CONTENTS

LEARNING OBJECTIVES

After completing this lecture, students should be able to:

1. **Define** transmission lines and explain their role in power distribution and communication systems.
2. **Identify** common types of transmission lines such as coaxial cables, microstrip lines, and two-wire lines.
3. **Describe** the primary transmission line parameters: resistance (R), inductance (L), conductance (G), and capacitance (C).
4. **Derive and interpret** the transmission line equations using distributed element models.
5. **Explain** wave propagation on transmission lines and distinguish between forward and backward traveling waves.
6. **Calculate** the characteristic impedance and propagation constant of a line.

7. **Differentiate** between lossless and distortionless transmission lines and apply their conditions.

8. **Compute** input impedance, standing wave ratio (SWR), and power flow on a transmission line.

9. **Analyze** transmission line performance under various load conditions.

1 INTRODUCTION

Transmission lines play a fundamental role in modern electrical and communication systems. They are used to transfer electromagnetic energy from a source to a load efficiently, whether in low-frequency power networks or high-frequency communication circuits. Common examples include coaxial cables used in television and networking, microstrip lines in integrated circuits, and two-wire lines used in various electronic systems.

A transmission line consists of two or more parallel conductors that guide electromagnetic waves. When the physical length of the line becomes comparable to the wavelength of the signal, the line exhibits distributed effects, making conventional circuit theory insufficient. In such cases, transmission line theory must be applied to understand voltage and current behavior along the line.

This lecture introduces the essential parameters that characterize transmission lines, derives the governing wave equations, and explains concepts such as characteristic impedance, wave propagation, and standing waves. Understanding these principles is crucial for analyzing power delivery, signal integrity, and high-frequency system performance.

2 TRANSMISSION LINES

Transmission lines are commonly used in power distribution (at low frequencies) and in communications (at high frequencies). Transmission lines such as twisted-pair and coaxial cables are used in computer networks such as the Ethernet and the Internet.

A transmission line basically consists of two or more parallel conductors used to connect a source to a load. The source may be a hydroelectric generator, a transmitter, or an oscillator; the load may be a factory, an antenna, or an oscilloscope. Typical transmission lines include coaxial cable, a two-wire line, a parallel-plate or planar line, a wire above the conducting plane, and a microstrip line. These lines are portrayed in Figure 1. Notice that each of these lines consists of two conductors in parallel. Coaxial cables are routinely used in electrical laboratories and in connecting TV sets to antennas. Microstrip lines (similar to that in Figure 1 e) are particularly important in integrated circuits, where metallic strips connecting electronic elements are deposited on dielectric substrates.

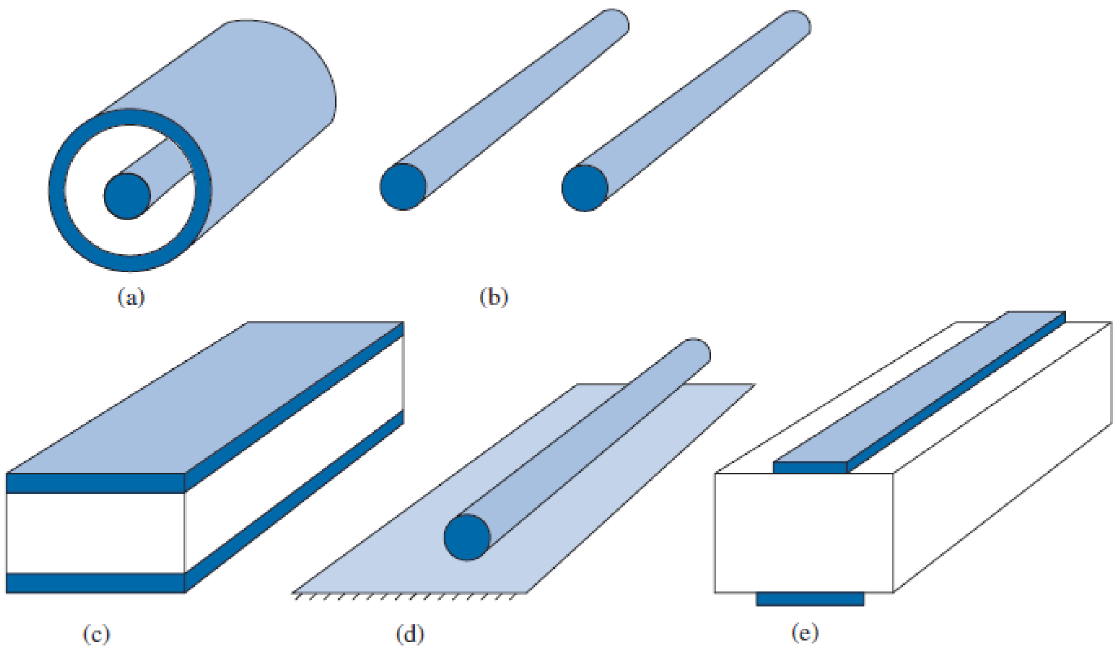


Figure 1: Typical transmission lines in cross-sectional view: (a) coaxial line, (b) two-wire line, (c) planar line, (d) wire above conducting plane, (e) microstrip line.

3 TRANSMISSION LINE PARAMETERS AND EQUATIONS

It is customary and convenient to describe a transmission line in terms of its line parameters, which are its resistance per unit length R, inductance per unit length L, conductance per unit length G, and capacitance per unit length C.

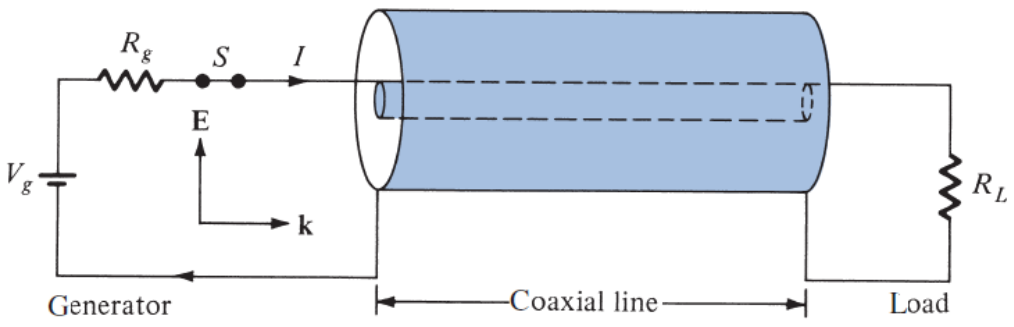


Figure 2: Coaxial line connecting the generator to the load.

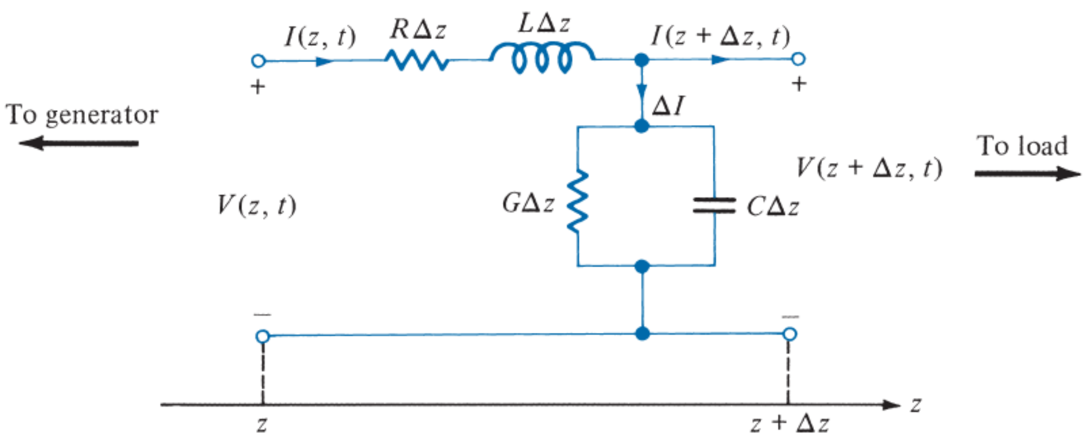


Figure 3: An L-type equivalent circuit model of a two-conductor transmission line of differential length Δz.

Let us examine an incremental portion of length Δz of a two-conductor transmission line. We intend to find an equivalent circuit for this line and derive the line equations. The model in Figure 3 is in terms of the line parameters R, L, G, and C, and may represent any of the two-conductor lines of Figure 3. The model is called the L-type equivalent circuit;

there are other possible types. In the model of Figure 3, we assume that the wave propagates along the $+z$ direction, from the generator to the load.

By applying Kirchhoff's voltage law to the outer loop of the circuit, we obtain

$$V(z,t) = R\Delta z I(z,t) + L\Delta z \frac{\partial I(z,t)}{\partial t} + V(z+\Delta z,t)$$

$$Or \qquad -\frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t} \tag{1}$$

Taking the limit of as $\Delta z \to 0$ leads to

$$-\frac{\partial V(z,t)}{\partial z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t} \tag{2}$$

Similarly, applying Kirchhoff's current law to the main node of the circuit gives:

$$-\frac{\partial I(z,t)}{\partial z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t} \tag{3}$$

If we assume harmonic time dependence so that:

$$\begin{aligned} V(z,t) &= Re\left[V_s(z)e^{j\omega t}\right] \\ I(z,t) &= Re\left[I_s(z)e^{j\omega t}\right] \end{aligned} \tag{4}$$

Where $V_s(z)$ and $I_s(z)$ are the phasor forms of $V(z,t)$ and $I(z,t)$, respectively, eqs. (3) and (4) become:

$$\begin{aligned} -\frac{dV_s}{dz} &= (R + j\omega L)I_s \\ -\frac{dI_s}{dz} &= (G + j\omega C)V_s \end{aligned} \tag{5}$$

The differential eqs. (5) are coupled. To separate them, we take the second derivative of V_s so that we obtain:

$$\frac{d^2V_s}{dz^2} = (R + j\omega L)(G + j\omega C)V_s \tag{6}$$

$$or$$

$$\frac{d^2V_s}{dz^2} - \gamma^2 V_s = 0$$

Where,

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (7)$$

By taking the second derivative of I_s we get a similar equation for I_s

$$\frac{d^2I_s}{dz^2} - \gamma^2 I_s = 0 \quad (8)$$

The solutions of the linear homogeneous differential equations (6) and (8) are namely,

$$\begin{aligned} V_s(z) =& V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z} \\ & \longrightarrow +z \quad -z \longleftarrow \end{aligned} \quad (9)$$

And

$$\begin{aligned} I_s(z) =& I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z} \\ & \longrightarrow +z \quad -z \longleftarrow \end{aligned} \quad (10)$$

Where V_o^+, V_o^-, I_o^+, and I_o^- are wave amplitudes; the + and - signs, respectively, denote waves traveling along $+z$ - and $-z$-directions, as is also indicated by the arrows. We obtain the instantaneous expression for voltage as:

$$\begin{aligned} V(z,t) &= Re\left[V_s(z)e^{j\omega t}\right] \\ &= V_o^+ e^{-\alpha z} cos\,(\omega t - \beta z) + V_o^- e^{\alpha z} cos\,(\omega t + \beta z) \end{aligned} \quad (11)$$

4 CHARACTERISTIC IMPEDANCE

The **characteristic impedance** Zo of the line is the ratio of the positively traveling voltage wave to the current wave at any point on the line.

The characteristic impedance Z_o is analogous to η, the intrinsic impedance of the medium of wave propagation.

$$Z_o = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-} = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C} \qquad (12)$$

Or

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_o + jX_o \qquad (13)$$

Where R_o and X_o are the real and imaginary parts of Z_o. Do not mistake R_o for R-while R is in ohms per meter, R_o is in ohms. The propagation constant γ and the characteristic impedance Z_o are important properties of the line because both depend on the line parameters R, L, G, and C and the frequency of operation. we may now consider two special cases: the lossless transmission line and the distortion less line:

4.1 LOSSLESS LINE $(R = 0 = G)$

For such a line:

$$\alpha = 0, \qquad \gamma = j\beta = j\omega\sqrt{LC}$$
$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda \qquad (14)$$
$$X_o = 0, \qquad Z_o = R_o = \sqrt{\frac{L}{C}}$$

4.2 DISTORTIONLESS LINE $(R/L = G/C)$

A distortion less line is one in which the attenuation constant α is frequency independent while the phase constant β is linearly dependent on frequency. A distortion less line results if the line parameters are such that. Thus, for a distortion less line,

$$\gamma = \sqrt{RG\left(1+\frac{j\omega L}{R}\right)\left(1+\frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG}\left(1+\frac{j\omega C}{G}\right) = \alpha + j\beta \tag{15}$$

or

$$\alpha = \sqrt{RG}, \qquad \beta = \omega\sqrt{LC}$$

Showing that α does not depend on frequency, whereas β is a linear function of frequency. Also:

$$Z_o = \sqrt{\frac{R(1+j\omega L/R)}{G(1+j\omega C/G)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} = R_o + jX_o \tag{16}$$

Or

$$R_o = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}, X_o = 0 \tag{17}$$

And

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda \tag{18}$$

EXAMPLE 1 : An lossless line has a characteristic impedance of 70Ω and a phase constant (β) of 3rad/m at 100 MHz. Calculate the inductance per meter and the capacitance per meter of the line.

Solution

$$R = 0 = G \text{ and } \alpha = 0$$

$$Z_o = R_o = \sqrt{\frac{L}{C}} = 70$$

$$\beta = \omega\sqrt{LC}$$

$$\frac{R_o}{\beta} = \frac{1}{\omega C}$$

$$C = \frac{\beta}{\omega R_o} = \frac{3}{2\pi \times 100 \times 10^6 (70)} = 68.2\text{pF/m}$$

$$L = R_o^2 C = (70)^2 (68.2 \times 10^{-12}) = 334.2\text{nH/m}$$

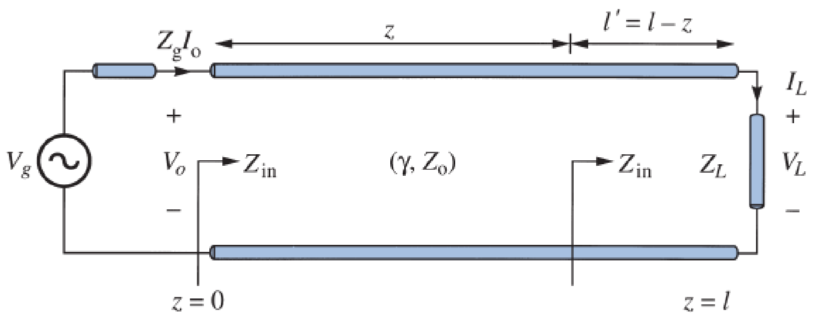
5 INPUT IMPEDANCE

Consider a transmission line of length ℓ, characterized by γ and Z_o, connected to a load Z_L as shown in Figure 4. Looking into the line, the generator sees the line with the load as an input impedance Z_{in} ..

Let the transmission line extend from $z = 0$ at the generator to $z = \ell$ at the load. First of all, we need the voltage and current waves:

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$$
$$I_s(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z} \quad (19)$$

To find V_o^+ and V_o^-, the terminal conditions must be given. For example, if we are given the conditions at the input, say



(a)

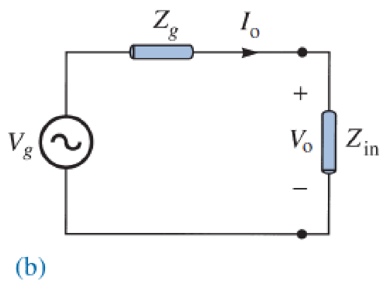


Figure 4: An L-type equivalent circuit model of a two-conductor transmission line of differential length Δz.

1. $V_o = V(z = 0), I_o = I(z = 0)$.

$$V_o^+ = \frac{1}{2}(V_o + Z_o I_o)$$
$$V_o^- = \frac{1}{2}(V_o - Z_o I_o) \tag{20}$$

If the input impedance at the input terminals is Z_{in} , the input voltage V_o and the input current I_o are easily obtained as:

$$V_o = \frac{Z_{in}}{Z_{in} + Z_g} V_g, \qquad I_o = \frac{V_g}{Z_{in} + Z_g} \tag{21}$$

and

$$Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{Z_o(V_o^+ + V_o^-)}{V_o^+ - V_o^-} \tag{22}$$

2. $V_L = V(z = \ell), I_L = I(z = \ell)$

$$V_o^+ = \frac{1}{2}(V_L + Z_o I_L)e^{\gamma\ell}$$
$$V_o^- = \frac{1}{2}(V_L - Z_o I_L)e^{-\gamma\ell} \tag{23}$$

And

$$Z_{in} = Z_o \left[\frac{Z_L + Z_o \tanh \gamma\ell}{Z_o + Z_L \tanh \gamma\ell}\right] \tag{24}$$

EXAMPLE 2 : A certain transmission line 2 m long operating at $\omega = 10^6 \text{rad/s}$ has $\alpha = 0.921 \text{ Np/m}$, $\beta = 1 \text{rad/m}$, and $Z_o = 60 + j40\Omega$. If the line is connected to a source of $10\angle 0° \text{V}$, $Z_g = 40\Omega$ and terminated by a load of $20 + j50\Omega$, determine

(a) The input impedance

(b) The sending-end current

(c) I_o and V_o.

Solution

(a)

$$\gamma = \alpha + j\beta = 0.921 + j1/\text{m}$$
$$\gamma\ell = 2(0.921 + j1) = 1.84 + j2$$

$$\tanh \gamma\ell = 1.033 - j0.03929$$

$$Z_{\text{in}} = Z_o\left(\frac{Z_L + Z_o \tanh \gamma\ell}{Z_o + Z_L \tanh \gamma\ell}\right)$$
$$= (60 + j40)\left[\frac{20 + j50 + (60 + j40)(1.033 - j0.03929)}{60 + j40 + (20 + j50)(1.033 - j0.03929)}\right]$$

$$Z_{\text{in}} = 60.25 + j38.79\Omega$$

(b) The sending-end current is $I(z = 0) = I_o$.

$$I(z = 0) = \frac{V_g}{Z_{\text{in}} + Z_g} = \frac{10}{60.25 + j38.79 + 40}$$
$$= 93.03/-21.15° \text{mA}$$

(c) To find the current at any point, we need V_o^+ and V_o^-. But

$$I_o = I(z = 0) = 93.03\angle -21.15° \text{mA}$$
$$V_o = Z_{\text{in}} I_o = (71.66\angle 32.77°)(0.09303\angle -21.15°) = 6.667\angle 11.62° \text{V}$$

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