Ministry of Higher Education and Scientific Research

Al-Mustaqbal University

College of Engineering Technologies

Medical Instrumentation Techniques Engineering Department

Electrical Circuits

First year

Basic Laws

2

2.1 Introduction

Chapter 1 introduced basic concepts such as current, voltage, and power in an electric circuit. To actually determine the values of these variables in a given circuit requires that we understand some fundamental laws that govern electric circuits. These laws, known as Ohm's law and Kirchhoff's laws, form the foundation upon which electric circuit analysis is built.

In this chapter, in addition to these laws, we shall discuss some techniques commonly applied in circuit design and analysis. These techniques include combining resistors in series or parallel, voltage division, current division, and delta-to-wye and wye-to-delta transformations. The application of these laws and techniques will be restricted to resistive circuits in this chapter. We will finally apply the laws and techniques to real-life problems of electrical lighting and the design of dc meters.

2.2 Ohm's Law

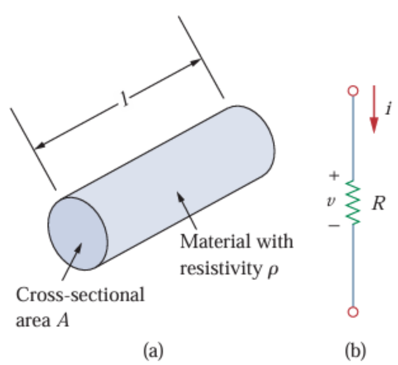

Figure 2.1
(a) Resistor, (b) Circuit symbol for resistance.

Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist current, is known as *resistance* and is represented by the symbol R. The resistance of any material with a uniform cross-sectional area A depends on A and its length ℓ, as shown in Fig. 2.1(a). We can represent resistance (as measured in the laboratory), in mathematical form,

$$R = \rho \frac{\ell}{A} \tag{2.1}$$

where ρ is known as the *resistivity* of the material in ohm-meters. Good conductors, such as copper and aluminum, have low resistivities, while insulators, such as mica and paper, have high resistivities. Table 2.1 presents the values of ρ for some common materials and shows which materials are used for conductors, insulators, and semiconductors.

The circuit element used to model the current-resisting behavior of a material is the *resistor*. For the purpose of constructing circuits, resistors are usually made from metallic alloys and carbon compounds. The circuit

Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

That is,

$$v \propto i \tag{2.2}$$

Ohm defined the constant of proportionality for a resistor to be the resistance, R. (The resistance is a material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.) Thus, Eq. (2.2) becomes

$$\boxed{v = iR} \tag{2.3}$$

which is the mathematical form of Ohm's law. R in Eq. (2.3) is measured in the unit of ohms, designated Ω. Thus,

The **resistance** R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

We may deduce from Eq. (2.3) that

$$R = \frac{v}{i} \tag{2.4}$$

so that

$$1\ \Omega = 1\ \text{V/A}$$

$$\boxed{G = \frac{1}{R} = \frac{i}{v}} \tag{2.7}$$

The conductance is a measure of how well an element will conduct electric current. The unit of conductance is the *mho* (ohm spelled backward) or reciprocal ohm, with symbol ℧, the inverted omega. Although engineers often use the mho, in this book we prefer to use the siemens (S), the SI unit of conductance:

$$1\ \text{S} = 1\ \mho = 1\ \text{A/V} \tag{2.8}$$

Thus,

> **Conductance** is the ability of an element to conduct electric current; it is measured in mhos (℧) or siemens (S).

The same resistance can be expressed in ohms or siemens. For example, 10 Ω is the same as 0.1 S. From Eq. (2.7), we may write

$$i = Gv \tag{2.9}$$

The power dissipated by a resistor can be expressed in terms of R. Using Eqs. (1.7) and (2.3),

$$p = vi = i^2R = \frac{v^2}{R} \tag{2.10}$$

The power dissipated by a resistor may also be expressed in terms of G as

$$p = vi = v^2G = \frac{i^2}{G} \tag{2.11}$$

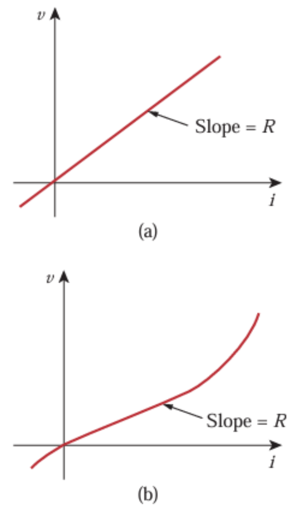

Figure 2.7
The i-v characteristic of: (a) a linear resistor, (b) a nonlinear resistor.

Example 2.1

An electric iron draws 2 A at 120 V. Find its resistance.

Solution:
From Ohm's law,

$$R = \frac{v}{i} = \frac{120}{2} = 60\ \Omega$$

Practice Problem 2.1

The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 10 Ω at 110 V?

Answer: 11 A.

Example 2.2

In the circuit shown in Fig. 2.8, calculate the current i, the conductance G, and the power p.

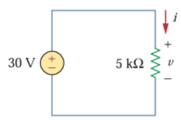

Figure 2.8
For Example 2.2.

Solution:
The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6\ \text{mA}$$

The conductance is

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2\ \text{mS}$$

We can calculate the power in various ways using either Eqs. (1.7), (2.10), or (2.11).

$$p = vi = 30(6 \times 10^{-3}) = 180\ \text{mW}$$

or

$$p = i^2R = (6 \times 10^{-3})^2 5 \times 10^3 = 180\ \text{mW}$$

or

$$p = v^2G = (30)^2 0.2 \times 10^{-3} = 180\ \text{mW}$$

Practice Problem 2.2

For the circuit shown in Fig. 2.9, calculate the voltage v, the conductance G, and the power p.

Answer: 20 V, 100 μS, 40 mW.

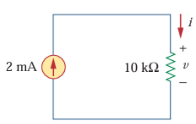

Figure 2.9
For Practice Prob. 2.2

Example 2.3

A voltage source of $20 \sin \pi t$ V is connected across a 5-kΩ resistor. Find the current through the resistor and the power dissipated.

Solution:

$$i = \frac{v}{R} = \frac{20 \sin \pi t}{5 \times 10^3} = 4 \sin \pi t\ \text{mA}$$

Hence,

$$p = vi = 80 \sin^2 \pi t\ \text{mW}$$

Practice Problem 2.3

A resistor absorbs an instantaneous power of $20 \cos^2 t$ mW when connected to a voltage source $v = 10 \cos t$ V. Find i and R.

Answer: $2 \cos t$ mA, 5 kΩ.