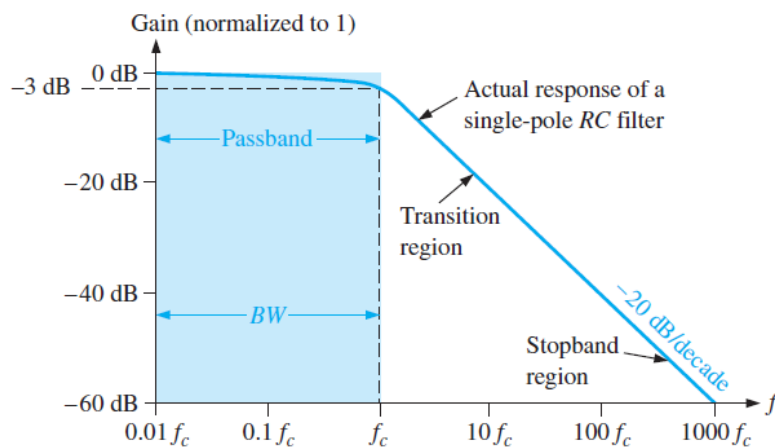
Fig6. 1: Ideal filter response: (a) low-pass; (b) high-pass; (c) bandpass

Low-Pass Filter Response

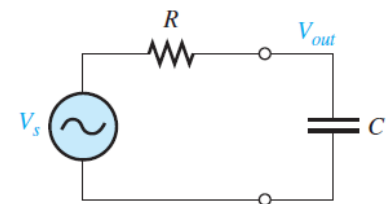
A **filter** is a circuit that passes certain frequencies and attenuates or rejects all other frequencies.

The **passband** of a filter is the range of frequencies that are allowed to pass through the filter with minimum attenuation (usually defined as less than -3 dB of attenuation).

The **critical frequency** (also called the *cutoff frequency*) defines the end of the *passband* and is normally specified at the point where the response drops (70.7%) from the passband response. Following the passband is a region called the *transition region* that leads into a region called the *stopband*. There is no precise point between the transition region and the stopband.



(a) Comparison of an ideal low-pass filter response (blue area) with actual response. Although not shown on log scale, response extends down to $fc = 0$.



(b) Basic low-pass circuit

Fig6. 2



A **low-pass filter** is one that passes frequencies from dc to f_c and significantly attenuates all other frequencies. The passband of the ideal low-pass filter is shown in the blue-shaded area of Figure 6–2(a); the response drops to zero at frequencies beyond the passband.

$$BW = f_c$$

The -20 dB/decade **roll-off** rate for the gain of a basic RC filter means that at a frequency of $10f_c$, the output will be -20 dB (10%) of the input. This roll-off rate is not a particularly good filter characteristic because too *much of the unwanted frequencies (beyond the passband) are allowed through the filter.*

The critical frequency of a low-pass RC filter occurs when $XC = R$, where

$$f_c = \frac{1}{2\pi RC}$$

Figure 6–3 illustrates three idealized low-pass response curves, including the basic one-pole response (-20 dB/decade). In order to produce a filter that has a steeper transition region, it is necessary to add additional circuitry to the basic filter. Responses that are steeper than -20 dB/decade in the transition region cannot be obtained by simply cascading identical RC stages (*due to loading effects*). However, by combining an op-amp with frequency-selective feedback circuits, filters can be designed with roll-off rates of -40,-60 or more dB/decade. Filters that include one or more op-amps in the design are called **active filters**. These filters can optimize the roll-off rate

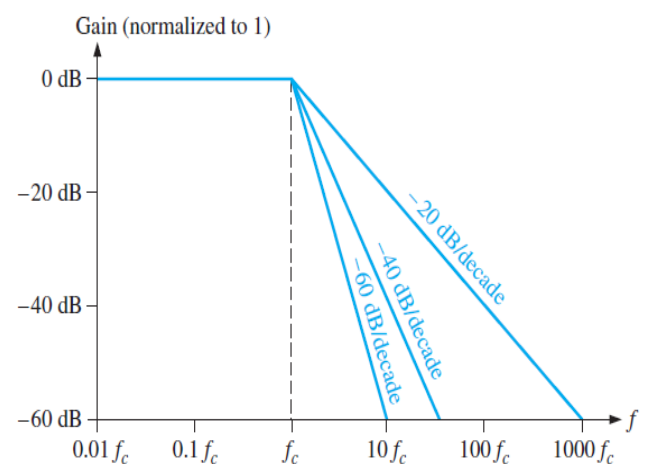
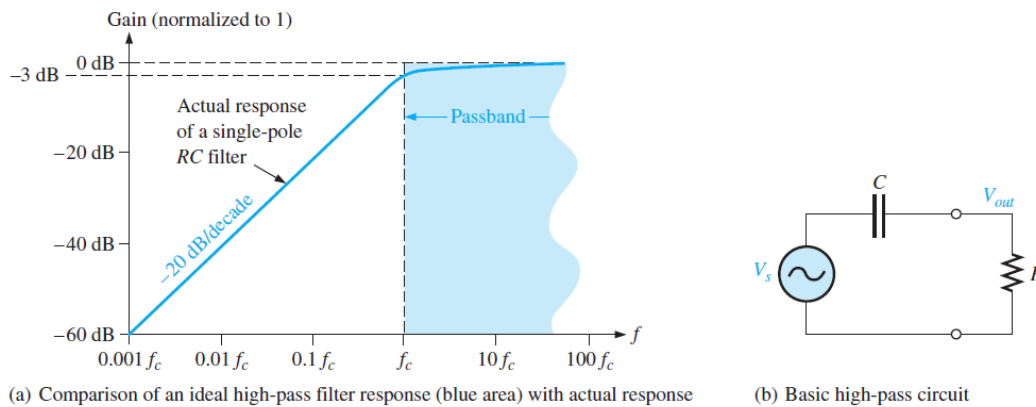


Fig6. 3: Idealized low-pass filter responses

High-Pass Filter Response

A **high-pass filter** is one that significantly attenuates or rejects all frequencies below f_c and passes all frequencies above f_c . The critical frequency is, again, the frequency at which the output is 70.7% of the input (or -3 dB) as shown in Figure 6-4(a). The ideal response, indicated by the blue-shaded area, has an instantaneous drop at f_c , which, of course, is not achievable. Ideally, the passband of a high-pass filter is all frequencies above the critical frequency.



A simple RC circuit consisting of a single resistor and a capacitor can be configured as a high-pass filter by taking the output across the resistor, as shown in Figure 6-4(b). As in the case of the low-pass filter, the basic RC circuit has a roll-off rate of -20 dB/decade as indicated by the blue line in Figure 6-4(a). Also, the critical frequency for the basic high-pass filter occurs when $XC = R$, where

$$f_c = \frac{1}{2\pi RC}$$

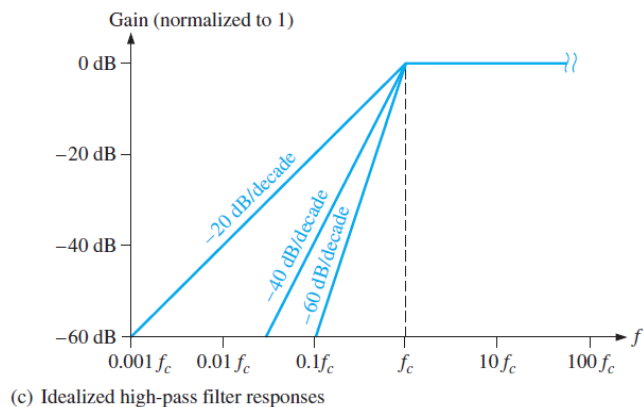


Fig. 4: High-pass filter responses.

Band-Pass Filter Response

A **band-pass filter** passes all signals lying within a band between a lower-frequency limit and an upper-frequency limit and essentially rejects all other frequencies that are outside this specified band. A generalized band-pass response curve is shown in Figure 6–5. The bandwidth (BW) is defined as the difference between the upper critical frequency (f_{c2}) and the lower critical frequency (f_{c1}).

$$BW = f_{c2} - f_{c1}$$

The critical frequencies are, of course, the points at which the response curve is 70.7% of its maximum. These critical frequencies are also called *3 dB frequencies*. The frequency about which the passband is centered is called the *center frequency*, f_0 , defined as the geometric mean of the critical frequencies

$$f_0 = \sqrt{f_{c1}f_{c2}}$$

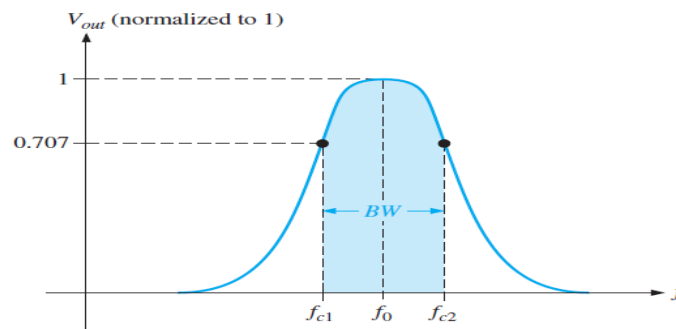


Fig6. 5: General band-pass response curve.

Band-Stop Filter Response

Another category of active filter is the **band-stop filter**, also known as a *notch*, *band-reject*, or *band-elimination* filter. You can think of the operation as the opposite of that of the bandpass filter because frequencies within a certain bandwidth are rejected, and frequencies outside the bandwidth are passed. A general response curve for a band-stop filter is shown in Figure 6–6.

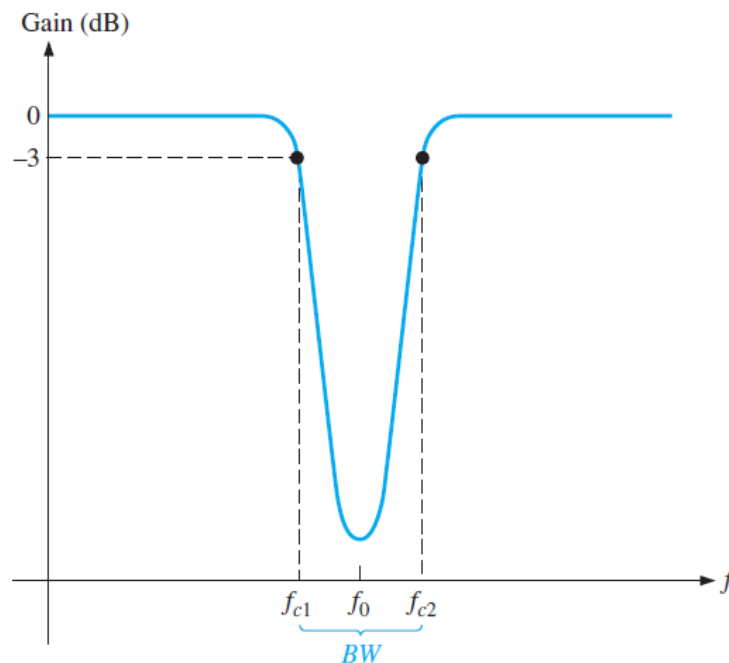


Fig6. 6: General band-stop filter response.

6.2 ACTIVE LOW-PASS FILTERS

Filters that use op-amps as the active element provide several advantages over passive filters (R , L , and C elements only). The op-amp provides gain, so the signal is not attenuated as it passes through the filter. The high input impedance of the op-amp prevents excessive loading of the driving source, and the low output impedance of the op-amp prevents the filter from being affected by the load that it is driving. Active filters are also easy to adjust over a wide frequency range without altering the desired response.

A Single-Pole Filter

Figure 6–7(a) shows an active filter with a single low-pass RC frequency-selective circuit that provides a roll-off of -20 dB/decade above the critical frequency, as indicated by the response curve in Figure 6–7(b). The critical frequency of the single-pole filter is $f_c = 1/(2\pi RC)$. The op-amp in this filter is connected as a noninverting amplifier with the closed-loop voltage gain in the passband set by the values of R_1 and R_2 .

$$A_{cl(NI)} = \frac{R_1}{R_2} + 1$$

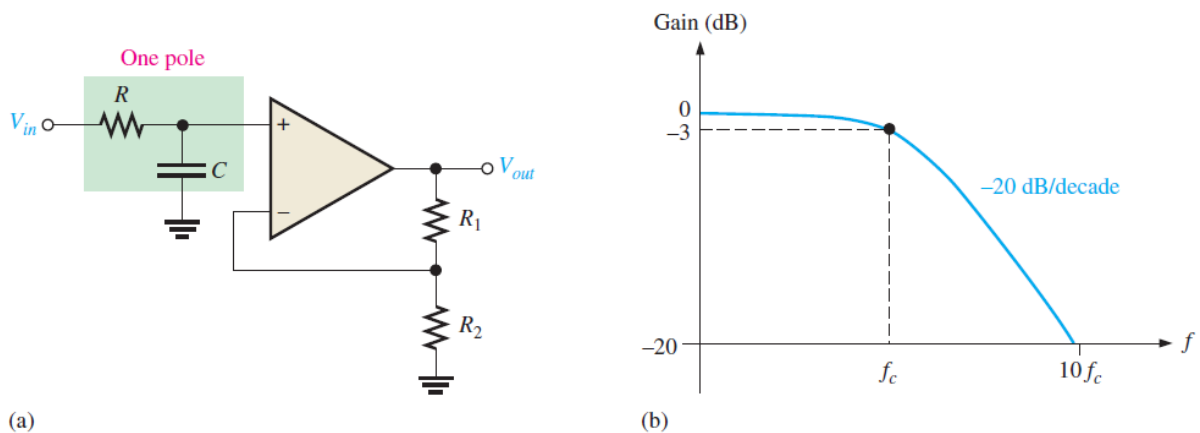


Fig6. 7: Single-pole active low-pass filter and response curve.

The Sallen-Key Low-Pass Filter

The Sallen-Key is one of the most common configurations for a second-order (two-pole) filter. It is also known as a VCVS (voltage-controlled voltage source) filter. A low-pass version of the Sallen-Key filter is shown in Figure 6–8. Notice that there are two low-pass RC circuits that provide a roll-off of -40 dB/decade above the critical frequency. The critical frequency for the Sallen-Key filter is

$$f_c = \frac{1}{2\pi \sqrt{R_A R_B C_A C_B}}$$

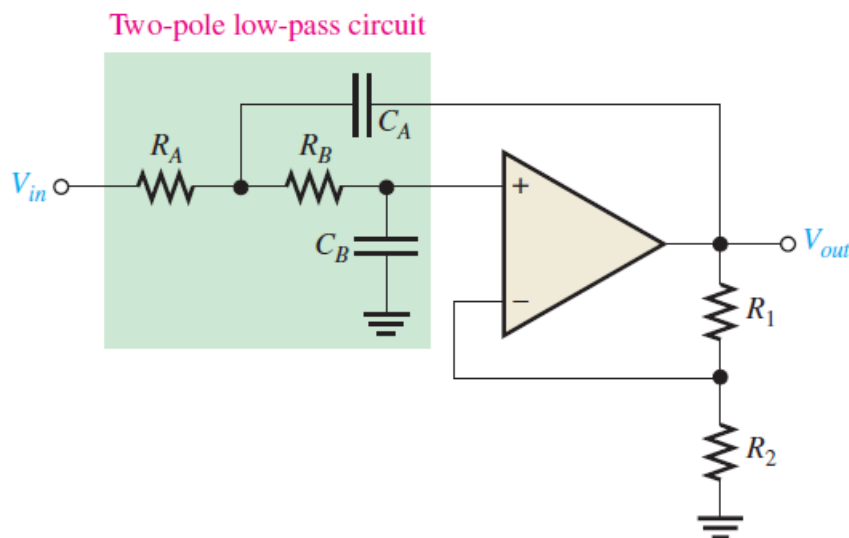


Fig 6-8: Basic Sallen-Key low-pass filter.

The component values can be made equal so that $R_A = R_B = R$ and $C_A = C_B = C$. In this case, the expression for the critical frequency simplifies to

$$f_c = \frac{1}{2\pi RC}$$

EXAMPLE 6–1: Determine the critical frequency of the Sallen-Key low-pass filter in Figure 6–9.

Solution

Since $R_A = R_B = R = 1\text{k}\Omega$ and $C_A = C_B = C = 0.022\ \mu\text{F}$,

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(1\text{k}\Omega)(0.022\ \mu\text{F})} = 7.23\text{kHz}$$

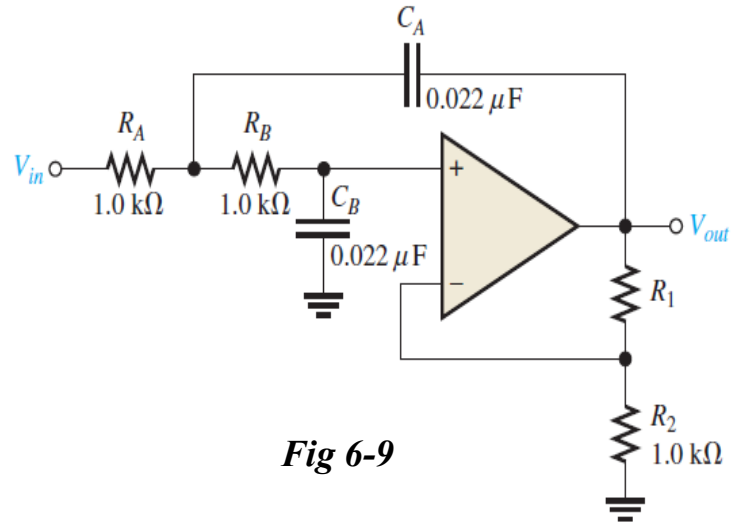
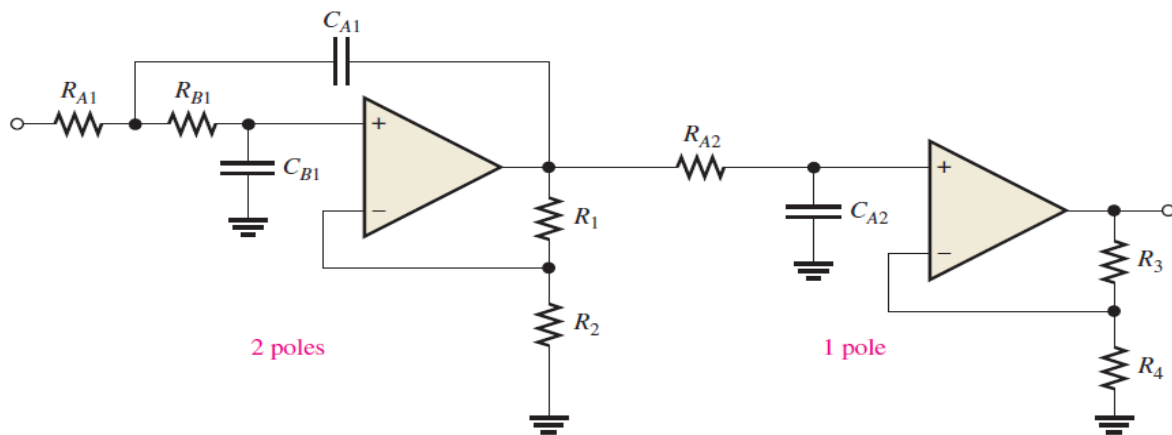


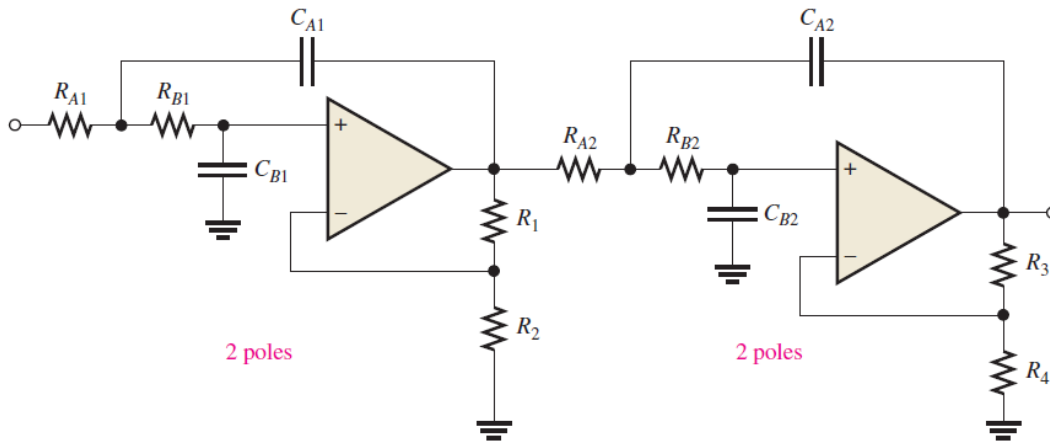
Fig 6-9

Cascaded Low-Pass Filters

A three-pole filter is required to get a third-order low-pass response (-60 dB/decade). This is done by cascading a two-pole Sallen-Key low-pass filter and a single-pole low-pass filter, as shown in Figure 6–10(a). Figure 6–10(b) shows a four-pole configuration obtained by cascading two Sallen-Key (2-pole) low-pass filters.



(a) Third-order configuration



(b) Fourth-order configuration

Fig 6-10: Cascaded low-pass filters.

EXAMPLE 6–2: For the four-pole filter in Figure 6–10(b), determine the capacitance values required to produce a critical frequency of 2680 Hz if all the resistors in the RC low-pass circuits are 1.8 kΩ.

Solution

Both stages must have the same f_c . Assuming equal-value capacitors

$$f_c = \frac{1}{2\pi RC}$$

$$C = \frac{1}{2\pi R f_c} = \frac{1}{2\pi(1.8 \text{ k}\Omega)(2680 \text{ Hz})} = 0.033 \text{ }\mu\text{F}$$

$$C_{A1} = C_{B1} = C_{A2} = C_{B2} = 0.033 \text{ }\mu\text{F}$$

ACTIVE HIGH-PASS FILTERS

In high-pass filters, the roles of the capacitor and resistor are reversed in the RC circuits. Otherwise, the basic parameters are the same as for the low-pass filters.

A Single-Pole Filter

A high-pass active filter with a -20 dB/decade roll-off is shown in Figure 6–11(a).

Notice that the input circuit is a single high-pass RC circuit. The negative feedback circuit is the same as for the low-pass filters previously discussed. The high-pass response curve is shown in Figure 6–11(b).

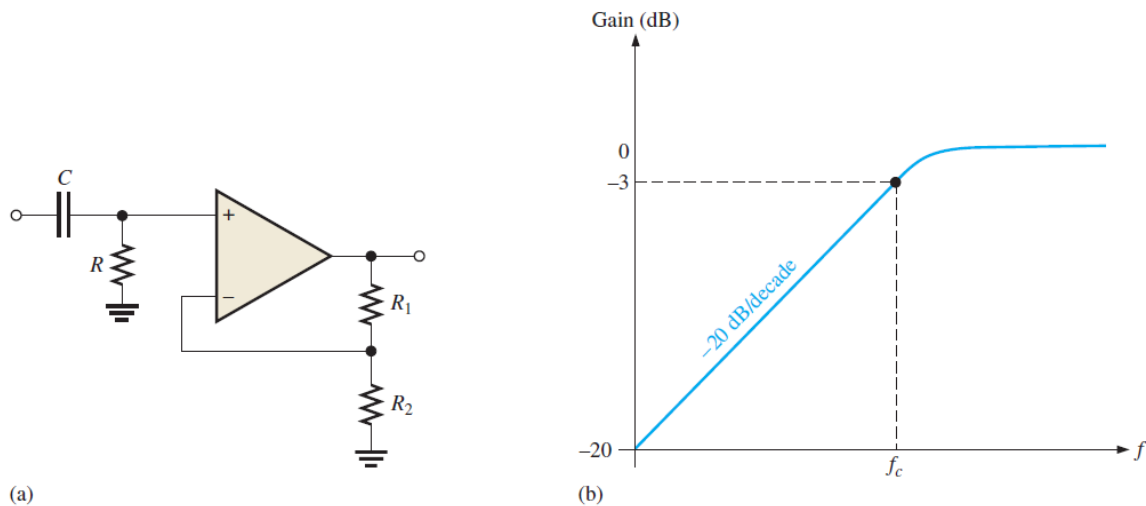


Fig 6-11: Single-pole active high-pass filter and response curve.

The Sallen-Key High-Pass Filter

A high-pass Sallen-Key configuration is shown in Figure 6–12. The components R_A , C_A , R_B , and C_B form the two-pole frequency-selective circuit. Notice that the positions of the resistors and capacitors in the frequency-selective circuit are opposite to those in the low-pass configuration. As with the other filters, the response characteristic can be optimized by proper selection of the feedback resistors, R_1 and R_2 .

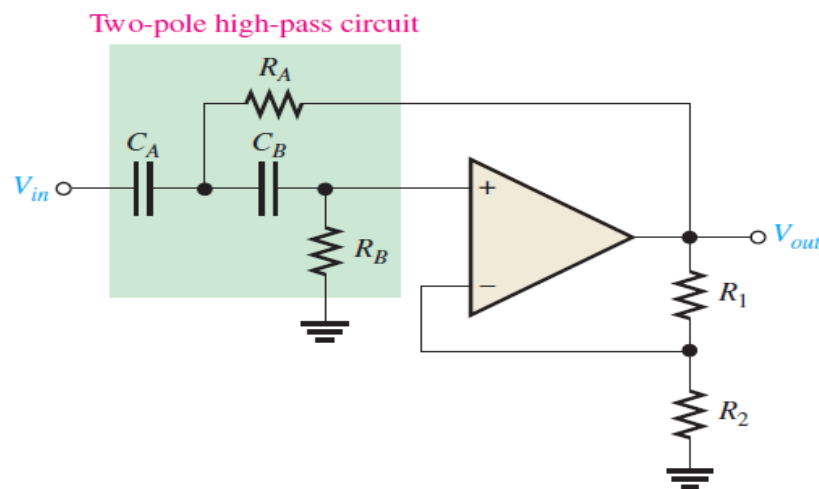


Fig 6-12: Basic Sallen-Key high-pass filter.

EXAMPLE 6–3: Calculate the voltage gain and the cutoff frequency of a second-order high-pass filter as in Fig. 6-12 for $R_A = R_B = 2.1 \text{ k}\Omega$, $C_A = C_B = 0.05 \text{ }\mu\text{F}$, and $R_2 = 10 \text{ k}\Omega$, $R_1 = 50 \text{ k}\Omega$.

Solution:

The voltage gain is
$$A_V = 1 + \frac{R_1}{R_2} = 1 + \frac{50\text{k}\Omega}{10\text{k}\Omega} = 6$$

The cutoff frequency is then
$$f_{OL} = \frac{1}{2\pi R_A C_A} = \frac{1}{2\pi(2.1 \text{ k}\Omega)(0.05 \text{ }\mu\text{F})} = 1.5\text{kHz}$$

Cascading High-Pass Filters

As with the low-pass configuration, first- and second-order high-pass filters can be cascaded to provide three or more poles and thereby create faster roll-off rates. Figure 6–13 shows a six-pole high-pass filter consisting of three Sallen-Key two-pole stages. With this configuration optimized for a Butterworth response, a roll-off of -120 dB/decade is achieved.

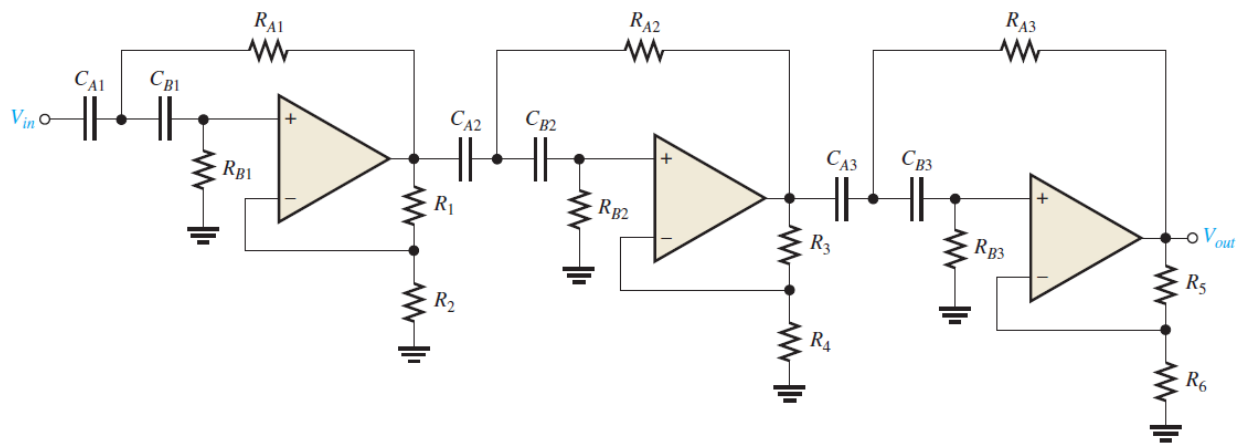


Fig 6-13: Sixth-order high-pass filter.

ACTIVE BAND-PASS FILTERS

As mentioned, band-pass filters pass all frequencies bounded by a lower-frequency limit and an upper-frequency limit, and reject all others lying outside this specified band. A band-pass response can be thought of as the overlapping of a low-frequency response curve and a high-frequency response curve.

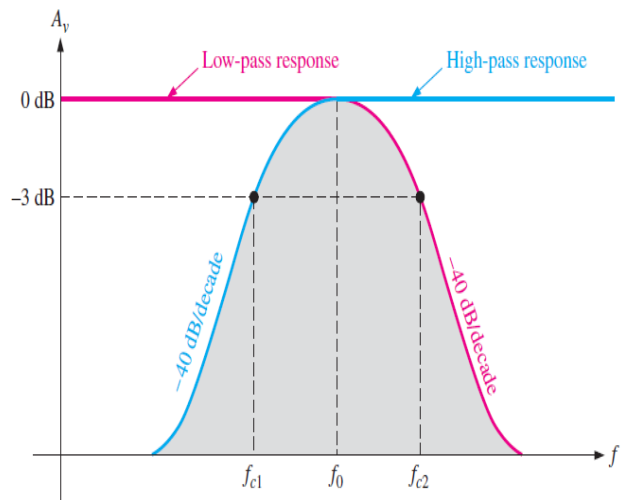


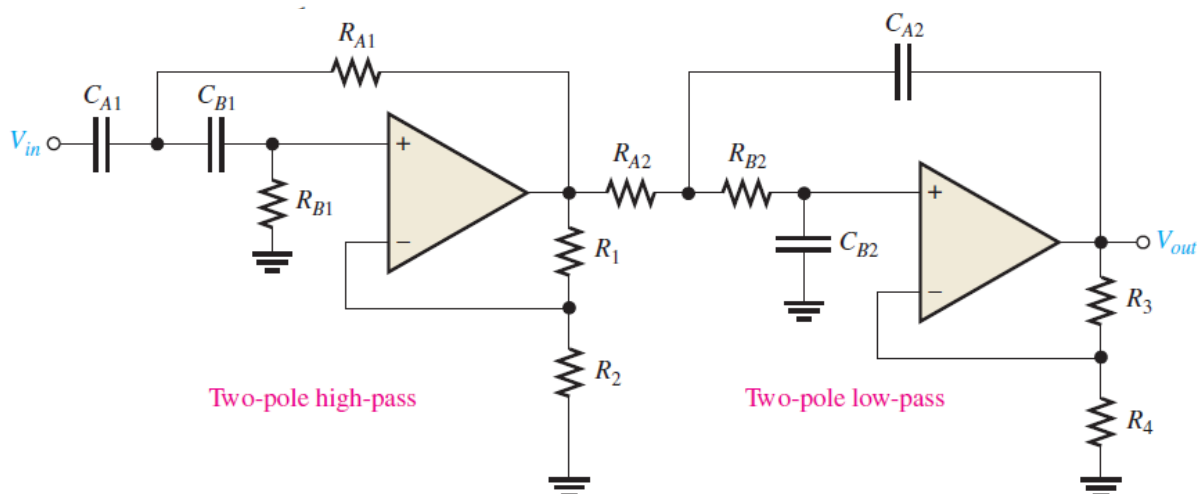
Fig 6-14: Band-pass filter.

Cascaded Low-Pass and High-Pass Filters

One way to implement a band-pass filter is a cascaded arrangement of a high-pass filter and a low-pass filter, as shown in Figure 6–14(a),

The lower frequency f_{c1} of the passband is the critical frequency of the high-pass filter. The upper frequency f_{c2} is the critical frequency of the low-pass filter

$$f_{c1} = \frac{1}{2\pi \sqrt{R_{A1}R_{B1}C_{A1}C_{B1}}} \quad f_{c2} = \frac{1}{2\pi \sqrt{R_{A2}R_{B2}C_{A2}C_{B2}}} \quad f_0 = \sqrt{f_{c1}f_{c2}}$$



(a) **Fig 6-15: cascading a two-pole high-pass and a two-pole low-pass filter**

EXAMPLE 6-4: Calculate the cutoff frequencies of the bandpass filter circuit of Fig. 6-16 with $R_1 = R_2 = 10 \text{ k}\Omega$, $C_1 = 0.1 \text{ mF}$, and $C_2 = 0.002 \text{ }\mu\text{F}$.

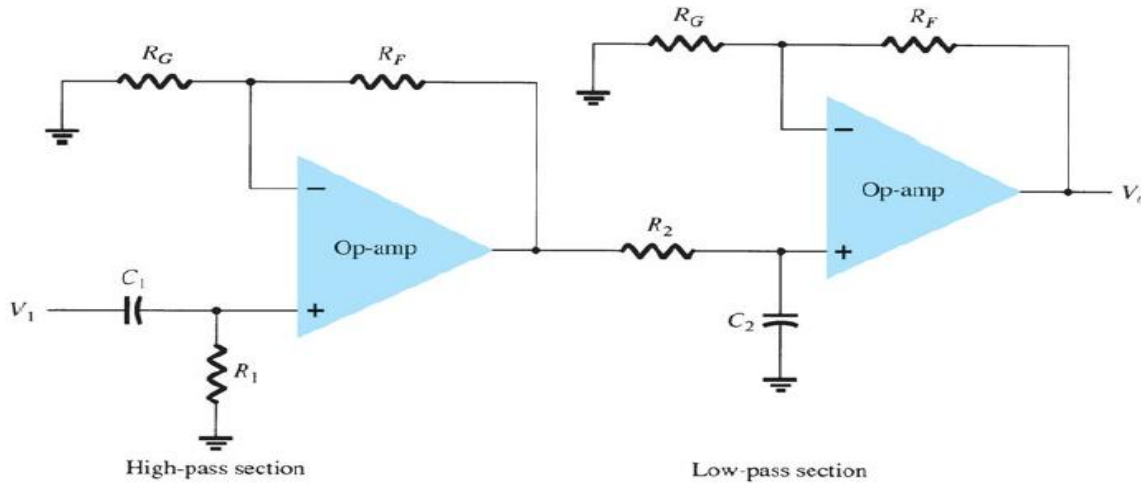


Fig 6-16

Solution:

$$f_{OL} = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi(10 \times 10^3)(0.1 \times 10^{-6})} = 159.15 \text{ Hz}$$

$$f_{OH} = \frac{1}{2\pi R_2 C_2} = \frac{1}{2\pi(10 \times 10^3)(0.002 \times 10^{-6})} = 7.96 \text{ kHz}$$

