Al-Mustaqbal University / College of Engineering & Technology
Class: first
Subject: Differential Mathematics/Code: UOMU024013
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Lecture name: limit and continuity
Lecture:3
1st term

(Continuous Functions) : الدوال المستمرة

The Continuity Test:

The function **y= f (x)** is continuous at **x=c** if and only if the following statements are true:

1- f (c) exists
2- $\lim_{x\to c} f(x)$ exists
3- $f(c) = \lim_{x\to c} f(x)$

Example: did the function $f(x) = 8 - x^3 - 2x^2$ is continuous at the x=2 ?

Sol:

$$f(2) = 8 - 2^3 - 2 * (2)^2 = 8$$

$$\lim_{x\to 2}[\,8 - x^3 - 2x^2\,] = 8 - 2^3 - 2 * (2)^2 = 8$$

$$f(2) = \lim_{x\to 2} f(x).$$

So the function is continuous at x=2.

THEOREM Properties of Continuous Functions

If the functions f and g are continuous at $x = c$, then the following combinations are continuous at $x = c$.

1. *Sums*: $f + g$
2. *Differences*: $f - g$
3. *Products*: $f \cdot g$
4. *Constant multiples*: $k \cdot f$, for any number k
5. *Quotients*: f/g provided $g(c) \neq 0$
6. *Powers*: $f^{r/s}$, provided it is defined on an open interval containing c, where r and s are integers

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$$\lim_{x\to c}(f+g)(x) = \lim_{x\to c}(f(x)+g(x)) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x) = f(c)+g(c) = (f+g)(c)$$

This shows that $f+g$ *is continuous.*

THEOREM: Composite of Continuous Functions

If f *is continuous at* c *and* g *is continuous at* c, *then the composite* $g \circ f$ *is continuous at* c.

The following types of functions are continuous at every point in their domains:

1 – *Polynomials.*

2 – *Rational functions: They have points of discontinuity at the zero of their denominators.*

3 – *Root functions:* ($y = \sqrt[n]{x}$, *n a positive integer greater than* 1).

4 – *Trigonometric functions.*

5 – *Inverse trigonometric functions.*

6 – *Exponential functions.*

7 – *Logarithmic functions.*

Note: The inverse function of any continuous function is continuous.

Example

Show that *f (x)* has a continuous extension to *x=2* and find that extension.

$$f(x) = \frac{x^2 + x - 6}{x^2 - 4}$$

Sol.

Although f (2) is not defined,

If $x \neq 2$ we have

$$f(x) = \frac{x^2 + x - 6}{x^2 - 4} = \frac{(x-2)(x+3)}{(x-2)(x+2)} = \frac{x+3}{x+2}$$

The function $g(x) = \frac{x+3}{x+2}$ is equivalent to $f(x) = \frac{x^2+x-6}{x^2-4}$, but $g(x)$ is continuous at $x=2$ having a $\lim_{x\to 2} g(x) = \frac{5}{4}$ and $g(2)=5/4$.

Examples:

$$1)\ f(x) = \begin{Bmatrix} \frac{2x^2+x-3}{x-1} & x \neq 1 \\ 2 & x = 1 \end{Bmatrix}$$

$1 - f(1) = 2$

$2 - \lim_{x\to 1} \frac{2x^2 + x - 3}{x - 1} = \lim_{x\to 1} \frac{(2x+3)(x-1)}{(x-1)} = 2(1) + 3 = 5$

$3 - \lim_{x\to 1} f(x) \neq f(1)$

$\therefore f(x)\ discontinuous\ at\ \ x = 1.$

$$2)\ f(x) = \begin{Bmatrix} 3 + x & x \leq 1 \\ 3 - x & x > 1 \end{Bmatrix}$$

$1 - f(1) = 3 + 1 = 4$

$2 - \lim_{x\to 1^-} 3 + x = 3 + 1 = 4$

$3 - \lim_{x\to 1^+} 3 - x = 3 - 1 = 2 \qquad \therefore \lim_{x\to 1^-} 3 + x \neq \lim_{x\to 1^+} 3 - x$

$\therefore f(x)$ *discontinuous at* $x = 1$.

3) $f(x) = \begin{cases} \frac{1}{x-2} & x \neq 2 \\ 3 & x = 2 \end{cases}$

$f(2) = 3$ & $\lim_{x\to 2} \frac{1}{x-2} = \frac{1}{0} = \infty$ $\quad \therefore$ *no limit* , $f(x)$ *discontinuous.*

4) $f(x) = \begin{cases} \frac{x^2-16}{x-4} & x \neq 4 \\ 9 & x = 4 \end{cases}$

$f(4) = 9$, $\lim_{x\to 4} \frac{x^2-16}{x-4} = \lim_{x\to 4} \frac{(x-4)(x+4)}{(x-4)} = 8,\ f(4) \neq \lim_{x\to 4} f(x)$

$\therefore f(x)$ *discontinuous at* $x = 4$.

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