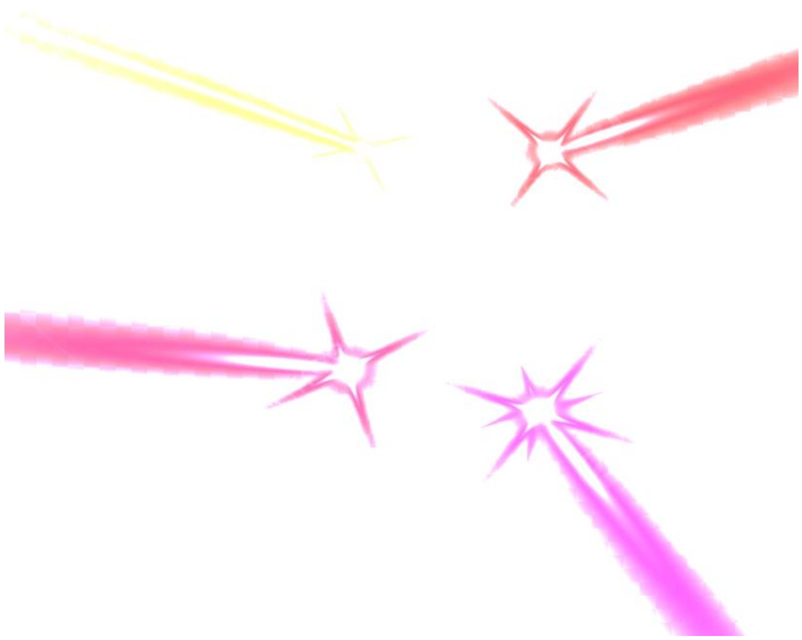
Al-Mustaqbal University / College of Engineering & Technology
Department of Medical Instrumentation Techniques Engineering
Class: 4th
Subject: Medical Laser Systems
Lecturer: MSc. Huda Wasfi Hassoon
1st term – Lecture No. 7 & Lecture Name: Optical Fiber - Part 2

Lecture 7
Optical Fiber – Part 2



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Optical Fiber's Numerical Aperture (NA)

Optical fibers guide light through the principle of **total internal reflection (TIR)**. However, not all incident light entering the fiber will propagate through the core. Only light rays entering within a specific angular range can be guided along the fiber. This angular range is known as the **acceptance cone**, and the parameter that defines this cone is called the **Numerical Aperture (NA)**.

The NA is a key property of an optical fiber because it determines how much light the fiber can collect, how efficiently it couples with light sources, and how many modes it can support.

Acceptance Cone and Acceptance Angle

When light from an external medium (usually air) strikes the entrance surface of the fiber, only rays entering within a certain half-angle—called the **acceptance angle**—will propagate by total internal reflection inside the core.

The acceptance cone is illustrated in the following figure:

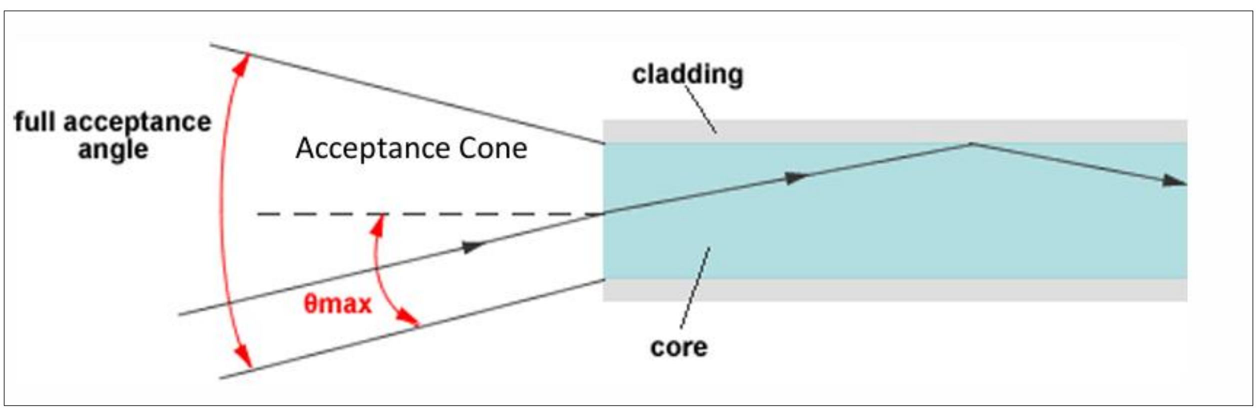

Fig. (1) The acceptance cone of optical fiber

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This cone represents all the possible angles for which light will be guided inside the fiber. The half-angle of this cone is the maximum acceptance angle $\theta_{\max}$.

Derivation of Numerical Aperture

Consider an optical fiber with:

- Core refractive index: n_1
- Cladding refractive index: n_2
- External medium refractive index: n_0 (equal to 1 for air)

A light ray enters the fiber at an incident angle $i$and refracts inside the core at an angle θ. The ray then strikes the core–cladding boundary at an internal angle θ'.

The geometry of this process is shown below:

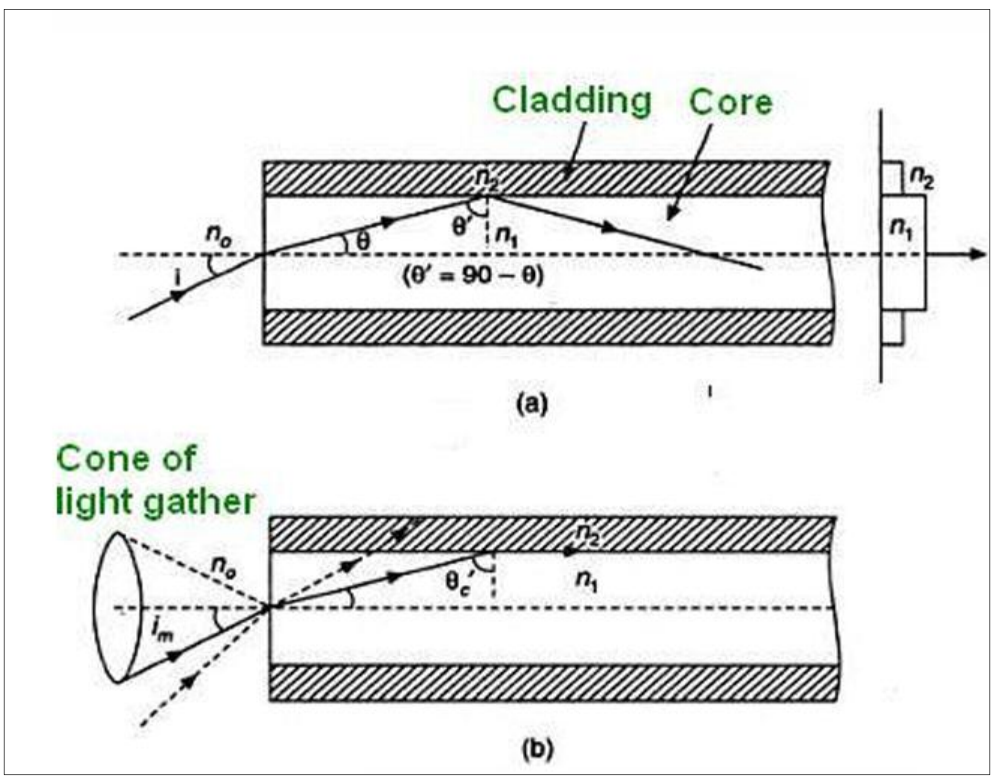

Fig. (2) Diagram showing incident angle, refracted angle, and core–cladding interaction.

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Snell's Law at the Fiber Entrance

$$n_0 \sin(i) = n_1 \sin(\theta)$$

Total Internal Reflection Requirement

At the core–cladding boundary, total internal reflection occurs if:

$$\theta' > \theta'_c$$

Where the critical angle is:

$$\theta'_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Maximum Acceptance Angle

By combining Snell's law and geometric relations, the maximum incidence angle $i = i_m$(or $\theta_{\max}$) that still allows TIR is:

$$n_0 \sin(i_m) = \sqrt{n_1^2 - n_2^2}$$

Thus, the **Numerical Aperture (NA)** is defined as:

$$\boxed{\mathrm{NA} = n_0 \sin(\theta_{\max}) = \sqrt{n_1^2 - n_2^2}}$$

For air ($n_0 = 1$):

$$\boxed{\mathrm{NA} = \sin(\theta_{\max})}$$

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Meaning and Importance of Numerical Aperture

The NA expresses the **light-gathering ability** of the fiber:

Higher NA

- Larger acceptance cone
- More light enters the fiber
- Easier coupling with LEDs and lasers
- Supports more propagation modes

Lower NA

- Smaller acceptance cone
- Less light enters the fiber
- Used in single-mode fibers
- Lower dispersion and higher performance in long-distance communication

Thus, NA controls both the **efficiency of light coupling** and the **propagation characteristics** of the fiber.

Spot Size at a Distance

When light emerges from the fiber into free space, the beam expands. If a detector or screen is placed at a distance **d** from the fiber end, the radius **r** of the resulting circular spot is:

$$r = d \tan(\theta_a)$$

This relationship is useful for determining detector size and fiber-to-detector alignment.

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Number of Modes in a Step-Index Multimode Fiber

In multimode step-index fibers, many different modes can propagate simultaneously. The approximate number of modes supported by the fiber depends on:

- Core diameter D
- Numerical Aperture NA
- Operating wavelength λ

The approximate formula for the number of guided modes is:

$$N_m \approx \frac{(\pi D \cdot \mathrm{NA})^2}{4\lambda^2}$$

This formula is valid only for **multimode step-index fibers**, and becomes inaccurate when the fiber supports only a few modes.

The V-Number (Cutoff Parameter)

The **V-Number** or **normalized frequency parameter** determines whether a fiber is single-mode or multimode. It is defined as:

$$V = \frac{2\pi a}{\lambda} \cdot \mathrm{NA}$$

Where:

- a: core radius
- λ: wavelength
- NA: numerical aperture

Mode Condition

- If $V < 2.405 \rightarrow$ the fiber supports **only one mode** (single-mode fiber)
- If $V > 2.405 \rightarrow$ the fiber becomes **multimode**

Example 1: Compute the NA, acceptance angle, and the critical angle of an optical fiber from the following data.

Core refractive index: $n_1 = 1.55$

Cladding refractive index: $n_2 = 1.50$

1. **Numerical Aperture**

$$\mathrm{NA} = \sqrt{n_1^2 - n_2^2}$$

$$= \sqrt{1.55^2 - 1.50^2} = 0.3905$$

2. **Acceptance Angle**

$$\theta_a = \sin^{-1}(0.3905) = 23.2°$$

3. **Critical Angle**

$$\theta_c = \sin^{-1}(\frac{n_2}{n_1}) = \sin^{-1}(\frac{1.50}{1.55}) = 75.4°$$

Example 2: Compute the cut-off parameter and the number of modes supported by a fiber, Given:

- Core refractive index: $n_1 = 1.54$
- Cladding refractive index: $n_2 = 1.50$
- Core radius: $a = 25\ \mu m$

- Wavelength: $\lambda = 1300\ nm$

1. **Numerical Aperture**

$$\text{NA} = \sqrt{n_1^2 - n_2^2}$$

$$= \sqrt{1.54^2 - 1.50^2} = 0.349$$

2. **V-Number**

$$V = \frac{2\pi a}{\lambda} \cdot \text{NA}$$

$$V = \frac{2\pi(25 \times 10^{-6})}{1300 \times 10^{-9}} \cdot 0.349 = 42.15$$

3. **Number of Modes**

$$N_m \approx \frac{V^2}{2} \approx \frac{42.15^2}{2} = 888$$

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