

Ministry of Higher Education and Scientific Research

Al-Mustaqbal University

College of Engineering Technology

Medical Instrumentation Techniques Engineering Department

Electrical Machines

Second Class



# Lectures 12 and 13

## DC Motor: EMF & Torque equation



**By Prof. Dr Bayan M Sabbar**

### **1. Motor Principle**

An electrical motor operates based on the fundamental principle of converting electrical energy into mechanical motion through the interaction of magnetic field.

In the motor's construction there are stationary magnetic field is established in the stator, which can be created using permanent magnets or electromagnets. This magnetic field serves as a foundation for the motors functionally.

Within the motor there is a component that known as rotor that is capable to rotate and it consist wires. When an electric current is flow through this coil so it transforms into an electromagnet. The key intersection occurs as the magnetic fields of the stator and the rotor coil interact, that leads to generate the Lorentz force and this force is crucial in initiating the rotational motion of the rotor.

The rotational motion is sustained by the continuous generation of electromotive force (EMF) in the coil as it rotates. This induced EMF ensures that the current flow persists and maintaining the rotational momentum of the motor.

As further the working of the electrical motor has the ability of the motor convert electrical energy into mechanical motion makes it a versatile and widely used components in various applications.

When the terminals of the motor are connected to an external source of D.C. supply:

- (i) The field magnets are excited developing alternate N and S poles;
- (ii) The armature conductors carry currents. All conductors under N-pole carry currents in one direction while all the conductors under S-pole carry currents in the opposite direction.

Since each armature conductor is carrying current and is placed in the magnetic field, mechanical force acts on it. It is clear that force on each conductor is tending to rotate the armature in anticlockwise direction. All these forces add together to produce a driving torque which sets the armature rotating.

When the conductor moves from one side of a brush to the other, the current in that conductor is reversed and at the same time it comes under the influence of next pole which is of opposite polarity. Consequently, the direction of force on the conductor remains the same.

## 2. Back E.M.F. and Voltage Equation of D.C. Motor

When the armature of a d.c. motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence e.m.f. is induced in them as in a generator. The induced e.m.f. acts in opposite direction to the applied voltage  $V$  (Lenz's law) and is known as back or counter e.m.f. ( $E_b$ ). The back EMF is always less than  $V$ , although this difference is small when the motor is running under normal conditions.

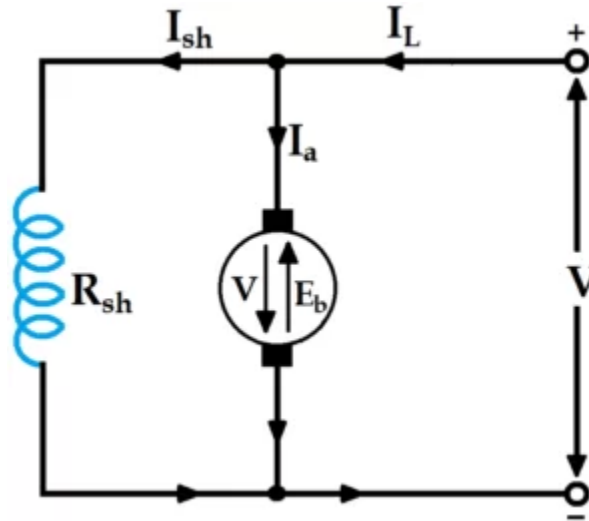


Fig. 1

$$E_b = \frac{Z P \phi N}{A 60}, N \text{ in rpm} \quad (1)$$

Consider the shunt wound motor shown in the figure above. When DC voltage  $V$  is applied across the motor terminals, the field magnets are excited and current is supplied to the armature conductors. Hence, the driving torque acts on the armature which starts rotating.

As the armature rotates, a back emf  $E_b$  is induced which opposes the applied voltage  $V$ . The applied voltage  $V$  is to force the current through the armature against the back emf  $E_b$ .

The electric work done in overcoming and causing the current to flow against  $E_b$  is converted into mechanical energy developed in the armature. Therefore, it follows that energy conversion in a DC motor is possible only due to the production of back emf  $E_b$ .

Net voltage in armature circuit =  $V - E_b$

If  $R_a$  is the armature circuit resistance, then,

$$I_a = \frac{V - E_b}{R_a} \quad (2)$$

$$V = I_a R_a + E_b \quad (3)$$

This is known as voltage equation of the D.C. motor. Now, multiplying both sides by  $I_a$ , we get:

$$VI_a = I_a^2 R_a + E_b I_a \quad (4)$$

$VI_a$  = Electrical input to the armature

$E_b I_a$  = Electrical equivalent of mechanical power developed in the armature

$I_a^2 R_a$  = Cu loss in the armature

### 3. Condition for Maximum Power

The gross mechanical power developed by a motor is

$$P_m = VI_a - I_a^2 R_a \quad (5)$$

Differentiating both sides with respect to  $I_a$  and equating the result to zero, we get

$$\frac{dP_m}{dI_a} = V - 2I_a R_a = 0$$

$$V = 2I_a R_a \quad (6)$$

Thus, gross mechanical power developed by a motor is maximum when back e.m.f. is equal to half the applied voltage. This condition is, however, not realized in practice, because in that case current would be much beyond the normal current of the motor. Moreover, half the input would be wasted in the form of heat and taking other losses (mechanical and magnetic) into consideration, the motor efficiency will be well below 50 percent.

**Example 1:** A 440-V, shunt motor has armature resistance of  $0.8 \, \Omega$  and field resistance of  $200 \, \Omega$ . Determine the back e.m.f. when giving an output of 7.46 kW at 85 % efficiency.

### **Solution**

$$\text{Motor input power} = 7.46 \times 10^3 / 0.85 = \mathbf{8776 \, W}$$

$$\text{Motor input current} = 8776 / 440 = \mathbf{19.95 \, A}$$

$$I_{sh} = 440 / 200 = \mathbf{2.2 \, A}$$

$$I_a = 19.95 - 2.2 = \mathbf{17.75 \, A}$$

$$E_b = V - I_a R_a = 440 - (17.75 \times 0.8) = \mathbf{425.8 \, V}$$

**Example 2:** A 4 pole, 32 conductor, lap-wound d.c. shunt generator with terminal voltage of 200 volts delivering 12 amps to the load has  $R_a = 2 \, \Omega$  and field circuit resistance of  $200 \, \Omega$ . It is driven at 1000 r.p.m. Calculate the flux per pole in the machine. If the machine has to be run as a motor with the same terminal voltage and drawing 5 A from the mains, maintaining the same magnetic field, find the speed of the machine.

### **Solution**

As a generator,

$$I_a = 13 \, A$$

$$E_g = 200 + (13 \times 2) = \mathbf{226 \, V}$$

$$\frac{Z P \phi N}{A 60} = 226 \text{ V} \Rightarrow \phi = \frac{226 \times 60}{1000 \times 32} = \mathbf{0.42375 \text{ wb}}$$

As a motor,  $I_a = 5 - 1 = 4 \text{ amp.}$

$$I_a = 4 \text{ A}$$

$$E_b = 200 - (4 \times 2) = 192$$

$$E_b = \frac{Z P \phi N}{A 60} \Rightarrow N = \frac{60 \times 192}{0.42375 \times 32} = 850 \text{ r.p.m.}$$

#### 4. Torque

By the term torque is meant the turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius at which this force acts. Consider a pulley of radius  $r$  meter acted upon by a circumferential force of  $F$  Newton which causes it to rotate at  $N$  r.p.m.

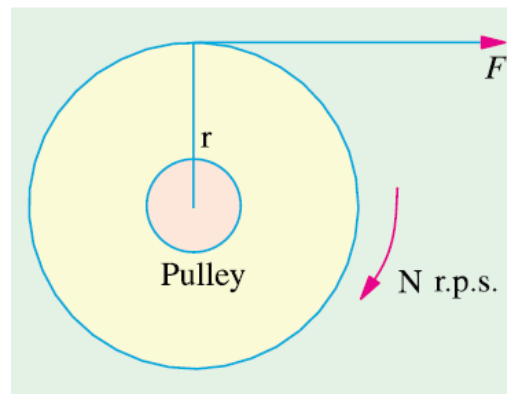


Fig. 2

Then torque

$$\mathbf{T = F \times r} \quad \text{Newton-meter} \quad (7)$$

Work done by this force in one revolution = Force  $\times$  distance =  $F \times 2\pi r$  Joule

$$\mathbf{\text{Power developed} = F \times 2 \pi r \times N/60} \quad \text{Joule/second or Watt} \quad (8)$$

or

$$\text{Power developed} = \frac{T \times 2\pi \times N}{60} = \frac{NT}{9.55} \quad \text{Joule/second or Watt} \quad (9)$$

#### 4.1 Armature Torque of a Motor

Let  $T_a$  be the torque developed by the armature of a motor running at  $N$  r.p.m. The power developed  $= T_a \times 2\pi \times N/60$  watt

We also know that electrical power converted into mechanical power in the armature  $= E_b I_a$  then

$$E_b I_a = T_a \times 2\pi \times N/60$$

But  $E_b = \frac{Z P \phi N}{A 60}$  then we have,

$$\frac{Z P \phi N}{A 60} I_a = \frac{T_a \times 2\pi \times N}{60}$$

$$T_a = \frac{0.159 \times I_a Z P \phi}{A} \quad \text{N-m} \quad (10)$$

Or

$$T_a = \frac{9.55 \times E_b I_a}{N} \quad \text{N-m} \quad (11)$$

#### 4.2 Shaft Torque ( $T_{sh}$ )

The whole of the armature torque, as calculated above, is not available for doing useful work, because a certain percentage of it is required for supplying iron and friction losses in the motor. The torque which is available for doing useful work is known as shaft torque  $T_{sh}$ . It is so called because it is available at the shaft. The motor output is given by:



$$\text{Output} = (T_{sh} \times 2\pi N) / 60 \quad \text{Watt} \quad (12)$$

Provided  $T_{sh}$  is in N-m and  $aN$  in r.p.m.

$$T_{sh} = \frac{9.55 \times \text{Output}}{N} \quad \text{N-m} \quad (13)$$

The difference ( $T_a - T_{sh}$ ) is known as lost torque and is due to iron and friction losses of the motor.

**Example 3:** Determine developed torque and shaft torque of 220-V, 4-pole series motor with 800 conductors' wave-connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mWb and its armature circuit resistance is 0.6  $\Omega$ .

### Solution

Developed torque or gross torque is the same thing as armature torque.

$$T_a = \frac{0.159 \times I_a Z P \phi}{A} = \frac{0.159 \times 45 \times 800 \times 4 \times 25 \times 10^{-3}}{2} = \mathbf{286.2 \text{ N.m}}$$

$$E_b = V - I_a R = 220 - 45 \times 0.6 = \mathbf{193 \text{ V}}$$

$$E_b = \frac{Z P \phi N}{A 60} \Rightarrow 193 = \frac{800 \times 4 \times 25 \times 10^{-3} \times N}{2 \times 60}$$

$$N = \mathbf{289.5 \text{ r.p.m.}}$$

$$\text{Output} = (T_{sh} \times 2\pi N) / 60 \Rightarrow T_{sh} = \frac{8200 \times 60}{2\pi \times 289.5} = \mathbf{270.5 \text{ N.m}}$$

**Example 4:** A 4-pole, 240 V, wave connected shunt motor gives 11.19 kW when running at 1000 r.p.m. and drawing armature and field currents of 50 A and 1.0 A respectively. It has 540 conductors. Its resistance is 0.1  $\Omega$ . Assuming a drop of 1 volt per brush, find (a) total torque (b) useful torque (c) useful flux / pole (d) Armature input, losses and rotational losses (e) efficiency.

### Solution

$$E_b = V - I_a R_a - \text{brush drop} = 240 - (50 \times 0.1) - 2 = \mathbf{233 \text{ V}}$$

$$(a) T_a = 9.55 \times E_b I_a / N = \mathbf{111 \text{ N.m}}$$

$$(b) T_{sh} = 9.55 \times \text{output} / N = \mathbf{106.9 \text{ N.m}}$$

$$(c) E_b = \frac{Z P \phi N}{A 60}$$

$$233 = \frac{540 \times 4 \times \phi \times 1000}{2 \times 60} \Rightarrow \phi = \mathbf{12.9 \text{ mWb}}$$

$$(d) \text{Armature input} = V I_a = 240 \times 50 = \mathbf{12000 \text{ W}}$$

$$\text{Armature Cu loss} = I_a^2 R_a = 50^2 \times 0.1 = \mathbf{250 \text{ W}}$$

$$\text{Brush contact loss} = 50 \times 2 = \mathbf{100 \text{ W}}$$

$$\text{Power developed} = 12000 - 250 - 100 = \mathbf{11650 \text{ W}}$$

$$\text{Output} = 11190 \text{ W}$$

$$\text{Rotational losses} = 11650 - 11190 = \mathbf{460 \text{ W}}$$

$$(e) \text{motor input} = V I = 240 \times 51 = \mathbf{12340 \text{ W}}$$

$$\text{Efficiency} = \frac{\text{motor output}}{\text{motor input}} \times 100\% = \frac{11190}{12340} \times 100\% = \mathbf{91.4\%}$$

### 4.3 Speed of a D.C. Motor

From the voltage equation of a motor, we get:

$$E_b = V - I_a R_a \quad (14)$$

$$\frac{Z P \phi N}{A 60} = V - I_a R_a \quad (15)$$

$$N = \frac{V - I_a R_a}{Z P \phi} \times A 60 \quad (16)$$

Or

$$N = \frac{E_b A 60}{Z P \phi} = k \frac{E_b}{\phi} \quad (17)$$

It shows that speed is directly proportional to back e.m.f.

#### 4.3.1 For Series Motor

$N_1$  = Speed in the 1st case

$I_{a1}$  = armature current in the 1st case

$\phi_1$  = flux/pole in the first case

$N_2, I_{a2}, \phi_2$  = corresponding quantities in the 2nd case.

Then, using the above relation, we get

$$N_1 = k \frac{E_{b1}}{\phi_1}, \quad N_2 = k \frac{E_{b2}}{\phi_2}$$

Then

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} \quad (18)$$

Prior to saturation of magnetic poles,  $\phi \propto I_a$  then

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} \quad (19)$$

#### 4.3.2 For Shunt Motor

In this case the same equation applies as in Eq 18.

If  $\phi_2 = \phi_1$  then

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad (20)$$

**Example 5:** A d.c. series motor operates at 800 r.p.m. with a line current of 100 A from 230-V mains. Its armature circuit resistance is  $0.15 \Omega$  and its field resistance  $0.1 \Omega$ . Find the speed at which the motor runs at a line current of 25 A, assuming that the flux at this current is 45 per cent of the flux at 100 A.

#### Solution

$$\phi_2 = 0.45 \phi_1$$

$$E_{b1} = V - I_{a1}(R_a + R_{sh}) = 230 - 100(0.15 + 0.1) = \mathbf{205 \text{ V}}$$

$$E_{b2} = V - I_{a2}(R_a + R_{sh}) = 230 - 25(0.15 + 0.1) = \mathbf{223.75 \text{ V}}$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} \Rightarrow \frac{N_2}{800} = \frac{223.75}{205} \times \frac{\phi_1}{0.45\phi_1}$$

$$N_2 = \mathbf{1940 \text{ r. p. m}}$$