Al-Mustaqbal University - College of Engineering & Technology
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1st term / Lecture: Project Evaluation and Review Technique (PERT)

Project Planning Techniques

Project Evaluation and Review Technique (PERT)

Part 1

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Introduction

The project evaluation and review technique (PERT) is a network model that allows for randomness in activity completion times. PERT uses three-time estimates-optimistic, pessimistic and most likely, which help in establishing the probability of completing a project within a specified time and take the calculated risk before commencing a project. It has the potential to reduce both the time and cost required to complete a project.

تقنية تقييم ومراجعة المشروع (PERT) هو نموذج من المخططات الشبكية التي تلائم الأنشطة ذات الأوقات العشوائية (غير المؤكدة) كما في مشاريع البحث والتطوير.
تستخدم هذه التقنية ثلاث أوقات مخمنة لكل نشاط هي:

- الوقت المتفائل (optimistic)
- الوقت المتشائم (pessimistic)
- الوقت الأكثر احتمالا (most likely)

What are the different steps involved in PERT planning?

ماهي خطوات التخطيط بطريقة PERT ؟

PERT planning involves the following steps:

1. **Identify the specific activities and milestones.** 1. تحديد الأنشطة بدقة

2. **Determine the interdependencies and proper sequence of the activities.**
2. تحديد الاعتماديات والتسلسل المحتمل للأنشطة

3. **Construct a network diagram.** 3. انشاء المخطط الشبكي

4. **Estimate the time (three-time estimates, if probabilities are to be computed) required for each activity.** 4. تخمين الوقت المطلوب لكل نشاط (3 أوقات اذا كان مطلوب حساب الاحتمالية)

5. **Determine the *critical path*.** 5. تحديد المسار الحرج

6. **Update the PERT chart as the project progresses.** 6. تحديث مخطط PERT مع تقدم المشروع

Estimate activity times : تخمين أوقات النشاط

A distinguishing feature of PERT is its ability to deal with uncertainty in activity completion times. For each activity, the model usually includes three-time estimates:

ميزة طريقة PERT هي إمكانية التعامل مع الأوقات غير المؤكدة لاكتمال النشاط ، ولهذا تتضمن ثلاث ازمنة مخمنة لكل نشاط هي :

- ***Optimistic time (T_o)*** **– generally, the shortest time in which the activity can be completed.**
  - الوقت المتفائل (T_o) : هو اقل وقت يمكن إكمال النشاط به
- ***Most likely time (T_m)*** **- the completion time having the highest probability. This is different from expected time. Seasoned managers have an amazing way of estimating very close to actual data from prior estimation errors.**
  - الوقت الأكثر احتمالا (T_m) : هو الوقت ذو الاحتمالية الأكبر ان ينجز به النشاط وهو يختلف عن الوقت المخمن ولكنه الأقرب اليه
- ***Pessimistic time (T_p)*** **- the longest time that an activity might require.**
  - الوقت المشائم (T_p) :هو أطول وقت يمكن ان ينجز النشاط به

The expected time for each activity can be approximated using the following weighted average:

الوقت المخمن لكل نشاط يحسب من المعادلة التالية :

Expected time: $$T_e = \frac{(T_o + 4T_m + T_p)}{6}$$

This expected time might be displayed on the network diagram.
The standard deviation and variance for each activity are given by:

الانحراف المعياري (Sd) لوقت كل نشاط يحسب من المعادلة التالية :

$$Sd = (T_p - T_o) / 6$$

التباين (V) في وقت كل نشاط يحسب من المعادلة التالية :

$$V = \left[(T_p - T_o) / 6\right]^2 = (Sd)^2$$

(وقت المسار الحرج في شبكة PERT) = مجموع متوسطات أوقات (T_e) أنشطة المسار

$$(T_e)\,for\ critical\ path = (\sum T_e)\ for\ critical\ activities$$

الانحراف المعياري للمسار الحرج = جذر مجموع مربعات الانحرافات المعيارية لأنشطة المسار الحرج ويحسب من المعادلة :

$$(Sd)\ for\ critical\ Path = \sqrt{\sum (Sd)_{ij}^2}$$

OR

$$Sd = \sum V_{ij}$$

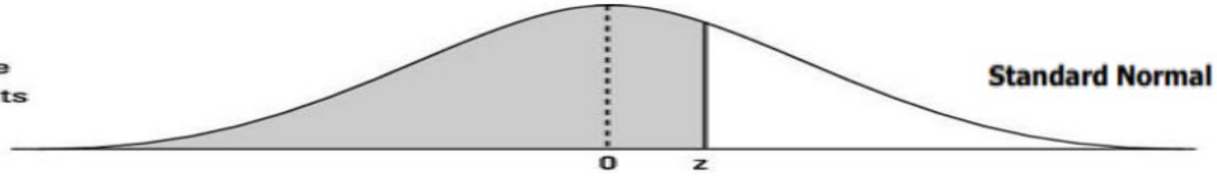
احتمالية تنفيذ المشروع بوقت اكبر او اقل من الوقت المحسوب (وقت المسار الحرج في المخطط الشبكي) تحسب من المعادلة :

$$Z = \frac{D - T_e}{\sqrt{Sd}}$$

اعتمادا على قيمة (Z) تستخرج الاحتمالية (P) من جدول الاحتمالات

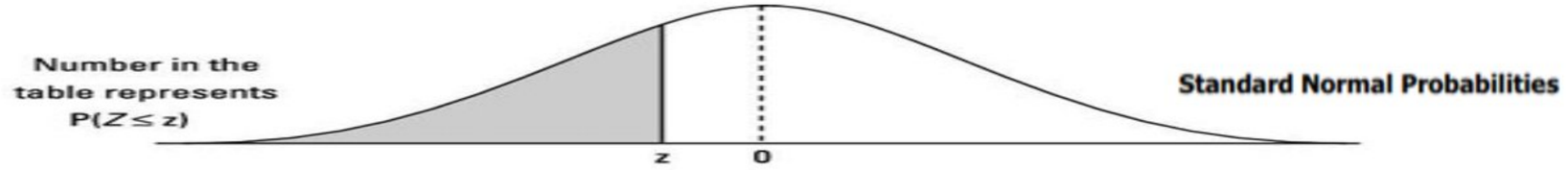
D = الوقت المطلوب حساب الاحتمالية فيه

Number in the table represents $P(Z \leq z)$



Standard Normal Probabilities

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999



Standard Normal Probabilities

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

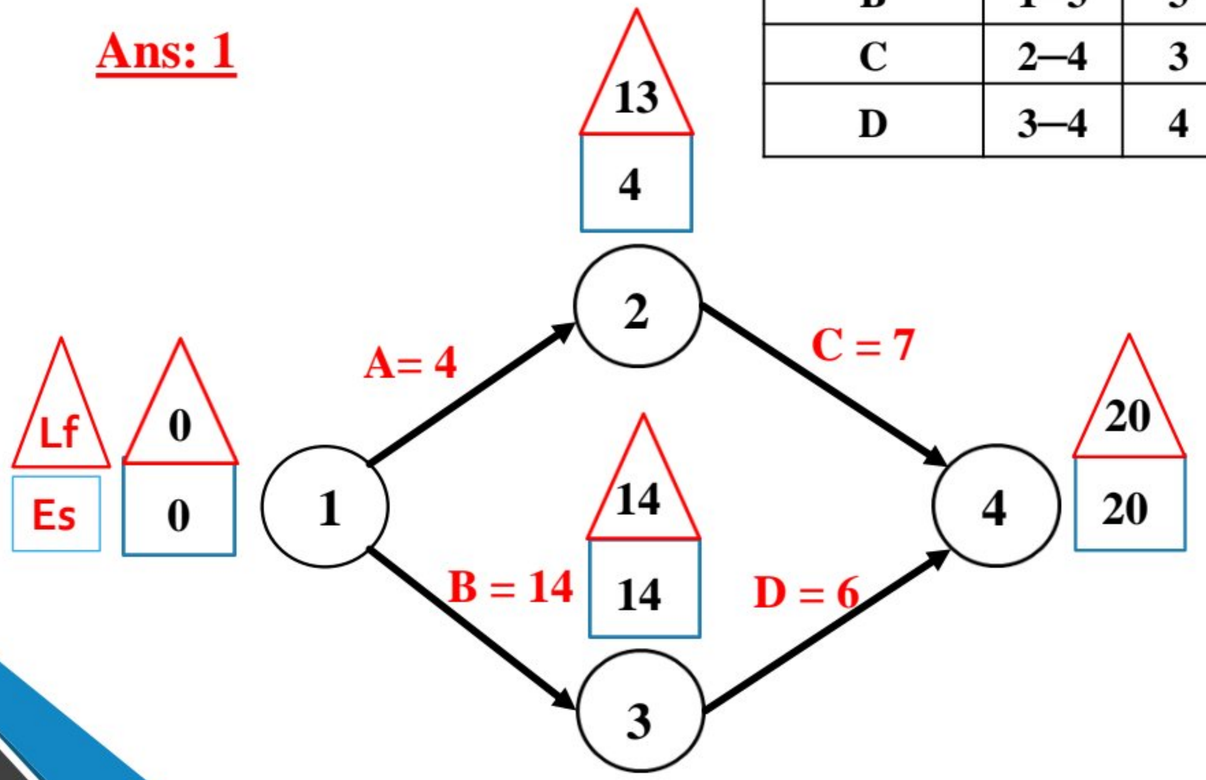
Ex 1: **Using PERT, construct the network diagram and determine:**

1- T_e, Sd, V, ES, LF *and* $C.P$

2- **The probability of completing the project on or before 23 weeks (using standard normal probabilities Table) , for the following Table.**

Activities	path	To	Tm	Tp	Te	Sd	V	ES	LF	C.P
A	1–2	2	4	6	4	0.6	0.36	0	13	
B	1–3	3	16	17	14	2.3	5.29	0	14	=
C	2–4	3	7	7	7	0.6	0.36	4	20	
D	3–4	4	6	8	6	0.6	0.36	14	20	=

Ans: 1



$$Expected\ Time\ (T_e) = \frac{(T_{o+4T_m+T_p})}{6}$$

$$Sd = (T_p - T_o) / 6$$

$$V = (Sd)^2$$

$CP = B + D = 14 + 6 = 20$ *time unit*

Ans:2

$$(Sd)_{\text{critical path}} = V_B + V_D = 5.29 + 0.36 = 5.65$$

$$Z = \frac{D - T_e}{\sqrt{Sd}} = \frac{23 - 20}{\sqrt{5.65}} = 1.26$$

From Probability table :

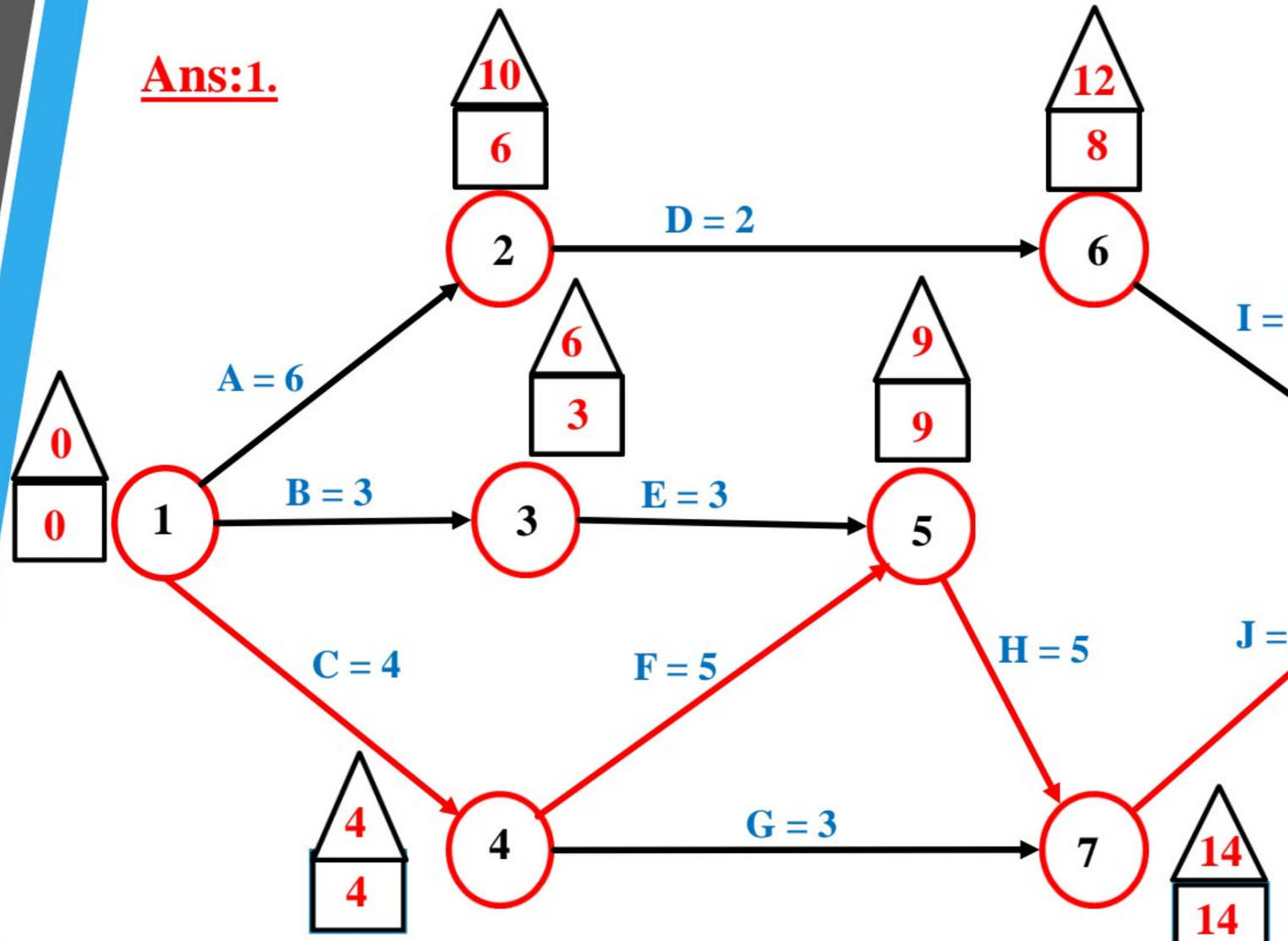
P[D ≤ 23] = 0.8962 = 89.62%

Ex 2: For the following project:

Activity	Preceded by	T_o	T_m	T_p	T_e	Sd	V	Es	Lf	CP
A	-	5	6	7	6	0.33	0.11	0	10	
B	-	1	3	5	3	0.66	0.44	0	6	
C	-	1	4	7	4	1	1	0	4	=
D	A	1	2	3	2	0.33	0.11	6	12	
E	B	1	2	9	3	1.33	1.78	3	9	
F	C	1	5	9	5	1.33	1.78	4	9	=
G	C	2	2	8	3	1	1	4	14	
H	E,F	4	4	10	5	1	1	9	14	=
I	D	2	5	8	5	1	1	8	17	
J	H,G	2	2	8	3	1	1	14	17	=

1. Construct the project network using PERT.
2. Find the expected time and variance for each activity.
3. Find the critical path and expected project completion time (ES and LF).
4. What the probability of completing the project on or before 22 weeks (using standard normal probabilities Table)

Ans:1.



Activity	Preceded by	T_o	T_m	T_p
A	-	5	6	7
B	-	1	3	5
C	-	1	4	7
D	A	1	2	3
E	B	1	2	9
F	C	1	5	9
G	C	2	2	8
H	E,F	4	4	10
I	D	2	5	8
J	H,G	2	2	8

2. In table.

3. C.P = C - F – H – J = 4 + 5 + 5 + 3 = 17 weeks

$$(T_e)_A = \frac{5+(4\times6)+7}{6} = \frac{36}{6} = 6$$

And so on ..

$$(Sd)_A = \frac{T_P - T_O}{6} = \frac{7-5}{6} = \frac{2}{6} = 0.33$$

$$V_A = (Sd)^2 = (0.33)^2 = 0.11$$

4.

$$(Sd)_{\text{critical path}} = \sum (V)_{\text{critical activities}} = 1 + 1.78 + 1 + 1 = 4.78$$

$$Z = \frac{D - T_e}{\sqrt{Sd}} = \frac{22 - 17}{\sqrt{4.78}} = 2.28$$

From Probability table :

$$P[D \leq 22] = 0.9887 = 98.87\%$$

Ex3 (H.W): For the following Project:

Activities	path	To	Tm	Tp	Te	Sd	V	ES	LF	C.P
A	1–2	2	5	14						
B	1–6	2	5	8						
C	2–3	5	11	29						
D	2–4	1	4	7						
E	3–5	5	11	17						
F	4–5	2	5	14						
G	6–7	3	9	27						
H	5–8	2	2	8						
I	7–8	7	13	31						

1. Construct the project network using PERT.
2. Find the expected time and variance for each activity.
3. Find the critical path and expected project completion time (ES and LF).
4. What the probability of completing the project on or before 38 weeks (using standard normal probabilities Table)

Ex4 (H.W): **Draw the network diagram of the following Project using PERT and determine $T_e, Sd, V, ES, LF, and\ C.P$. Find the probability P if D = 40 weeks.**

Activities	Prec. by	To	Tm	Tp	Te	Sd	V	ES	LF	C.P
A	--	6	9	12						
B	A	6	8	10						
C	--	2	5	8						
D	C	4	7	10						
E	D	4	6	8						
F	D	5	9	13						
G	F	2	3	4						
H	B, E	1	2	3						
I	G, H	2	3	4						
J	B	2	4	6						
K	J	3	4	5						
L	K, I	2	3	4						