



Steady-State Error

Type 0: $G(s) = \frac{K(T_a s + 1)(T_b s + 1) \cdots}{s^0(T_1 s + 1)(T_2 s + 1) \cdots} = \frac{K(1 + b_1 s + b_2 s^2 + \cdots)}{s^0(1 + a_1 s + a_2 s^2 + \cdots)}$

$$e_{ss} = e(\infty) = \lim_{s \rightarrow 0} \frac{sR(s)}{1 + G(s)}$$

- **Unit-Step Input:** $R(s) = 1/s$ $r(t) = 1$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{s(1/s)}{1 + G(s)} = \frac{1}{1 + G(0)}$$

- **Static Position Error Constant, K_p :** $K_p = G(0)$

$$e_{ss} = \frac{1}{1 + K_p} \quad \text{Constant Value}$$

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- **Unit-Ramp Input:** $R(s) = 1/s^2$ $r(t) = t$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{s(1/s^2)}{1 + G(s)} = \lim_{s \rightarrow 0} \frac{1}{s + sG(s)} = \frac{1}{\lim_{s \rightarrow 0} sG(s)}$$

- **Static Velocity Error Constant, K_v :** $K_v = \lim_{s \rightarrow 0} sG(s) = 0$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$$

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- **Unit-Parabolic Input (Acceleration Input):** $R(s) = 1/s^3$ $r(t) = \frac{1}{2}t^2$

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- **Static Acceleration Error Constant, K_a :** $K_a = \lim_{s \rightarrow 0} s^2 G(s) = 0$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$



Type 1: $G(s) = \frac{K(T_a s + 1)(T_b s + 1) \cdots}{s^1(T_1 s + 1)(T_2 s + 1) \cdots} = \frac{K(1 + b_1 s + b_2 s^2 + \cdots)}{s^1(1 + a_1 s + a_2 s^2 + \cdots)}$

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Type 2: $G(s) = \frac{K(T_a s + 1)(T_b s + 1) \cdots}{s^2(T_1 s + 1)(T_2 s + 1) \cdots} = \frac{K(1 + b_1 s + b_2 s^2 + \cdots)}{s^2(1 + a_1 s + a_2 s^2 + \cdots)}$

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- **Static Position Error Constant, K_p :** $K_p = G(0) = \infty$

$$e_{ss} = \frac{1}{1 + K_p} = \frac{1}{1 + \infty} = 0$$

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- **Static Acceleration Error Constant, K_a :** $K_a = \lim_{s \rightarrow 0} s^2 G(s) = K$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{K}$$



Steady-State Errors for Unity Feedback Systems

$$K_p = G(0) \quad R(s) = 1/s$$

$$K_v = \lim_{s \rightarrow 0} sG(s) \quad R(s) = 1/s^2$$

$$K_a = \lim_{s \rightarrow 0} s^2 G(s) \quad R(s) = 1/s^3$$

$$G(s) = \frac{K(T_a s + 1)(T_b s + 1) \dots}{s^q (T_1 s + 1)(T_2 s + 1) \dots}$$

	Step Input $R(s) = \frac{1}{s}$	Ramp Input $R(s) = \frac{1}{s^2}$	Acceleration Input $R(s) = \frac{1}{s^3}$
Type 0 System	$\frac{1}{1+K}$	∞	∞
Type 1 System	0	$\frac{1}{K}$	∞
Type 2 System	0	0	$\frac{1}{K}$

Example: What is the steady state error due to a unit step-input to a type 1 system?

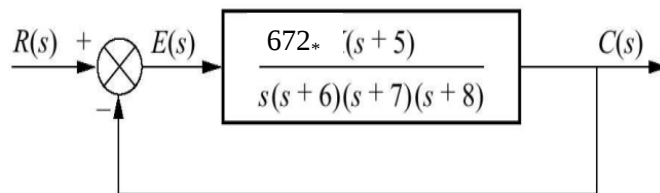
Ans.

$$\frac{1}{1+K_p}$$

Example: What is the type and order of the system with the open loop transfer function $1/s(1+s)$?

Ans. Type 1, second order.

Example: What is the steady state error ($e_{ss}(\infty)$) of the following system for a unit ramp input?



Solution:

$$K = \lim_{s \rightarrow 0} s \cdot G(s) = \frac{672 * (0 + 5)}{(0 + 6)(0 + 7)(0 + 8)} = \frac{3360}{336} = 10$$

$$e_{ss} = \frac{1}{K} = \frac{1}{10} = 0.1$$



Example: A unity feedback system with $G(s) = \frac{48(s+5)}{s(s+4)(s+2)(s+3)}$, For a unit ramp input, what is the steady state error?

Solution:

$$K = \lim_{s \rightarrow 0} s \cdot G(s) = \frac{48 * (0 + 5)}{(0 + 4)(0 + 2)(0 + 3)} = \frac{240}{24} = 10$$

$$e_{ss} = \frac{1}{K} = \frac{1}{10} = 0.1$$

Example: What is the step error coefficient of a system $G(s) = \frac{1}{(s+2)(s+3)}$ with unity feedback?

Solution: $K_p = \frac{1}{(0+2)(0+3)} = \frac{1}{6}$