Al-Mustaqbal University / College of Engineering & Technology
Class: first
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Lecturer: M.Sc. Alaa Khalid
Lecture name:
Lecture:1
1st term

Functions; Domain and Range

The temperature at which water boils depends on the elevation above sea level (the boiling point drops as you ascend). The interest paid on a cash investment depends on the length of time the investment is held. The area of a circle depends on the radius of the circle. The distance an object travels from an initial location along a straight line path depends on its speed.

In each case, the value of one variable quantity, which we might call y, depends on the value of another variable quantity, which we might call x. Since the value of y is completely determined by the value of x, we say that y is a function of x. Often the value of y is given by a *rule* or formula that says how to calculate it from the variable x. For instance, the equation $A = \pi r^2$ is a rule that calculates the area A of a circle from its radius r.

In calculus we may want to refer to an unspecified function without having any particular formula in mind. A symbolic way to say "y is a function of x" is by writing

$$y = f(x) \qquad (\text{"}y \text{ equals } f \text{ of } x\text{"})$$

In this notation, the symbol f represents the function. The letter x, called the **independent variable**, represents the input value of f, and y, the **dependent variable**, represents the corresponding output **value** of f at x.

DEFINITION Function

A **function** from a set D to a set Y is a rule that assigns a *unique* (single) element $f(x) \in Y$ to each element $x \in D$.

The set D of all possible input values is called the **domain** of the function. The set of all values of $f(x)$ as x varies throughout D is called the **range** of the function. The range may not include every element in the set Y.

The domain and range of a function can be any sets of objects, but often in calculus they are sets of real numbers. (In Chapters 13–16 many variables may be involved.)

Think of a function f as a kind of machine that produces an output value $f(x)$ in its range whenever we feed it an input value x from its domain (Figure 1.22). The function

FIGURE 1.22 A diagram showing a function as a kind of machine.

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keys on a calculator give an example of a function as a machine. For instance, the $\sqrt{x}$ key on a calculator gives an output value (the square root) whenever you enter a nonnegative number x and press the $\sqrt{x}$ key. The output value appearing in the display is usually a decimal approximation to the square root of x. If you input a number $x < 0$, then the calculator will indicate an error because $x < 0$ is not in the domain of the function and cannot be accepted as an input. The $\sqrt{x}$ key on a calculator is not the same as the exact mathematical function f defined by $f(x) = \sqrt{x}$ because it is limited to decimal outputs and has only finitely many inputs.

A function can also be pictured as an **arrow diagram** (Figure 1.23). Each arrow associates an element of the domain D to a unique or single element in the set Y. In Figure 1.23, the arrows indicate that $f(a)$ is associated with a, $f(x)$ is associated with x, and so on.

The domain of a function may be restricted by context. For example, the domain of the area function given by $A = \pi r^2$ only allows the radius r to be positive. When we define a function $y = f(x)$ with a formula and the domain is not stated explicitly or restricted by context, the domain is assumed to be the largest set of real x-values for which the formula gives real y-values, the so-called **natural domain**. If we want to restrict the domain in some way, we must say so. The domain of $y = x^2$ is the entire set of real numbers. To restrict the function to, say, positive values of x, we would write "$y = x^2, x > 0$."

Changing the domain to which we apply a formula usually changes the range as well. The range of $y = x^2$ is $[0, \infty)$. The range of $y = x^2, x \geq 2$, is the set of all numbers obtained by squaring numbers greater than or equal to 2. In set notation, the range is $\{x^2 | x \geq 2\}$ or $\{y | y \geq 4\}$ or $[4, \infty)$.

When the range of a function is a set of real numbers, the function is said to be **real-valued**. The domains and ranges of many real-valued functions of a real variable are intervals or combinations of intervals. The intervals may be open, closed, or half open, and may be finite or infinite.

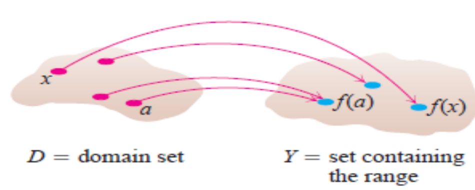

FIGURE 1.23 A function from a set D to a set Y assigns a unique element of Y to each element in D.

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Domain represents values of (x)

Range represents values of (y)

Example1: Find the domain and range of $y=x^2+1$

Solution//

Domain function=R or $-\infty \leq x \leq +\infty$

$y=x^2+1 \rightarrow x=\pm\sqrt{y-1}$

Range function =$y \geq 1$

Example 2: Find the domain and range of $y=\frac{2x}{x-1}$

Solution//

$X-1=0 \rightarrow x=1 \rightarrow$ domain function=R except {1}

$$y=\frac{2x}{x-1} \rightarrow yx-y=2x$$

$$x=\frac{y}{y-2}$$

$$y-2=0 \rightarrow y=2$$

Range function=R except {2}

Find the domain and range of $y=\frac{1}{x+1}-\frac{1}{x-1}$

Solution//

$X+1=0 \rightarrow x=-1$

$X-1=0 \rightarrow x=1$

Domain function=R except {-1,1}

$y=\frac{1}{x+1}-\frac{1}{x-1}$

$y=\frac{(x-1)-(x+1)}{(x+1)(x-1)}$

$y=\frac{-2}{(x^2-1)}$

$yx^2-y=-2$

$\rightarrow x=\frac{\sqrt{-2+y}}{\sqrt{y}}$

$$y>0 \text{ and } y \geq 2$$

Range function= $y>0 \cup y \geq 2$

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Find the domain and range of $y=\sqrt{\frac{x-1}{x+2}}$

Solution/

$x-1\geq 0 \rightarrow x \geq 1$

$x+2\geq 0 \rightarrow x > -2$

Domain function=$\{x:x\geq 1\} \cup \{x:x> -2\}$

$y=\sqrt{\frac{x-1}{x+2}}$

$Y^2=\frac{x-1}{x+2} \rightarrow xy^2 + 2y^2 = x - 1$

$\rightarrow xy^2 - x = -1 - 2y^2$

$\rightarrow x = \frac{-1-2y^2}{y^2-1} \rightarrow y^2 - 1 = 0 \rightarrow y = \pm 1$

Range function=R except $\{+1. -1\}$

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