Al-Mustaqbal University / College of Technical Engineering
Department: Medical Instrumentation Techniques Engineering
Class: 1st
Subject: integral Mathematics / Code: UOMU024024
Lecturer: M.Sc. Alaa Khalid
2st term–Lecture No. 2/ Integral of trigonometric functions

العلاقات بين الدوال المثلثية

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin x = \frac{1}{\csc x} \quad , \quad \cos x = \frac{1}{\sec x} \quad , \quad \cot x = \frac{1}{\tan x}$$

جيب ضعف الزاوية

$$\sin 2x = 2\sin x \cos x$$

أمثله

$$\sin 4x = 2\sin 2x \cos 2x \quad , \quad \sin 8x = 2\sin 4x \cos 4x$$

$$\sin^2 2x = (2\sin x \cos x)^2 = 4\sin^2 x \cos^2 x$$

مثال / جد $2\sin 3x \cos 3x$

$$2\sin 3x \cos 3x = \sin 6x$$

جيب تمام ضعف الزاوية

$$Cos2x = \cos^2 x - \sin^2 x \quad or \quad Cos2x = 2\cos^2 x - 1$$

$$\text{Cos } 2x = 1 - 2\sin^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \quad , \quad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

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Integrals of trigonometric functions:

1) $\int \sin u \ du = -\cos u + c$
2) $\int \cos u \ du = \sin u + c$
3) $\int \tan u \, du = -\ln|\cos u| + c$
4) $\int \cot u \, du = \ln|\sin u| + c$
5) $\int \sec u \, du = \ln|\sec u + \tan u| + c$
6) $\int \csc u \, du = -\ln|\csc u + \cot u| + c$
7) $\int \sec^2 u \, du = \tan u + c$
8) $\int \csc^2 u \, du = -\cot u + c$
9) $\int \sec u \cdot \tan u \ du = \sec u + c$
10) $\int \csc u \cdot \cot u \ du = -\csc u + c$

Example/

1) $\int \cos(3\theta - 1)\, d\theta = \frac{3}{3}\int \cos(3\theta - 1)\, d\theta = \frac{1}{3}\sin(3\theta - 1) + c$

2) $$\int \mathbf{x + \sec x \tan x \, dx} = \frac{\mathbf{x^2}}{\mathbf{2}} + \mathbf{\sec x + c}$$

3 $$\int x^2 Sinx^3 dx = \frac{1}{3}\int Sinx^3 (3x^2 dx) = \frac{-1}{3} cosx^3 + c$$

4) $$\int \acute{sin}^4 x \acute{cos}\, x \, dx = \frac{1}{5} sin^5 x + c$$

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EXAMPLE 5 Evaluating Integrals

(a) $\int_0^{\pi} \cos x\,dx = \sin x\Big]_0^{\pi} = \sin\pi - \sin 0 = 0 - 0 = 0$

(b) $\int_{-\pi/4}^{0} \sec x\tan x\,dx = \sec x\Big]_{-\pi/4}^{0} = \sec 0 - \sec\left(-\frac{\pi}{4}\right) = 1 - \sqrt{2}$

(c) $\int_1^4 \left(\frac{3}{2}\sqrt{x} - \frac{4}{x^2}\right)dx = \left[x^{3/2} + \frac{4}{x}\right]_1^4$

$= \left[(4)^{3/2} + \frac{4}{4}\right] - \left[(1)^{3/2} + \frac{4}{1}\right]$

$= [8 + 1] - [5] = 4.$ ■

$$\int \frac{2\sin\sqrt[3]{x}}{\sqrt[3]{x^2}} = 2\int \sin\sqrt[3]{x}\ \ x^{-\frac{2}{3}}\,dx$$

$$= 2\times 3\int \sin\sqrt[3]{x}\ \ \left(\frac{1}{3}\right)x^{-\frac{2}{3}}\,dx$$

$$= -6\cos\sqrt[3]{x} + c$$

3) $\int \frac{\cot^2\sqrt{x}}{\sqrt{x}}\,dx = \int \frac{\csc^2\sqrt{x}-1}{\sqrt{x}}dx = 2\int \frac{\csc^2\sqrt{x}}{2\sqrt{x}}\,dx - \int x^{-1/2}\,dx$

$= -2\cot\sqrt{x} - \frac{x^{1/2}}{1/2} + c = -2\cot\sqrt{x} - 2\sqrt{x} + c$

$\cot^2 x + 1 = \csc^2 x$

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