Ministry of Higher Education and Scientific Research

Al-Mustaqbal University

College of Engineering Technologies

Medical Instrumentation Techniques Engineering Department

Electrical Circuits

First year

Series Resistors and Voltage Division

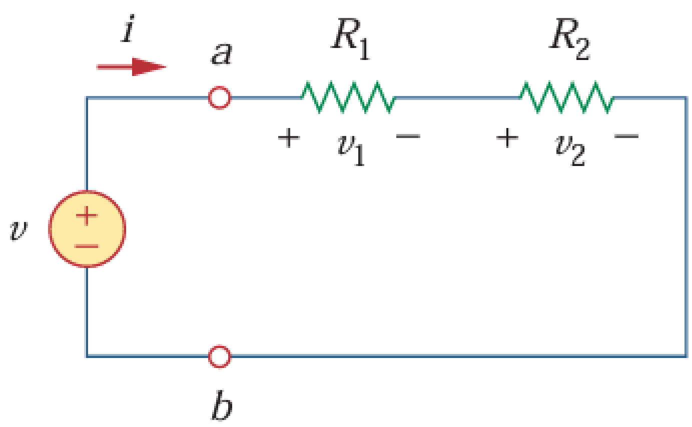

Figure 2.29
A single-loop circuit with two resistors in series.

I n the circuit shown in figure 2.29 the in the circuit is given the following equation:

$$i = \frac{v}{R_1 + R_2}$$

and

$$v_1 = iR_1, \qquad v_2 = iR_2$$

$$-v + v_1 + v_2 = 0$$

$$v = v_1 + v_2 = i(R_1 + R_2)$$

$$v = iR_{eq}$$

Therefore, in series combination

$$R_{eq} = R_1 + R_2$$

The **equivalent resistance** of any number of resistors connected in series is the sum of the individual resistances.

For N resistors in series then,

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

To determine the voltages across each resistor in circuit shown in figure 2.29 are as follows:

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

However, for resistor n in the combination of N series resistors the voltage across R_n is

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

Parallel Resistors and Current Division

Consider the circuit in Fig. 2.31, where two resistors are connected in parallel and therefore have the same voltage across them. From Ohm's law,

$$v = i_1 R_1 = i_2 R_2$$

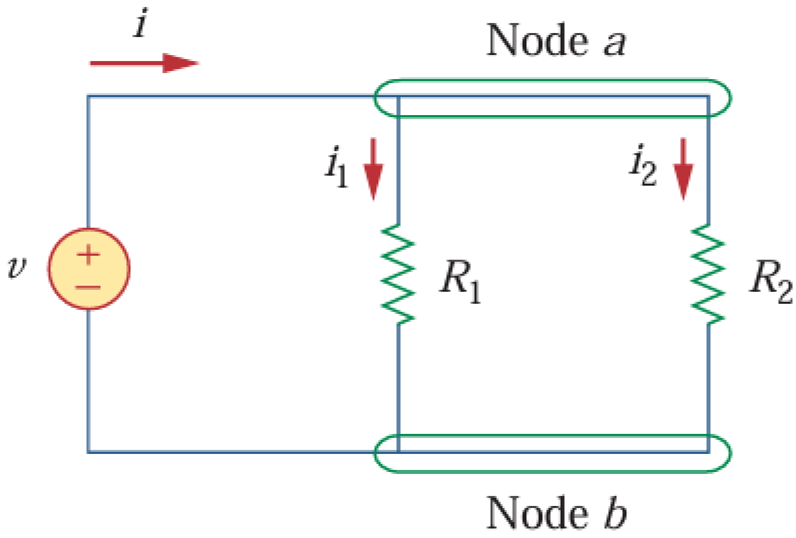

Figure 2.31
Two resistors in parallel.

or

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

Applying KCL at node a gives the total current i as

$$i = i_1 + i_2$$

Then

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

Therefore,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

or

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Thus,

The **equivalent resistance** of two parallel resistors is equal to the product of their resistances divided by their sum.

For N resistors connected in parallel, the equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

And for equal resistors connected in parallel the equivalent resistance is:

$$R_{eq} = \frac{R}{N}$$

Or can be written by another form as:

$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

where $G_{eq} = 1/R_{eq}$, $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_3 = 1/R_3$, ... , $G_N = 1/R_N$.

The **equivalent conductance** of resistors connected in parallel is the sum of their individual conductances.

Example 2.9

Find R_{eq} for the circuit shown in Fig. 2.34.

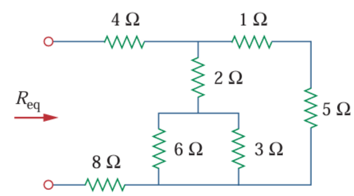

Figure 2.34
For Example 2.9.

Solution:
To get R_{eq}, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$6\,\Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\,\Omega$$

(The symbol ‖ is used to indicate a parallel combination.) Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

$$1\,\Omega + 5\,\Omega = 6\,\Omega$$

Thus the circuit in Fig. 2.34 is reduced to that in Fig. 2.35(a). In Fig. 2.35(a), we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$2\,\Omega + 2\,\Omega = 4\,\Omega$$

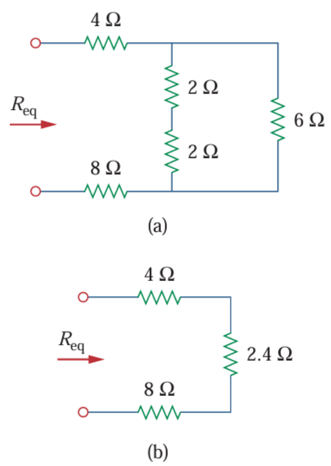

Figure 2.35
Equivalent circuits for Example 2.9.

This 4-Ω resistor is now in parallel with the 6-Ω resistor in Fig. 2.35(a); their equivalent resistance is

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$

The circuit in Fig. 2.35(a) is now replaced with that in Fig. 2.35(b). In Fig. 2.35(b), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

Practice Problem 2.9

By combining the resistors in Fig. 2.36, find R_{eq}.

Answer: 6 Ω.

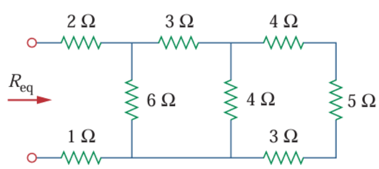

Figure 2.36
For Practice Prob. 2.9.

Example 2.10

Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

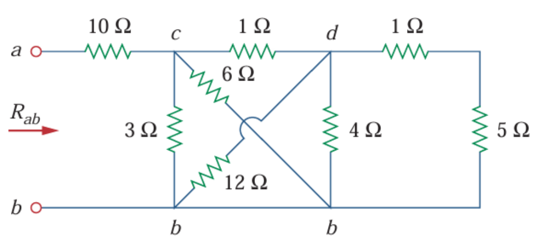

Figure 2.37
For Example 2.10.

Example 2.12

Find i_o and v_o in the circuit shown in Fig. 2.42(a). Calculate the power dissipated in the 3-Ω resistor.

Practice Problem 2.10

Find R_{ab} for the circuit in Fig. 2.39.

Answer: 11 Ω.

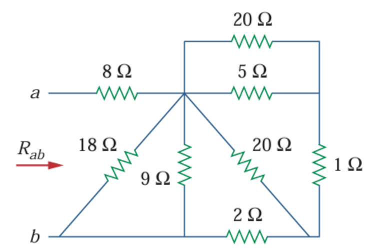

Figure 2.39
For Practice Prob. 2.10.

Practice Problem 2.11

Calculate G_{eq} in the circuit of Fig. 2.41.

Answer: 4 S.

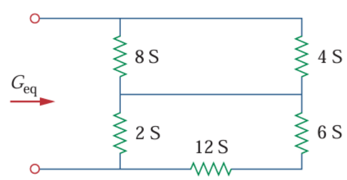

Figure 2.41
For Practice Prob. 2.11.

Example 2.12

Find i_o and v_o in the circuit shown in Fig. 2.42(a). Calculate the power dissipated in the 3-Ω resistor.

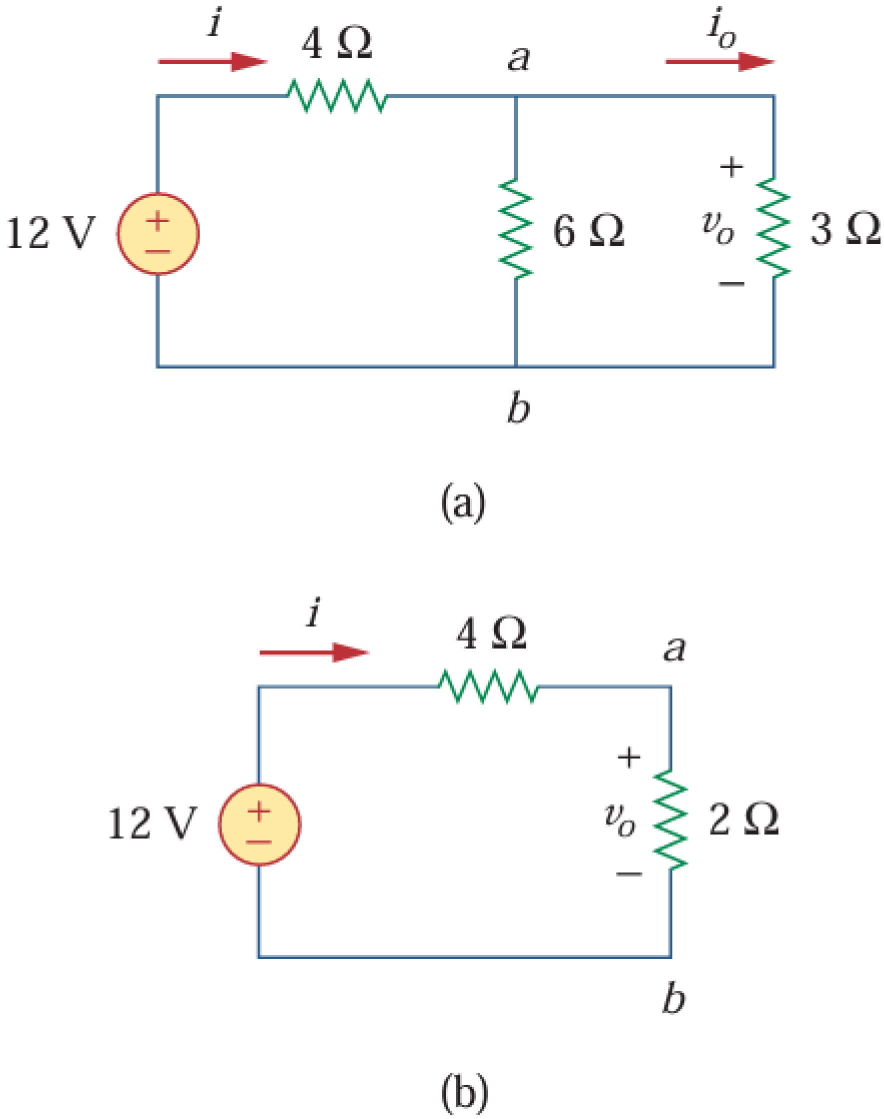

Figure 2.42