CONTENTS

LEARNING OBJECTIVES

After completing this lecture, students should be able to:

1. Explain the concept of waves as functions of both space and time.
2. Describe the mathematical form of the one-dimensional scalar wave equation.
3. Identify key wave parameters such as **wavelength**, **frequency**, **period**, **phase**, and **propagation velocity**.
4. Determine the direction of propagation of electromagnetic (EM) plane waves using the sign of the phase term.
5. Analyze EM wave propagation in different media, including **free space**, **lossless dielectrics**, **lossy dielectrics**, and **good conductors**.
6. Solve Maxwell's equations to derive wave equations for electric and magnetic fields.

7. Distinguish between the **propagation constant**, **intrinsic impedance**, and their significance in wave behavior.

8. Compute wave attenuation, phase shift, and velocity in lossy dielectric media.

9. Describe the relationship between electric and magnetic field components in uniform plane waves.

10. Apply formulas to calculate field magnitudes, propagation constants, intrinsic impedances, and wave velocities in given media.

1 INTRODUCTION

Electromagnetic waves play a fundamental role in all branches of electrical engineering, particularly in communication systems, medical instrumentation, wireless technologies, and wave-based sensing devices. Understanding how these waves propagate through different media is essential for analyzing and designing modern engineering systems.

This lecture focuses on **plane wave propagation**, a fundamental concept derived from **Maxwell's equations**. By solving these equations under various conditions, we can describe how electromagnetic waves behave in free space, dielectrics, and conducting materials.

We begin by reviewing the general properties of waves, emphasizing how a wave depends simultaneously on **time** and **space**. Key parameters such as wavelength, frequency, phase, and propagation speed are introduced to build a strong foundation. We then explore how electromagnetic waves propagate in **lossy dielectrics**, which represent the most general case and form the basis for understanding simpler media such as free space or ideal dielectrics.

This introduction provides the necessary groundwork for analyzing electric and magnetic field components, identifying directions of propagation, and calculating the intrinsic impedance and propagation constant for different materials. The concepts discussed here are essential for understanding electromagnetic behavior in practical engineering applications.

2 WAVES IN GENERAL

A **wave** is a function of both space and time.

In one dimension, a scalar wave equation takes the form of

$$\frac{\partial^2 E}{\partial t^2} - u^2 \frac{\partial^2 E}{\partial z^2} = 0 \tag{1}$$

where u is the wave velocity, in which the medium is source free ($\boldsymbol{\rho}_v = 0, \mathbf{J} = \mathbf{0}$). Its solutions are of the form:

$$\begin{gathered} E^+ = f(z - ut) \\ E^- = g(z + ut) \\ or \\ E = f(z - ut) + g(z + ut) \end{gathered} \tag{2}$$

where f and g denote any function of $z - ut$ and $z + ut$, respectively. Examples of such functions include $z \pm ut, \sin\, k(z \pm ut), \cos\, k(z \pm ut)$, and $e^{jk(z \pm ut)}$, where k is a constant. It can easily be shown that these functions all satisfy.

For the moment let's take the imaginary part of this equation, we have:

$$E = A\sin\,(\omega t - \beta z) \tag{3}$$

This is a sine wave chosen for simplicity. Note the following characteristics of the wave:

1. It is time harmonic because we assumed time dependence of the form $e^{j\omega t}$.

2. The amplitude of the wave A has the same units as E.
3. The phase (in radians) of the wave depends on time t and space variable z, it is the term $(\omega t - \beta z)$.
4. The angular frequency ω is given in radians per second; β, the phase constant or wave number, is given in radians per meter.

Because E varies with both time t and the space variable z, we may plot E as a function of t by keeping z constant and vice versa.

2.1 PARAMETER OF WAVE EQUATION

The wave length λ: is the distance that the wave takes to repeat itself and measured in meters. Since the wave takes time T to repeat itself consequently T is known as the period, in seconds. Since it takes time T for the wave to travel distance λ at the speed u, we expect:

$$\lambda = uT \qquad (m) \tag{4}$$

But $T = 1/f$, where f is the frequency (the number of cycles per second) of the wave in hertz (Hz). Hence,

$$u = f\lambda \qquad (\frac{m}{s}) \tag{5}$$

Because of this fixed relationship between wavelength and frequency, one can identify the position of a radio station within its band by either the frequency or the wavelength. Usually, the frequency is preferred. Also, because

$$\begin{aligned} \omega &= 2\pi f \qquad (rad) \\ \beta &= \frac{\omega}{u} \qquad (\frac{rad}{m}) \\ \beta &= \frac{2\pi}{\lambda} \end{aligned} \tag{6}$$

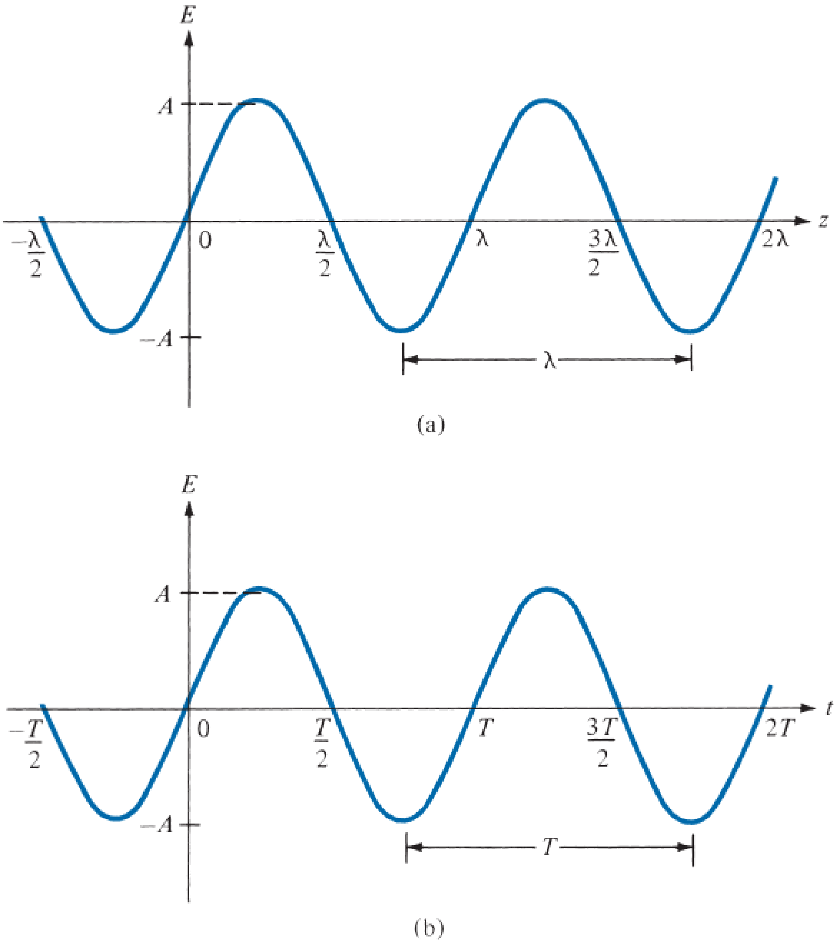

Figure 1: Plot of $E(z,t) = A\sin(\omega t - \beta z)$: (a) with constant t, (b) with constant z.

We will now show that the wave represented by $E = A\sin(\omega t - \beta z)$ is traveling with a velocity u in the $+z$-direction. To do this, we consider a fixed point P on the wave. We sketch the equation at times $t = 0, T/4$, and $T/2$ as in Figure 10.2. From the figure, it is evident that as the wave advances with time, point P moves along the $+z$-direction. Point P is a point of constant phase, therefore:

$$\omega t - \beta z = constant$$

$$or \qquad \frac{dz}{dt} = \frac{\omega}{\beta} = u \qquad (7)$$

Similarly, it can be shown that the wave $B\sin(\omega t+\beta z)$ is traveling with velocity u in the $-z$-direction. In summary, we note the following:

1. A wave is a function of both time and space.
2. Though time t = 0 is arbitrarily selected as a reference for the wave, a wave is
3. without beginning or end.
4. A negative sign in $(\omega t\pm\beta z)$ is associated with a wave propagating in the $+z$-direction (forward-traveling or positive-going wave), whereas a positive sign indicates that a wave is traveling in the $-z$-direction (backward-traveling or negative-going wave).

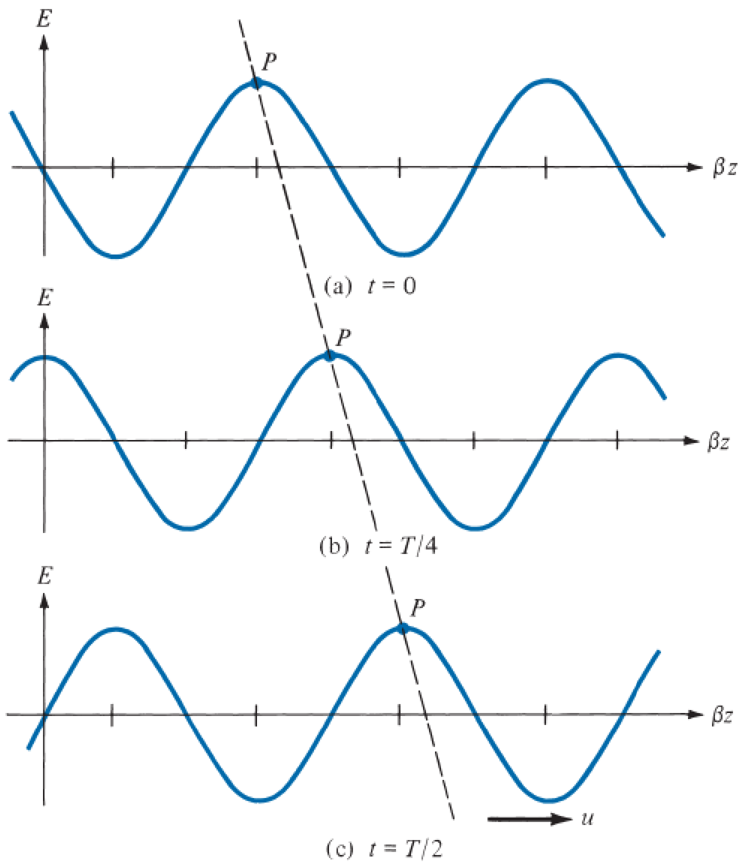

Figure 2: Plot of $E(z,t)=A\sin(\omega t-\beta z)$ at time (a) $t=0$, (b) $t=T/4$, (c) $t=T/2$; P moves in the $+z$-direction with velocity u.

5. Since $\sin(-\psi) = -\sin\psi = \sin(\psi \pm \pi)$, whereas $\cos(-\psi) = \cos\psi$,

$$\begin{aligned} \sin(\psi \pm \pi/2) &= \pm\cos\psi \\ \sin(\psi \pm \pi) &= -\sin\psi \\ \cos(\psi \pm \pi/2) &= \mp\sin\psi \\ \cos(\psi \pm \pi) &= -\cos\psi \end{aligned} \tag{8}$$

where $\psi = \omega t \pm \beta z$.

6. E and H are called uniform waves if they lie in a plane and are constant over such planes.

EXAMPLE 1 : An electric field in free space is given by

$$\mathbf{E} = 50\cos(10^8 t + \beta x)\mathbf{a}_y \text{ V/m}$$

(a) Find the direction of wave propagation.

(b) Calculate β and the time it takes to travel a distance of $\lambda/2$.

Solution

(a) From the positive sign in $(\omega t + \beta x)$, we infer that the wave is propagating along $-\mathbf{a}_x$.

(b) In free space, $u = c$:

$$\beta = \frac{\omega}{c} = \frac{10^8}{3 \times 10^8} = \frac{1}{3}$$

or

$$\beta = 0.3333\text{rad/m}$$

If T is the period of the wave, it takes T seconds to travel a distance λ at speed c. Hence to travel a distance of $\lambda/2$ will take

$$t_1 = \frac{T}{2} = \frac{1}{2}\frac{2\pi}{\omega} = \frac{\pi}{10^8} = 31.42 \text{ ns}$$

3 EM WAVES IN MEDIA

Our major goal is to solve Maxwell's equations and describe EM wave motion in the following media:

1. Free space ($\sigma = 0, \varepsilon = \varepsilon_o, \mu = \mu_o$).
2. Lossless dielectrics ($\sigma \simeq 0, \varepsilon = \varepsilon_r \varepsilon_o, \mu = \mu_r \mu_o$, or $\sigma \ll \omega\varepsilon$).
3. Lossy dielectrics ($\sigma \neq 0, \varepsilon = \varepsilon_r \varepsilon_0, \mu = \mu_r \mu_0$).
4. Good conductors ($\sigma \simeq \infty, \varepsilon = \varepsilon_o, \mu = \mu_r \mu_o$, or $\sigma \gg \omega\varepsilon$).

Where ω is the angular frequency of the wave. Case 3, for lossy dielectrics, is the most general case and will be considered first. Once this general case has been solved, we simply derive the other cases (1,2, and 4) from it as special cases by changing the values of σ, μ, and ε. However, before we consider wave motion in those different media, it is appropriate that we study the characteristics of waves in general. This is important for proper understanding of EM waves.

3.1 WAVE PROPAGATION IN LOSSY DIELECTRICS

A **lossy dielectric** is a medium in which an EM wave, as it propagates, loses power owing to imperfect dielectric.

Consider a linear, isotropic, homogeneous, lossy dielectric medium that is charge free (macroscopic $\boldsymbol{\rho}_v = 0$). Assuming and suppressing the time factor $e^{j\omega t}$, Maxwell's equations become:

$$\begin{aligned} \nabla \cdot \boldsymbol{E}_s &= 0 \\ \nabla \cdot \boldsymbol{H}_s &= 0 \\ \nabla \times \boldsymbol{E}_s &= -j\omega\mu \boldsymbol{H}_s \\ \nabla \times \boldsymbol{H}_s &= (\sigma + j\omega\varepsilon)\boldsymbol{E}_s \end{aligned} \tag{9}$$

Taking the curl of both sides of $\nabla \times \mathrm{E}_s$ gives:

$$\nabla \times \nabla \times \boldsymbol{E}_s = -j\omega\mu(\boldsymbol{\nabla} \times \boldsymbol{H}_s) \tag{10}$$

Applying the vector identity:

$$\nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A} \tag{11}$$

We obtain:

$$\nabla(\nabla \cdot \mathbf{E}_s) - \nabla^2 \mathbf{E}_s = -j\omega\mu(\sigma + j\omega\varepsilon)\mathbf{E}_s \tag{12}$$

Or

$$\nabla^2 \boldsymbol{E}_s - \gamma^2 \boldsymbol{E}_s = 0$$

Where,

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) \tag{13}$$

γ, in reciprocal meters, is called the **propagation constant** of the medium. By a similar procedure, it can be shown that for the **H** field,

$$\nabla^2 \boldsymbol{H}_s - \gamma^2 \boldsymbol{H}_s = 0$$

Where,

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) \tag{14}$$

Equations (13) and (14) are known as homogeneous vector Helmholtz's equations or simply vector wave equations. In Cartesian coordinates, is equivalent to three scalar wave equations, one for each component of **E** along $\mathbf{a}_x, \mathbf{a}_y$, and $\mathbf{a}_z$.

Since γ is a complex quantity, we may let $\gamma = \alpha + j\beta$. We obtain α and β by noting that:

$$Re(\gamma^2) = -(\beta^2 - \alpha^2) = -\omega^2\mu\varepsilon$$

and

$$|\gamma^2| = \beta^2 + \alpha^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2} \tag{15}$$

Then, α, β can be obtained as follows:

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1\right]}$$
$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1\right]} \quad (16)$$

Without loss of generality, if we assume that a wave propagates along $+\mathbf{a}_z$ and that $\mathbf{E}_s$ has only an x-component, then

$$\boldsymbol{E}_s = E_{xs}(z)\boldsymbol{a}_x \quad (17)$$

Without loss of generality, if we assume that a wave propagates in an unbounded medium along $\mathbf{a}_z$ and that $\mathbf{E}$ has only an x-component that does not vary with x and y, then:

$$\frac{\partial^2 E_{xs}(z)}{\partial x^2}+\frac{\partial^2 E_{xs}(z)}{\partial\partial y^2}+\frac{\partial^2 E_{xs}(z)}{\partial z^2}-\gamma^2 E_{xs}(z)=0$$

Or

$$\frac{\partial^2 E_{xs}(z)}{\partial z^2}-\gamma^2 E_{xs}(z)=0 \quad (18)$$

This is a scalar wave equation, a linear homogeneous differential equation, with solution:

$$E_{xs}(z) = E_o e^{-\gamma z} + E_o' e^{\gamma z} \quad (19)$$

Where E_o and E_o' are constants. The fact that the field must be finite at infinity requires that $E_o' = 0$. Alternatively, because $e^{\gamma z}$ denotes a wave traveling along $-\mathbf{a}_z$, whereas we assume wave propagation along $\mathbf{a}_z, E_o' = 0$. Then

$$E_{xs}(z) = E_o e^{-\gamma z} \quad (20)$$

Considering the time factor $e^{j\omega t}$, we obtain:

$$E(z,t) = Re\left[\boldsymbol{E}_s e^{j\omega t}\right] = Re\left[E_{xs}(z) e^{j\omega t} \boldsymbol{a}_x\right] = Re\left[E_o e^{-\alpha z} e^{j\omega t - j\beta z} \boldsymbol{a}_x\right] \quad (21)$$

Or

$$\boldsymbol{E}(z,t) = E_o e^{-\alpha z} cos\,(\omega t - \beta z) \boldsymbol{a}_x \quad (22)$$

Having obtained $\mathbf{E}(z,t)$, we obtain $\mathbf{H}(z,t)$ by taking similar steps. We will eventually arrive at:

$$\boldsymbol{H}(z,t) = Re\left(H_o e^{-\alpha z} e^{j(\omega t - \beta z)} \boldsymbol{a}_y\right) \quad (23)$$

Where;

$$H_o = \frac{E_o}{\eta} \quad (24)$$

And η is a complex quantity known as the *intrinsic impedance,* in ohms, of the medium.

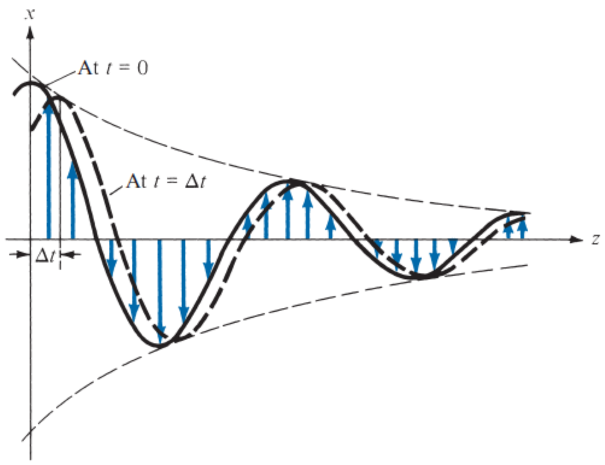

Figure 3: An E-field with an x-component traveling in the $+z$-direction at times $t = 0$ and $t = \Delta t$; arrows indicate instantaneous values of E.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta|\angle\theta_\eta = |\eta| e^{j\theta_\eta} \quad (25)$$

With;

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}, tan\ 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} \qquad (26)$$

Where $0 \leq \theta_\eta \leq 45°$. The Magnetic field equation:

$$\boldsymbol{H} = Re\left[\frac{E_o}{|\eta|e^{j\theta_\eta}} e^{-\alpha z} e^{j(\omega t-\beta z)} \boldsymbol{a}_y\right] \qquad (27)$$

Or

$$\boldsymbol{H} = \frac{E_o}{|\eta|} e^{-\alpha z} cos\left(\omega t - \beta z - \theta_\eta\right)\boldsymbol{a}_y \qquad (28)$$

EXAMPLE 2 : A lossy dielectric has an intrinsic impedance of $200\angle 30°\Omega$ at a particular radian frequency ω. If, at that frequency, the plane wave propagating through the dielectric has the magnetic field component

$$\mathbf{H} = 10e^{-\alpha x}\cos\left(\omega t - \frac{1}{2}x\right)\mathbf{a}_y \text{ A/m}$$

find **E** and α.

Solution

The given wave travels along $\mathbf{a}_x$ so that $\mathbf{a}_k = \mathbf{a}_x$; $\mathbf{a}_H = \mathbf{a}_y$, so

$$-\mathbf{a}_E = \mathbf{a}_k \times \mathbf{a}_H = \mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$$

or

$$\mathbf{a}_E = -\mathbf{a}_z$$

Also $H_o = 10$, so

$$\frac{E_o}{H_o} = \eta = 200\angle 30° = 200e^{j\pi/6} \rightarrow E_o = 2000e^{j\pi/6}$$

Except for the amplitude and phase difference, **E** and **H** always have the same form. Hence

$$\mathbf{E} = \text{Re}\left(2000e^{j\pi/6}e^{-\gamma x}e^{j\omega t}\mathbf{a}_E\right)$$

$$\mathbf{E} = -2e^{-\alpha x}\cos\left(\omega t - \frac{x}{2} + \frac{\pi}{6}\right)\mathbf{a}_z\text{kV/m}$$

Knowing that $\beta = 1/2$, we need to determine α. Since

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1\right]}$$

and

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1\right]}$$

$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1}{\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1}\right]^{1/2}$$

But $\frac{\sigma}{\omega\varepsilon} = \tan\ 2\theta_\eta = \tan\ 60^\circ = \sqrt{3}$. Hence,

$$\frac{\alpha}{\beta} = \left[\frac{2-1}{2+1}\right]^{1/2} = \frac{1}{\sqrt{3}}$$

or

$$\alpha = \frac{\beta}{\sqrt{3}} = \frac{1}{2\sqrt{3}} = 0.2887\ \text{Np/m}$$

3.2 PLANE WAVES IN LOSSLESS DIELECTRICS

In a lossless dielectric, $\sigma \ll \omega\varepsilon$. Then

$$\sigma \simeq 0, \qquad \varepsilon = \varepsilon_o\varepsilon_r, \qquad \mu = \mu_o\mu_r$$
$$\alpha = 0, \qquad \beta = \omega\sqrt{\mu\varepsilon}$$
$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}, \qquad \lambda = \frac{2\pi}{\beta} \tag{29}$$

Also

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\angle 0^\circ$$

Thus **E** and **H** are in time phase with each other.

3.3 PLANE WAVES IN FREE SPACE

The parameters are defined as follows:

$$\sigma = 0, \qquad \varepsilon = \varepsilon_o, \qquad \mu = \mu_o$$
$$\alpha = 0, \qquad \beta = \omega\sqrt{\mu_o \varepsilon_o} = \frac{\omega}{c}$$
$$u = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = c, \qquad \lambda = \frac{2\pi}{\beta} \tag{30}$$

where $c \simeq 3 \times 10^8$ m/s, the speed of light in a vacuum. The fact that EM waves travel in free space at the speed of light is significant. It provides some evidence that light is the manifestation of an EM wave. In other words, light is characteristically electromagnetic.

Intrinsic impedance of free space and is given by:

$$\eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi \simeq 377\Omega \tag{31}$$

Then

$$\boldsymbol{E} = E_o cos\,(\omega t - \beta z)\boldsymbol{a}_x$$
$$\boldsymbol{H} = H_o cos\,(\omega t - \beta z)\boldsymbol{a}_y = \frac{E_o}{\eta_o} cos\,(\omega t - \beta z)\boldsymbol{a}_y \tag{32}$$

In general, if $\mathbf{a}_E, \mathbf{a}_H$, and $\mathbf{a}_k$ are unit vectors along the **E** field, the **H** field, and the direction of wave propagation; it can be shown that:

$$\boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_H$$
$$or$$
$$\boldsymbol{a}_k \times \boldsymbol{a}_H = -\boldsymbol{a}_E \tag{33}$$
$$or$$
$$\boldsymbol{a}_E \times \boldsymbol{a}_H = \boldsymbol{a}_k$$

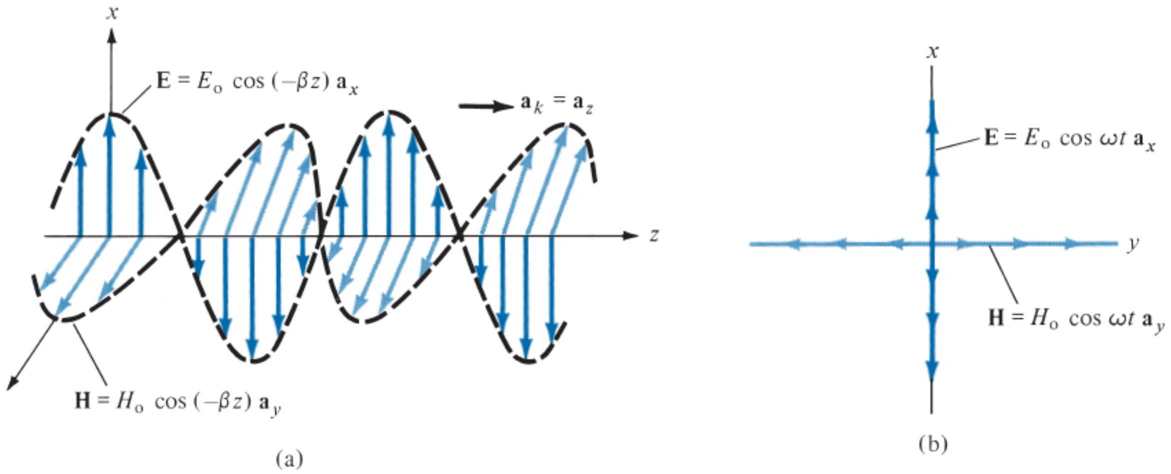

Figure 4: Plots of E and H(a) as functions of z at $t = 0$; and (b) at $z = 0$. The arrows indicate instantaneous values.

EXAMPLE 3 : In a lossless dielectric for which $\eta = 60\pi, \mu_r = 1$, and $\mathbf{H} = -0.1\cos(\omega t - z)\mathbf{a}_x + 0.5\sin(\omega t - z)\mathbf{a}_y$ A/m, calculate ε_r, ω, and $\mathbf{E}$.

Solution

In this case, $\sigma = 0, \alpha = 0$, and $\beta = 1$, so

$$\eta = \sqrt{\mu/\varepsilon} = \sqrt{\frac{\mu_o}{\varepsilon_o}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{120\pi}{\sqrt{\varepsilon_r}}$$

or

$$\sqrt{\varepsilon_r} = \frac{120\pi}{\eta} = \frac{120\pi}{60\pi} = 2 \rightarrow \varepsilon_r = 4$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o}\sqrt{\mu_r\varepsilon_r} = \frac{\omega}{c}\sqrt{4} = \frac{2\omega}{c}$$

or

$$\omega = \frac{\beta c}{2} = \frac{1(3 \times 10^8)}{2} = 1.5 \times 10^8 \text{rad/s}$$

$$\text{H} = \text{H}_1 + \text{H}_2$$

where $\mathbf{H}_1 = -0.1\cos(\omega t - z)\mathbf{a}_x$ and $\mathbf{H}_2 = 0.5\sin(\omega t - z)\mathbf{a}_y$ and the corresponding electric field

$$\text{E} = \text{E}_1 + \text{E}_2$$

where $\mathbf{E}_1 = E_{1o}\cos(\omega t - z)\mathbf{a}_{E_1}$ and $\mathbf{E}_2 = E_{2o}\sin(\omega t - z)\mathbf{a}_{E_2}$. Notice that although $\mathbf{H}$ has components along $\mathbf{a}_x$ and $\mathbf{a}_y$,.

For $\mathbf{E}_1$,

$$\mathbf{a}_{E_1} = -(\mathbf{a}_k \times \mathbf{a}_{H_1}) = -(\mathbf{a}_z \times -\mathbf{a}_x) = \mathbf{a}_y$$
$$E_{1o} = \eta H_{1o} = 60\pi(0.1) = 6\pi$$

Hence

$$\mathbf{E}_1 = 6\pi\cos(\omega t - z)\mathbf{a}_y$$

For $\mathbf{E}_2$,

$$\mathbf{a}_{E_2} = -(\mathbf{a}_k \times \mathbf{a}_{H_2}) = -(\mathbf{a}_z \times \mathbf{a}_y) = \mathbf{a}_x$$
$$E_{2o} = \eta H_{2o} = 60\pi(0.5) = 30\pi$$

Hence

$$\mathbf{E}_2 = 30\pi\sin(\omega t - z)\mathbf{a}_x$$

Adding $\mathbf{E}_1$ and $\mathbf{E}_2$ gives $\mathbf{E}$; that is,

$$\mathbf{E} = 94.25\sin(1.5\times10^8 t - z)\mathbf{a}_x + 18.85\cos(1.5\times10^8 t - z)\mathbf{a}_y \text{ V/m}$$

3.4 PLANE WAVES IN GOOD CONDUCTORS

A perfect, or good conductor, is one in which $\sigma \gg \omega\varepsilon$, so that $\frac{\sigma}{\omega\varepsilon,} \gg 1$; that is,

$$\sigma \simeq \infty, \qquad \varepsilon = \varepsilon_0, \qquad \mu = \mu_0\mu_r$$
$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$$
$$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}, \qquad \lambda = \frac{2\pi}{\beta}$$
$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ \tag{34}$$
$$\mathbf{E} = E_o e^{-\alpha z}\cos(\omega t - \beta z)\mathbf{a}_x$$
$$\mathbf{H} = \frac{E_o}{\sqrt{\frac{\omega\mu}{\sigma}}} e^{-\alpha z}\cos(\omega t - \beta z - 45^\circ)\mathbf{a}_y$$

Therefore, as the **E** (or **H**) wave travels in a conducting medium, its amplitude is attenuated by the factor $e^{-\alpha z}$. The distance δ, through which the wave amplitude decreases to a factor e^{-1} (about 37% of the original value) is called skin depth or penetration depth of the medium; that is,

$$E_o e^{-\alpha\delta} = E_o e^{-1}$$

or

$$\delta = \frac{1}{\alpha} \tag{35}$$

The skin depth is a measure of the depth to which an EM wave can penetrate the medium. For good conductors

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\alpha} \tag{36}$$

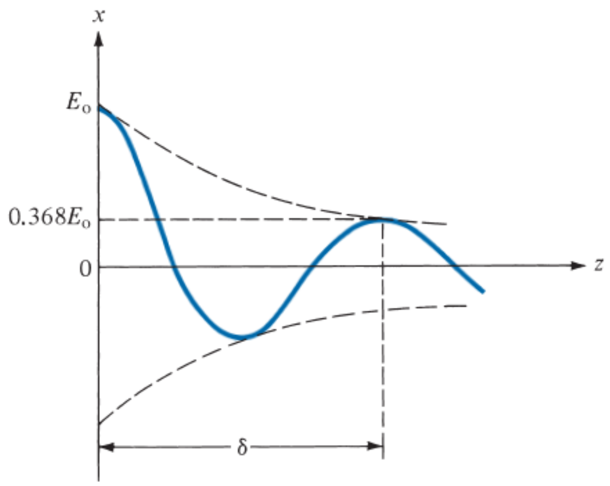

Figure 5: Illustration of skin depth.

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