Ministry of Higher Education and Scientific Research

Al-Mustaqbal University

College of Engineering Technologies

Medical Instrumentation Techniques Engineering Department

Electrical Circuits

First year

2.7 †Wye-Delta Transformations

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in Fig. 2.46. How do we combine resistors R_1 through R_6 when the resistors are neither in series nor in parallel? Many circuits of the type shown in Fig. 2.46 can be simplified by using three-terminal equivalent networks. These are

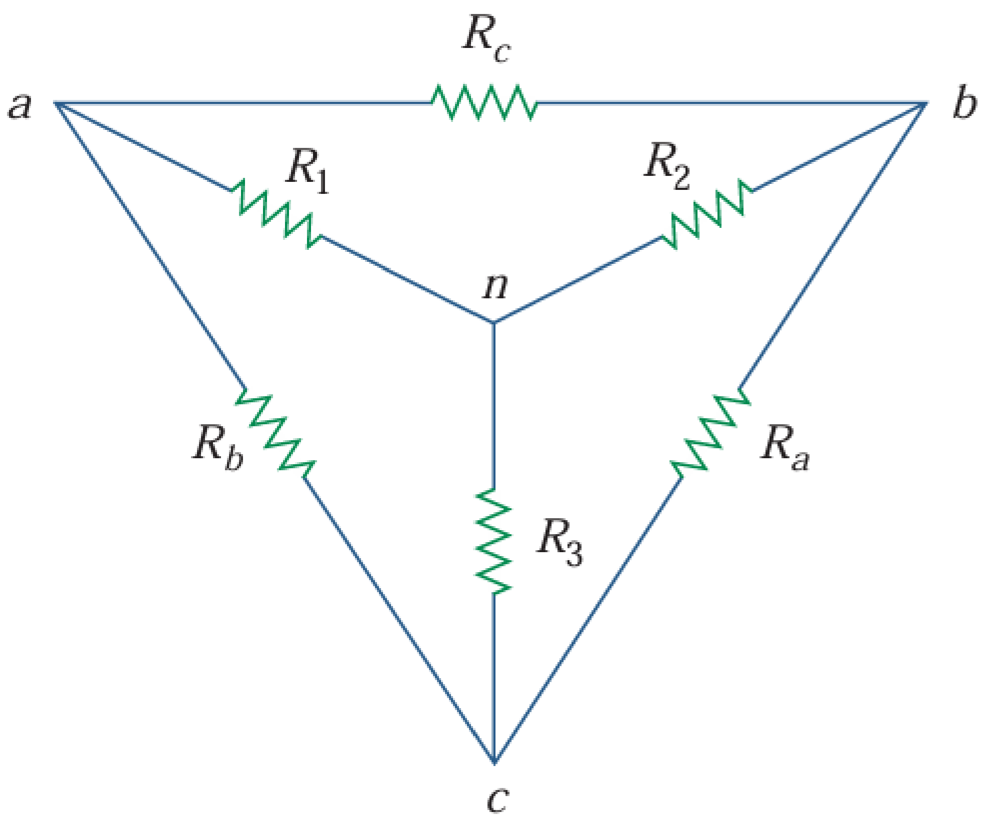


Figure 2.49
Superposition of Y and Δ networks as an aid in transforming one to the other.

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \tag{2.49}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \qquad (2.50)$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \qquad (2.51)$$

The above equations are used to convert from Delta to Star configurations.

Example:

Example 2.14

Convert the Δ network in Fig. 2.50(a) to an equivalent Y network.

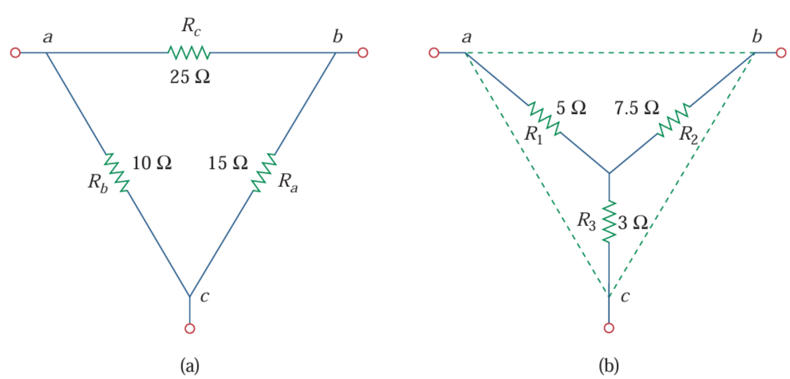


Figure 2.50
For Example 2.14: (a) original Δ network, (b) Y equivalent network.

Solution:

Using Eqs. (2.49) to (2.51), we obtain

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

Now, Star to Delta Conversion:

$$\boxed{R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}} \qquad \textbf{(2.53)}$$

$$\boxed{R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}} \qquad \textbf{(2.54)}$$

$$\boxed{R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}} \qquad \textbf{(2.55)}$$

Example:

We can use these equations on the above example to find R_a , R_b , and R_c . Note I will solve it on white board.

Example 2.15

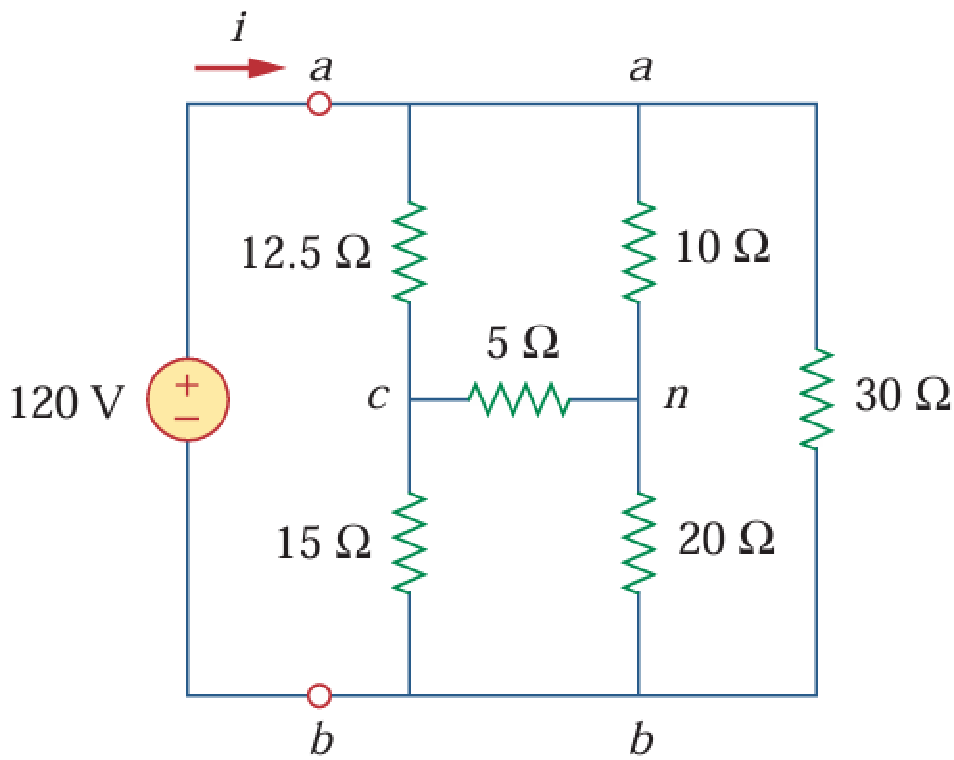


Figure 2.52
For Example 2.15.

In the above circuit in figure 2.52, find the current indicated in the circuit.

First of all, we select

$$R_1 = 10\ \Omega, \qquad R_2 = 20\ \Omega, \qquad R_3 = 5\ \Omega$$

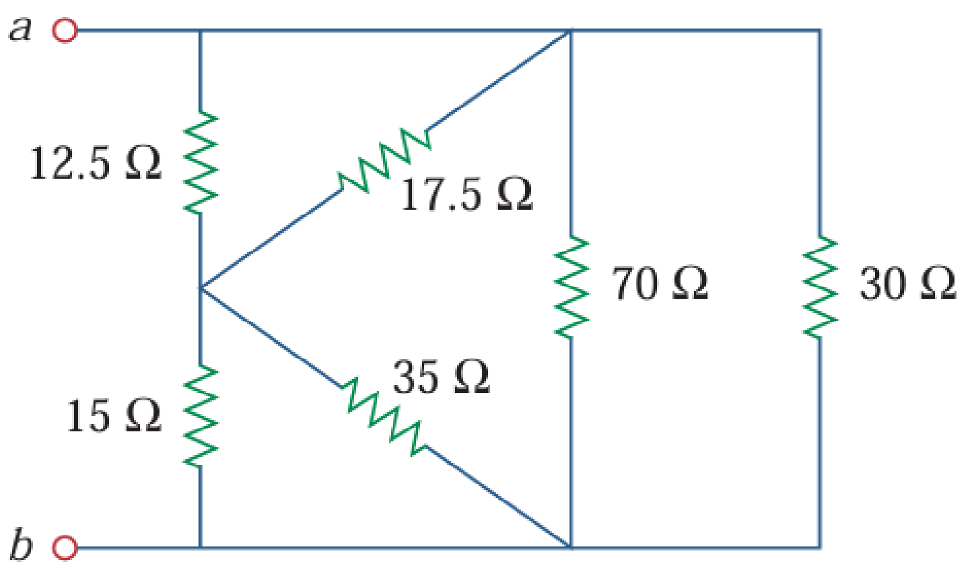
Thus from Eqs. (2.53) to (2.55) we have

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$

$$= \frac{350}{10} = 35\ \Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

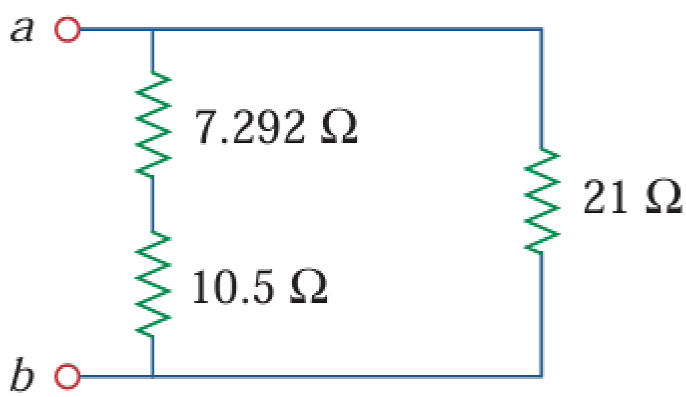
$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$



(a)

Figure 2.53

Then, simplify it to



(b)

Finally, we simplify it to the following relations

7.2.92 +10.5= 17.792 ohm which is parallel with 21 ohm and the resultant resistance R= 21*17.792/(21+17.792)=9.6317 ohm

Now i=v/R = 120/9.6317=12.4589 ampere

Home work: Find the current in the following circuit

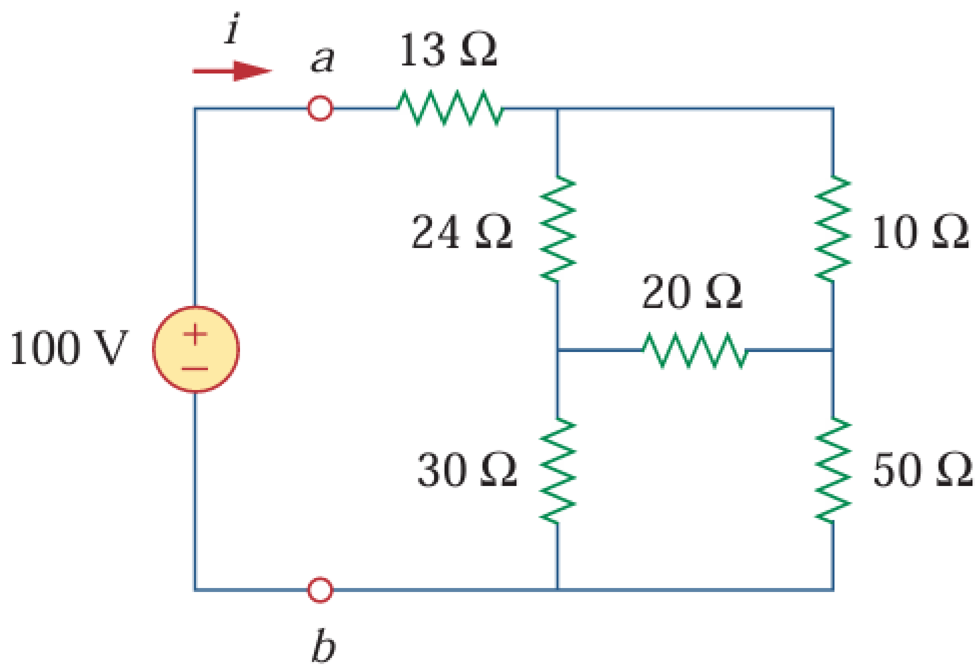


Figure 2.54
For Practice Prob. 2.15.