



Al-Mustaqbal University / College of Engineering & Technology  
Department of Medical Instrumentation Techniques Engineering

Class: 4<sup>th</sup>

Subject: Medical Laser Systems

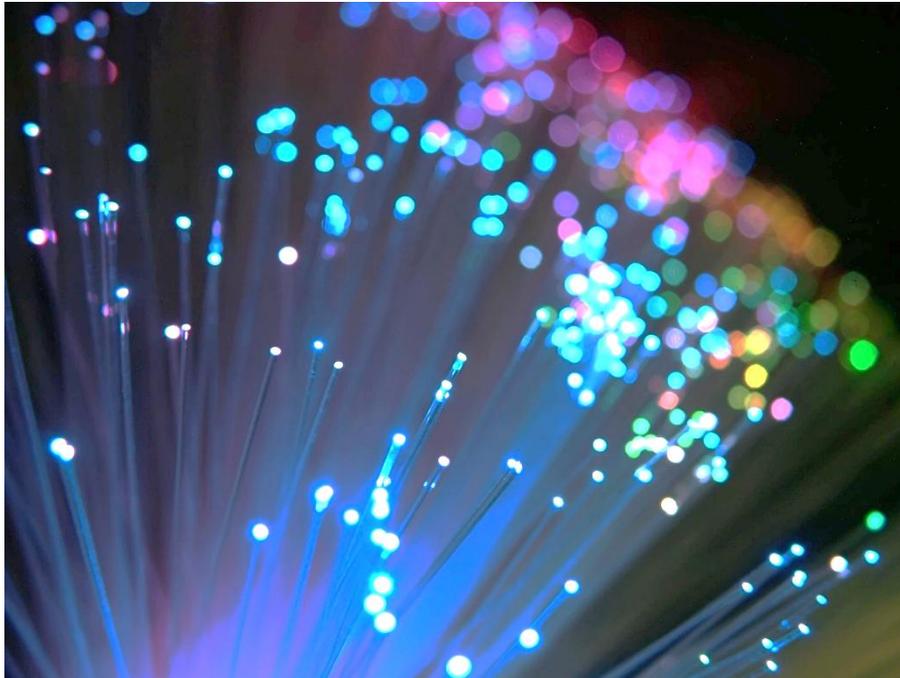
Lecturer: MSc. Huda Wasfi Hassoon

2<sup>nd</sup> term – Lecture No. 8 & Lecture Name: Optical Fiber - Numerical Aperture



## Lecture 8

# Optical Fiber – Numerical Aperture



Lecturer:

MSc. Huda Wasfi Hassoon



“Do all light rays entering an optical fiber propagate through the core?”

If not, what factors determine whether a light ray will be guided or lost?”



### Optical Fiber’s Numerical Aperture (NA)

Optical fibers guide light through the principle of **total internal reflection (TIR)**. However, not all incident light entering the fiber will propagate through the core. Only light rays entering within a specific angular range can be guided along the fiber. This angular range is known as the **acceptance cone**, and the parameter that defines this cone is called the **Numerical Aperture (NA)**.

The NA is a key property of an optical fiber because it determines how much light the fiber can collect, how efficiently it couples with light sources, and how many modes it can support.

### Acceptance Cone and Acceptance Angle

When light from an external medium (usually air) strikes the entrance surface of the fiber, only rays entering within a certain half-angle—called the **acceptance angle**—will propagate by total internal reflection inside the core.

The acceptance cone is illustrated in the following figure:

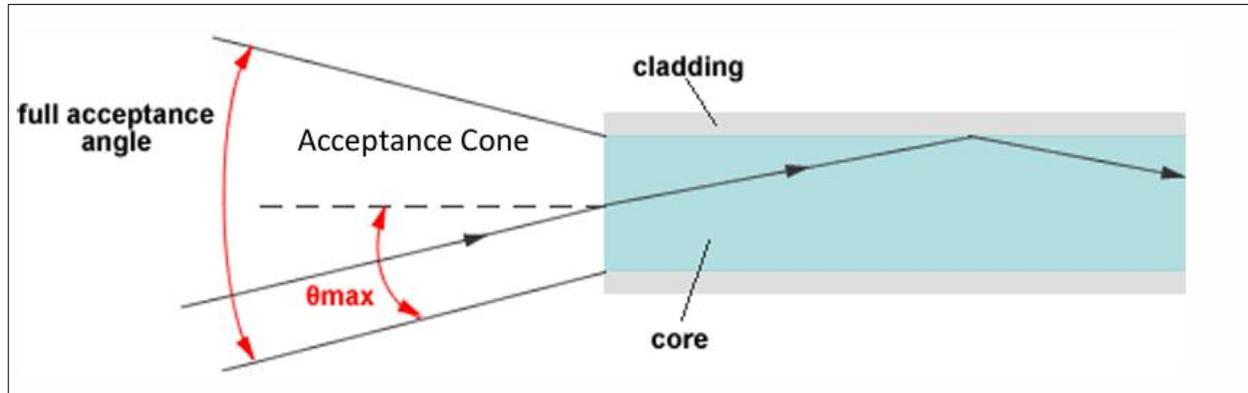


Fig. (1) The acceptance cone of optical fiber

This cone represents all the possible angles for which light will be guided inside the fiber. The half-angle of this cone is the maximum acceptance angle  $\theta_{\max}$ .

### Derivation of Numerical Aperture

Consider an optical fiber with:

- Core refractive index:  $n_1$
- Cladding refractive index:  $n_2$
- External medium refractive index:  $n_0$  (equal to 1 for air)

A light ray enters the fiber at an incident angle  $i$  and refracts inside the core at an angle  $\theta$ . The ray then strikes the core–cladding boundary at an internal angle  $\theta'$ .

The geometry of this process is shown below:

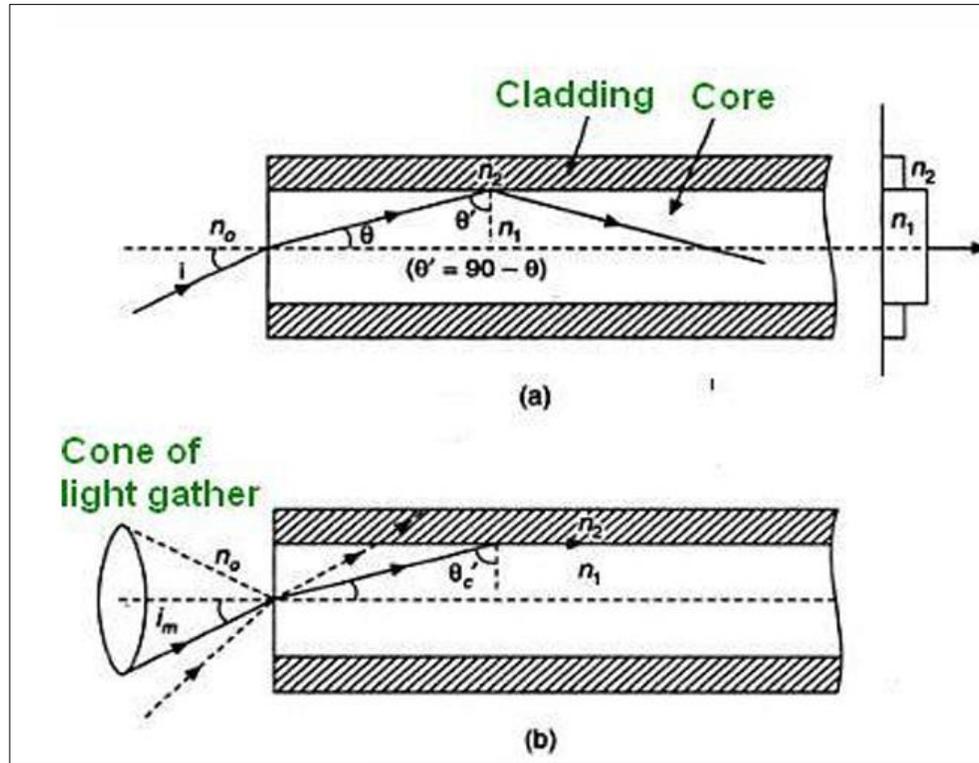


Fig. (2) Diagram showing incident angle, refracted angle, and core-cladding interaction.

### Snell's Law at the Fiber Entrance

$$n_0 \sin (i) = n_1 \sin (\theta)$$

### Total Internal Reflection Requirement

At the core-cladding boundary, total internal reflection occurs if:

$$\theta' > \theta'_c$$

Where the critical angle is:

$$\theta'_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$$



## Maximum Acceptance Angle

By combining Snell's law and geometric relations, the maximum incidence angle

$i = i_m$  (or  $\theta_{\max}$ ) that still allows TIR is:

$$n_0 \sin(i_m) = \sqrt{n_1^2 - n_2^2}$$

Thus, the **Numerical Aperture (NA)** is defined as:

$$\text{NA} = n_0 \sin(\theta_{\max}) = \sqrt{n_1^2 - n_2^2}$$

For air ( $n_0 = 1$ ):

$$\text{NA} = \sin(\theta_{\max})$$

## Meaning and Importance of Numerical Aperture

The NA expresses the **light-gathering ability** of the fiber:

### Higher NA

- Larger acceptance cone
- More light enters the fiber
- Easier coupling with LEDs and lasers
- Supports more propagation modes

### Lower NA

- Smaller acceptance cone
- Less light enters the fiber
- Used in single-mode fibers



- Lower dispersion and higher performance in long-distance communication

Thus, NA controls both the **efficiency of light coupling** and the **propagation characteristics** of the fiber.

### Spot Size at a Distance

When light emerges from the fiber into free space, the beam expands. If a detector or screen is placed at a distance **d** from the fiber end, the radius **r** of the resulting circular spot is:

$$r = d \tan (\theta_a)$$

This relationship is useful for determining detector size and fiber-to-detector alignment.

### Number of Modes in a Step-Index Multimode Fiber

In multimode step-index fibers, many different modes can propagate simultaneously. The approximate number of modes supported by the fiber depends on:

- Core diameter  $D$
- Numerical Aperture NA
- Operating wavelength  $\lambda$

The approximate formula for the number of guided modes is:

$$N_m \approx \frac{(\pi D \cdot NA)^2}{4\lambda^2}$$



This formula is valid only for **multimode step-index fibers**, and becomes inaccurate when the fiber supports only a few modes.

### The V-Number (Cutoff Parameter)

The **V-Number** or **normalized frequency parameter** determines whether a fiber is single-mode or multimode. It is defined as:

$$V = \frac{2\pi a}{\lambda} \cdot \text{NA}$$

Where:

- $a$ : core radius
- $\lambda$ : wavelength
- NA: numerical aperture

### Mode Condition

- If  $V < 2.405$  → the fiber supports **only one mode** (single-mode fiber)
- If  $V > 2.405$  → the fiber becomes **multimode**

**Example 1:** Compute the NA, acceptance angle, and the critical angle of an optical fiber from the following data.

Core refractive index:  $n_1 = 1.55$

Cladding refractive index:  $n_2 = 1.50$

#### 1. Numerical Aperture

$$\begin{aligned} \text{NA} &= \sqrt{n_1^2 - n_2^2} \\ &= \sqrt{1.55^2 - 1.50^2} = 0.3905 \end{aligned}$$



## 2. Acceptance Angle

$$\theta_a = \sin^{-1}(0.3905) = 23.2^\circ$$

## 3. Critical Angle

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.50}{1.55}\right) = 75.4^\circ$$

**Example 2:** Compute the cut-off parameter and the number of modes supported by a fiber, Given:

- Core refractive index:  $n_1 = 1.54$
- Cladding refractive index:  $n_2 = 1.50$
- Core radius:  $a = 25 \mu m$
- Wavelength:  $\lambda = 1300 nm$

### 1. Numerical Aperture

$$\begin{aligned} NA &= \sqrt{n_1^2 - n_2^2} \\ &= \sqrt{1.54^2 - 1.50^2} = 0.349 \end{aligned}$$

### 2. V-Number

$$\begin{aligned} V &= \frac{2\pi a}{\lambda} \cdot NA \\ V &= \frac{2\pi(25 \times 10^{-6})}{1300 \times 10^{-9}} \cdot 0.349 = 42.15 \end{aligned}$$

### 3. Number of Modes

$$N_m \approx \frac{(\pi D \cdot NA)^2}{4\lambda^2}$$



Al-Mustaqbal University / College of Engineering & Technology  
Department of Medical Instrumentation Techniques Engineering

Class: 4<sup>th</sup>

Subject: Medical Laser Systems

Lecturer: MSc. Huda Wasfi Hassoon

2<sup>nd</sup> term – Lecture No. 8 & Lecture Name: Optical Fiber - Numerical Aperture



$$D = 2a = 50\mu m$$

$$N_m \approx \frac{(\pi \times 50 \times 0.3487)^2}{4 \times (1.3)^2}$$
$$\approx \frac{(54.77)^2}{4 \times 1.69} \approx \frac{2999.8}{6.76} \approx 443.8$$