BJT and JFET Frequency Response-

We will now investigate the frequency effects introduced by the larger capacitive elements of the network at low frequencies and the smaller capacitive elements of the active device at the high frequencies

Since the analysis will extend through a wide frequency range, the logarithmic scale will be defined and used throughout the analysis

In addition, since industry typically uses a decibel scale on its frequency plots, the concept of the decibel is introduced in some detail

LOGARITHMS

$$a = b^x, \qquad x = \log_b a \tag{11.1}$$

$$a = b^x = (10)^2 = 100$$

but

$$x = \log_b a = \log_{10} 100 = 2$$

fact for the vast majority of scientific research, the base in the logarithmic equation is chosen as either 10 or the number e = 2.71828

$$\text{Common logarithm:} \quad x = \log_{10} a \tag{11.2}$$

$$\text{Natural logarithm:} \quad y = \log_e a \tag{11.3}$$

The two are related by

$$\log_e a = 2.3 \log_{10} a \tag{11.4}$$

Because the remaining analysis of this chapter employs the common logarithm, we review a few properties of logarithms using solely the common logarithm. In general, however, the same relationships hold true for logarithms to any base. First, note that

$$\log_{10} 1 = 0 \tag{9.5}$$

as clearly revealed by Table 9.1, because $10^0 = 1$. Next,

$$\log_{10} \frac{a}{b} = \log_{10} a - \log_{10} b \tag{9.6}$$

which for the special case of $a = 1$ becomes

$$\log_{10} \frac{1}{b} = -\log_{10} b \tag{9.7}$$

which shows that for any b greater than 1, the logarithm of a number less than 1 is always negative. Finally,

$$\log_{10} ab = \log_{10} a + \log_{10} b \tag{9.8}$$

In each case, the equations employing natural logarithms have the same format.

Tab 9.1 shows how the logarithm of a number increases only as the exponent of the number. If the antilogarithm of a number is desired, the 10 x or e x calculator function is employed

$\log_{10} 10^0$	= 0
$\log_{10} 10$	= 1
$\log_{10} 100$	= 2
$\log_{10} 1{,}000$	= 3
$\log_{10} 10{,}000$	= 4
$\log_{10} 100{,}000$	= 5
$\log_{10} 1{,}000{,}000$	= 6
$\log_{10} 10{,}000{,}000$	= 7
$\log_{10} 100{,}000{,}000$	= 8
etc.	

Table 9.1

Semilog graph paper

The log of 2 to the base 10 is approximately 0.3. The distance from 1 (log10 1 = 0) to 2 is therefore 30% of the span. The log of 3 to the base 10 is 0.4771 or almost 48% of the span (very close to one-half the distance between power-of-10 increments on the (log scale).

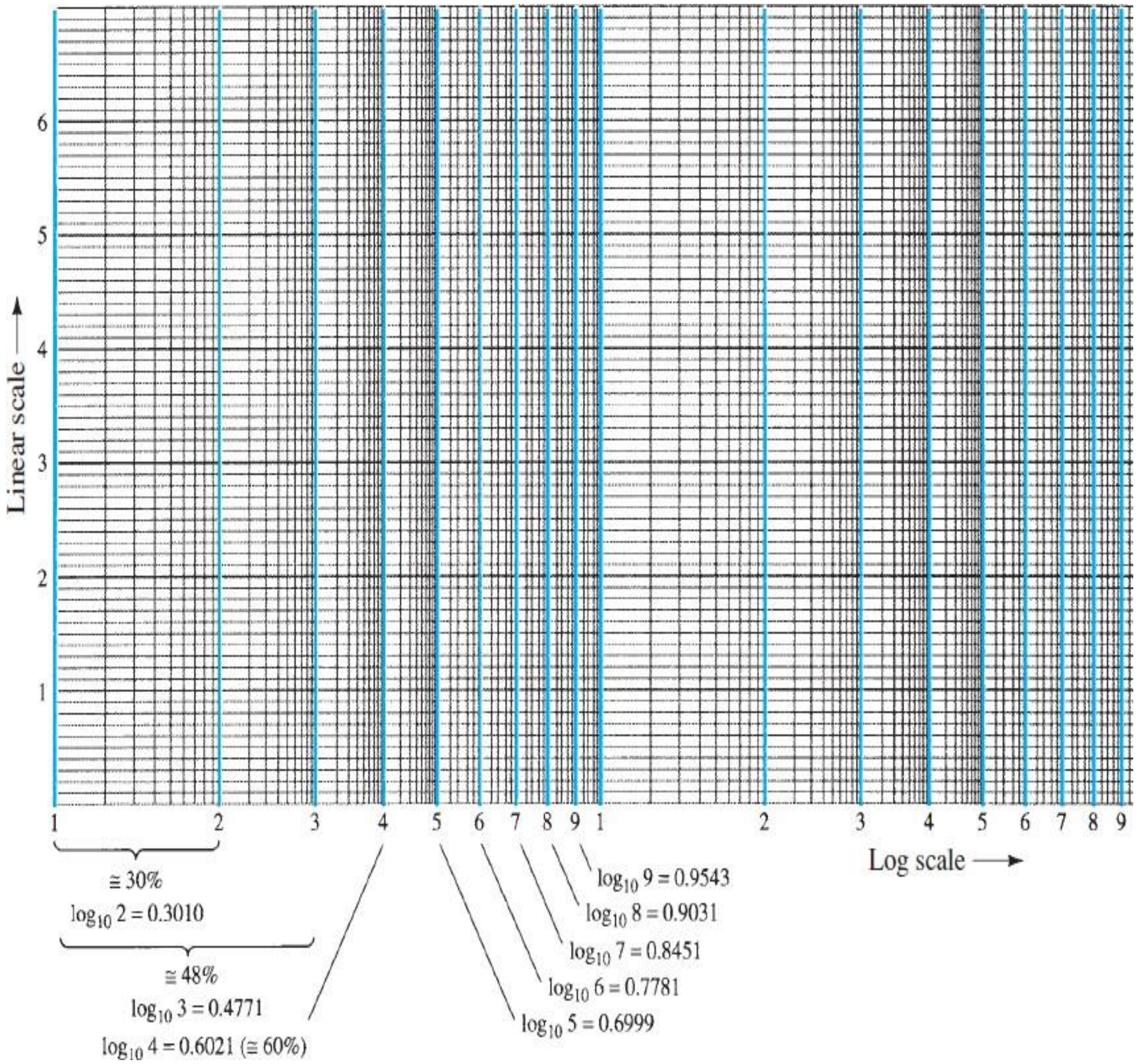
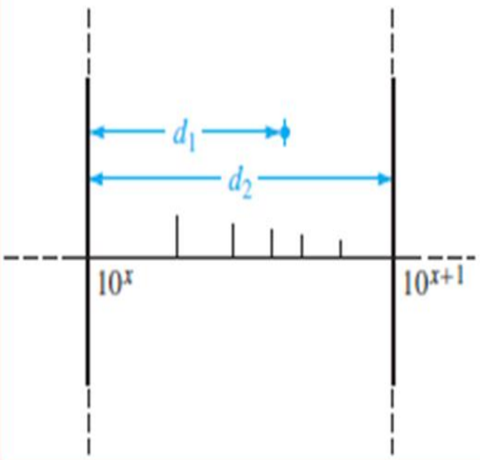


FIG. 9.1 Semilog graph paper

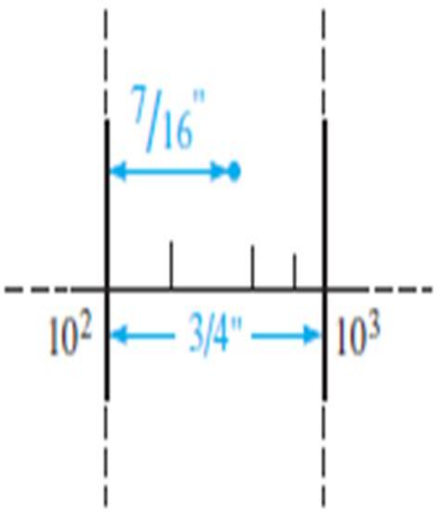
The following equation can be used to determine the logarithmic level at a particular point between known levels using a ruler or simply estimating the distances.

يمكن استخدام المعادلة التالية لتحديد المستوى اللوغاريتمي عند نقطة معينة بين المستويات المعروفة باستخدام مسطرة أو ببساطة تقدير المسافات.



$$\text{Value} = 10^x \times 10^{d_1/d_2}$$

EXAMPLE 9.5 Determine the value of the point appearing on the logarithmic plot in Fig.below using the measurements made by a ruler (linear).



$$\frac{d_1}{d_2} = \frac{7/16''}{3/4''} = \frac{0.438''}{0.750''} = 0.584$$

Using a calculator:

$$10^{d_1/d_2} = 10^{0.584} = 3.837$$

Applying Eq. (9.9):

$$\text{Value} = 10^x \times 10^{d_1/d_2} = 10^2 \times 3.837$$
$$= 383.7$$

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DECIBELS

The term bel is derived from the surname of Alexander Graham Bell. For standardization, the bel (B) is defined by the following equation relating two power levels, P 1 and P 2

$$G_{dB} = 10 \log_{10} \frac{P_2}{P_1} \quad \text{dB} \qquad (11.10)$$

$$G_{dBm} = 10 \log_{10} \frac{P_2}{1\text{ mW}}\bigg|_{600\,\Omega} \quad \text{dBm} \qquad (11.11)$$

$$G_{dB} = 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{V_2^2/R_i}{V_1^2/R_i} = 10 \log_{10}\left(\frac{V_2}{V_1}\right)^2$$

and

$$G_{dB} = 20 \log_{10} \frac{V_2}{V_1} \quad \text{dB} \qquad (11.12)$$

هي وحدة لوغاريتمية تعطي النسبة بين كميتين فيزيائيتين ، مثل القدرة أو الشدة وذلك بالنسبة إلى قيمة معيارية . يستخدم في الصوت وفي الإلكترونيات . تعني هذه الوحدة اللوغاريتمية أنه إذا زادت القدرة أو الشدة إلى الضعف ، يزداد الديسيبل بمقدار ٣ dB .

Voltage Gains versus dB Levels

EXAMPLE 9.6 Find the magnitude gain corresponding to a voltage gain of 100 dB.

Solution: By Eq. (9.13),

$$G_{dB} = 20 \log_{10} \frac{V_2}{V_1} = 100 \text{ dB} \Rightarrow \log_{10} \frac{V_2}{V_1} = 5$$

so that

$$\frac{V_2}{V_1} = 10^5 = \mathbf{100{,}000}$$

EXAMPLE 9.7 The input power to a device is 10,000 W at a voltage of 1000 V. The output power is 500 W and the output impedance is 20 Ω. a. Find the power gain in decibels. b. Find the voltage gain in decibels. c. Explain why parts (a) and (b) agree or disagree.

Solution:

a. $$G_{dB} = 10 \log_{10} \frac{P_o}{P_i} = 10 \log_{10} \frac{500 \text{ W}}{10 \text{ kW}} = 10 \log_{10} \frac{1}{20} = -10 \log_{10} 20$$

$$= -10(1.301) = \mathbf{-13.01 \text{ dB}}$$

b. $$G_v = 20 \log_{10} \frac{V_o}{V_i} = 20 \log_{10} \frac{\sqrt{PR}}{1000} = 20 \log_{10} \frac{\sqrt{(500 \text{ W})(20\ \Omega)}}{1000 \text{ V}}$$

$$= 20 \log_{10} \frac{100}{1000} = 20 \log_{10} \frac{1}{10} = -20 \log_{10} 10 = \mathbf{-20 \text{ dB}}$$

c. $$R_i = \frac{V_i^2}{P_i} = \frac{(1 \text{ kV})^2}{10 \text{ kW}} = \frac{10^6}{10^4} = 100\ \Omega \neq R_o = 20\ \Omega$$

GENERAL FREQUENCY CONSIDERATIONS

. At low frequencies, we shall find that the coupling and bypass capacitors can no longer be replaced by the short-circuit approximation because of the increase in reactance of these elements

The frequency-dependent parameters of the small-signal equivalent circuits and the stray capacitive elements associated with the active device and the network will limit the high-frequency response of the system

An increase in the number of stages of a cascaded system will also limit both the high- and low-frequency responses

Low-Frequency Range

the larger capacitors of a system will have an important impact on the response of a system in the low-frequency range and can be ignored for the high-frequency region

Variation in $X_C = \frac{1}{2\pi f_C}$ *with frequency for a 1-μF capacitor*

f	X_C	
10 Hz	15.91 kΩ	Range of possible effect
100 Hz	1.59 kΩ	
1 kHz	159 Ω	
10 kHz	15.9 Ω	
100 kHz	1.59 Ω	Range of lesser concern (≅ short-circuit equivalence)
1 MHz	0.159 Ω	
10 MHz	15.9 mΩ	
100 MHz	1.59 mΩ	

High-Frequency Range

the smaller capacitors of a system will have an important impact on the response of a system in the high-frequency range and can be ignored for the low-frequency region.

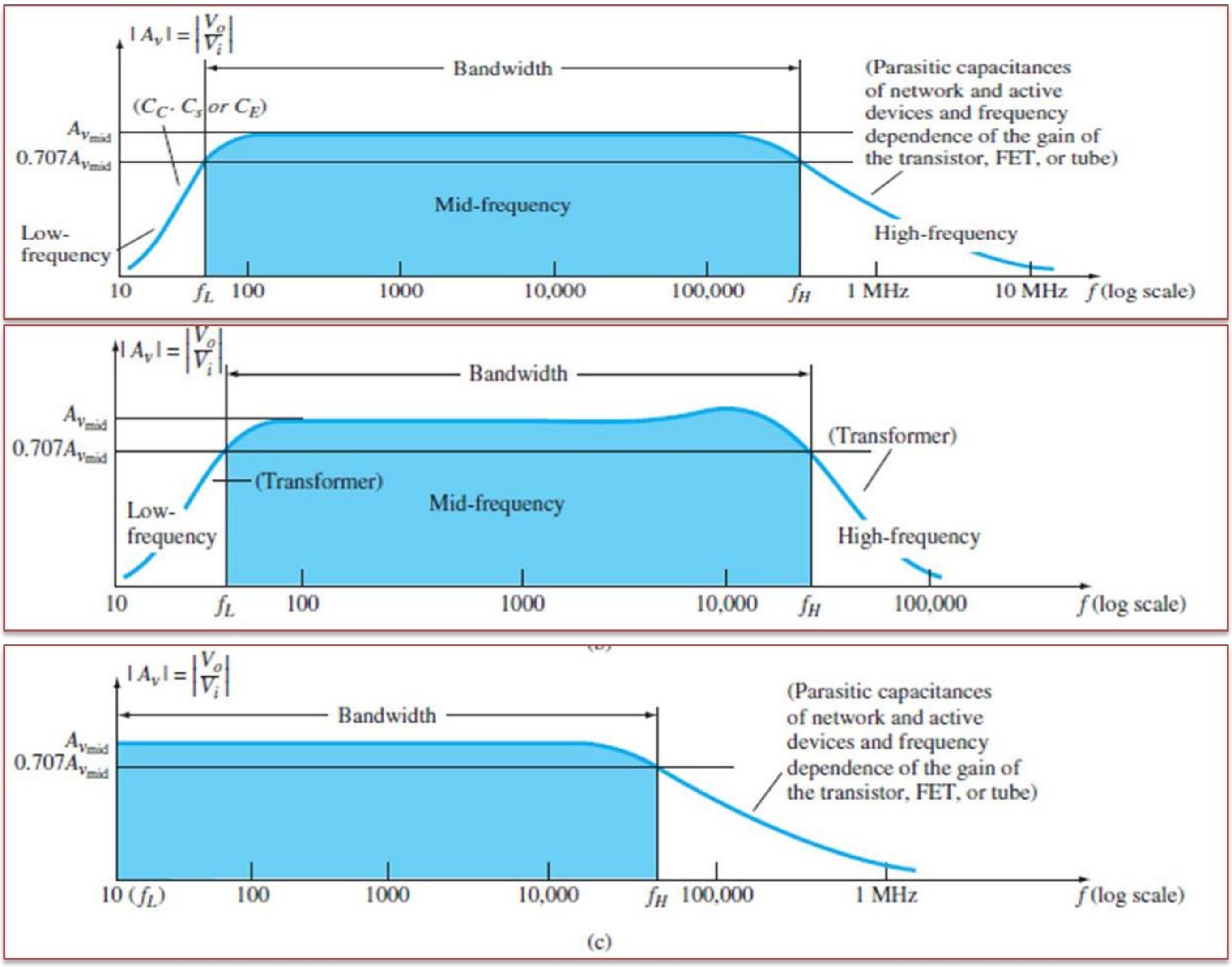
Variation in $X_C = \frac{1}{2\pi fC}$ *with frequency for a 5 pF capacitor*

f	X_C	
10 Hz	3,183 MΩ	Range of lesser concern (≅ open-circuit equivalent)
100 Hz	318.3 MΩ	
1 kHz	31.83 MΩ	
10 kHz	3.183 MΩ	
100 kHz	318.3 kΩ	Range of possible effect
1 MHz	31.83 kΩ	
10 MHz	3.183 kΩ	
100 MHz	318.3 Ω	

Mid-frequency range

the effect of the capacitive elements in an amplifier are ignored for the mid-frequency range when important quantities such as the gain and impedance levels are determined

Typical Frequency Response



Band of frequencies

For each system of Fig. 9.8 , there is a band of frequencies in which the magnitude of the gain is either equal or relatively close to the midband value. To fix the frequency boundaries of relatively high gain, 0.707Avmid was chosen to be the gain at the cutoff levels. The corresponding frequencies f 1 and f 2 are generally called the corner, cutoff, band, break, or half-power frequencies. The multiplier 0.707 was chosen because at this level the output power is half the midband power output, that is, at mid frequencies.

$$P_{o_{\text{mid}}} = \frac{|V_o^2|}{R_o} = \frac{|A_{v_{\text{mid}}} V_i|^2}{R_o}$$

and at the half-power frequencies,

$$P_{o_{HPF}} = \frac{|0.707\, A_{v_{\text{mid}}} V_i|^2}{R_o} = 0.5\,\frac{|A_{v_{\text{mid}}} V_i|^2}{R_o}$$

and

$$P_{o_{HPF}} = 0.5\, P_{o_{\text{mid}}} \tag{9.18}$$

The bandwidth (or passband) of each system is determined by f_H and f_L, that is,

$$\text{bandwidth (BW)} = f_H - f_L \tag{9.19}$$

with f_H and f_L defined in each curve of Fig. 9.8.

the effect of the capacitive elements in an amplifier are ignored for the mid-frequency range when important quantities such as the gain and impedance levels are determined.

The band frequencies define a level where the gain or quantity of interest will be 70.7% or its maximum value.

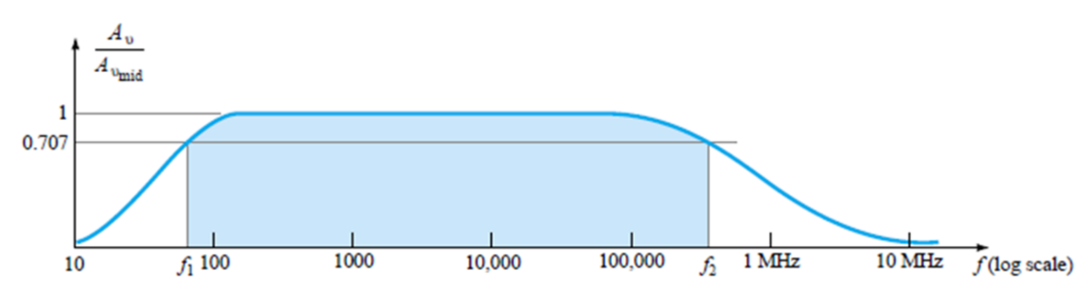


Figure 11.5 Normalized gain versus frequency plot.

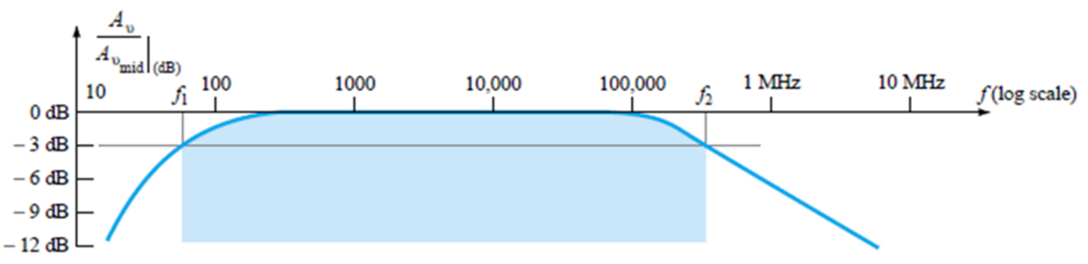
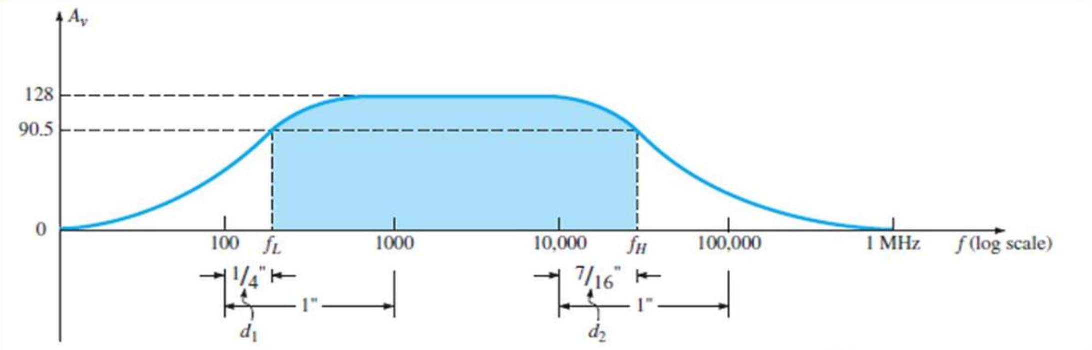


Figure 11.6 Decibel plot of the normalized gain versus frequency plot of Fig. 11.5.

EXAMPLE 9.9 Given the frequency response of Fig. 9.10 :



a. Find the cutoff frequencies f_L and f_H using the measurements provided.
b. Find the bandwidth of the response.
c. Sketch the normalized response.

Solution:

a. For f_L: $\frac{d_1}{d_2} = \frac{1/4''}{1''} = 0.25$

$10^{d_1/d_2} = 10^{0.25} = 1.7783$

Value $= 10^x \times 10^{d_1/d_2} = 10^2 \times 1.7783 =$ **177.83 Hz**

For f_H: $\frac{d_1}{d_2} = \frac{7/16''}{1''} = 0.438$

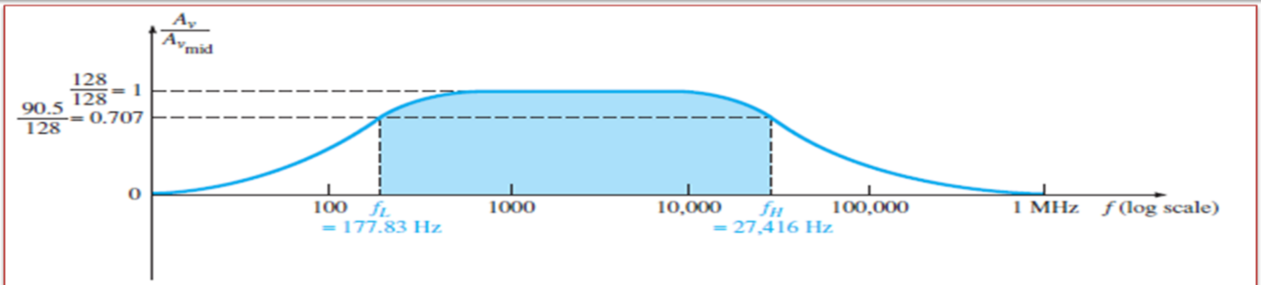
$10^{d_1/d_2} = 10^{0.438} = 2.7416$

Value $= 10^x \times 10^{d_1/d_2} = 10^4 \times 2.7416 =$ **27,416 Hz**

b. The bandwidth:

$BW = f_H - f_L = 27{,}416\text{ Hz} - 177.83\text{ Hz} \cong$ **27.24 KHz**

c. The normalized response is determined by simply dividing each level of Fig. 9.10 by the midband level of 128, as shown in Fig. 9.11. The result is a maximum value of 1 and cutoff levels of 0.707.



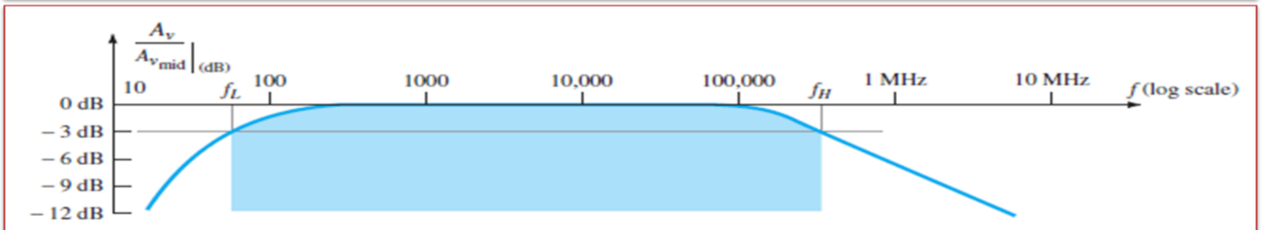


FIG. 9.12
Decibel plot of the normalized gain versus frequency plot of Fig. 9.9.

LOW-FREQUENCY ANALYSIS- BODE PLOT

In the low-frequency region of the single-stage BJT or FET amplifier, it is the RC combinations formed by the network capacitors CC, CE, and C s and the network resistive parameters that determine the cutoff frequencies. In fact, an RC network similar to Fig. 9.14 can be established for each capacitive element, and the frequency at which the output voltage drops to 0.707 of its maximum value can be determined. Once the cutoff frequencies due to each capacitor are determined, they can be compared to establish which will determine the low-cutoff frequency for the system

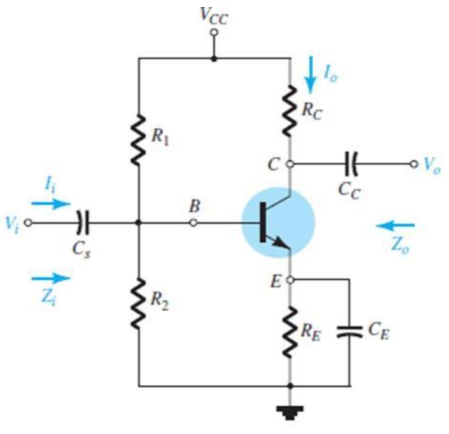


FIG. 9.15
Voltage-divider bias configuration.

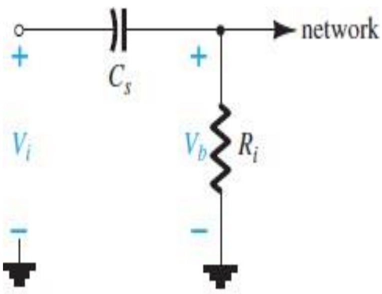


FIG. 9.16
Equivalent input circuit for the network of Fig. 9.15.

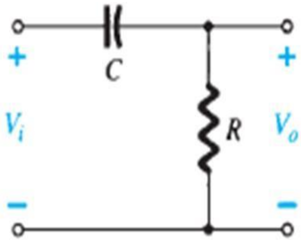


FIG. 9.14
RC combination that will define a low-cutoff frequency.

LOW-FREQUENCY ANALYSIS - BODE PLOT

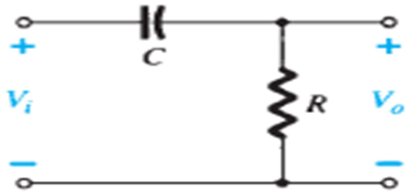


FIG. 9.14
RC combination that will define a low-cutoff frequency.

At very high frequencies,

$$X_C = \frac{1}{2\pi fC} \cong 0\ \Omega$$

$$V_o \cong V_i$$

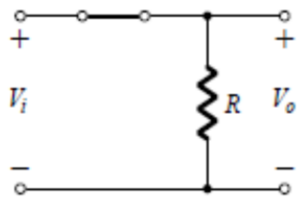


Figure 11.9 R-C circuit of Figure 11.8 at very high frequencies.

At $f = 0$ Hz,

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(0)C} = \infty\ \Omega$$

$$V_o = 0\ \text{V}.$$

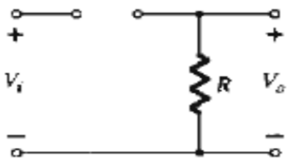


Figure 11.10 R-C circuit of Figure 11.8 at $f = 0$ Hz.

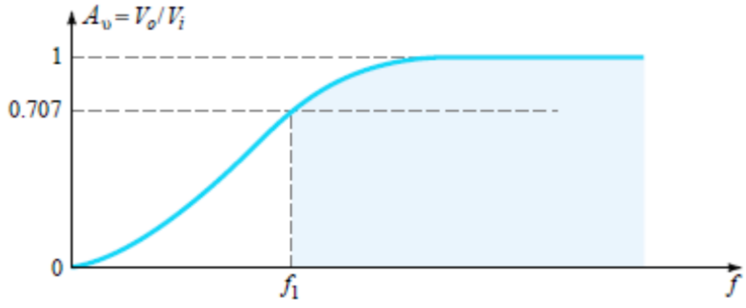


Figure 11.11 Low frequency response for the R-C circuit of Figure 11.8.

The output and input voltages are related by the voltage-divider rule in the following manner:

$$\mathbf{V}_o = \frac{\mathbf{R}\mathbf{V}_i}{\mathbf{R} + \mathbf{X}_C}$$

with the magnitude of V_o determined by

$$V_o = \frac{RV_i}{\sqrt{R^2 + X_C^2}}$$

$$V_o = \frac{RV_i}{\sqrt{R^2 + X_C^2}} = \frac{RV_i}{\sqrt{R^2 + R^2}} = \frac{RV_i}{\sqrt{2R^2}} = \frac{RV_i}{\sqrt{2}R} = \frac{1}{\sqrt{2}} V_i$$

and

$$|A_v| = \frac{V_o}{V_i} = \frac{1}{\sqrt{2}} = 0.707\Big|_{X_C = R} \qquad (11.19)$$

$$X_C = \frac{1}{2\pi f_1 C} = R$$

and

$$f_1 = \frac{1}{2\pi RC} \qquad (11.20)$$

In terms of logs,

$$G_v = 20 \log_{10} A_v = 20 \log_{10} \frac{1}{\sqrt{2}} = -3 \text{ dB}$$

while at $A_v = V_o/V_i = 1$ or $V_o = V_i$ (the maximum value),

$$G_v = 20 \log_{10} 1 = 20(0) = 0 \text{ dB}$$

If the gain equation is written as

$$A_v = \frac{V_o}{V_i} = \frac{R}{R - jX_C} = \frac{1}{1 - j(X_C/R)} = \frac{1}{1 - j(1/\omega CR)} = \frac{1}{1 - j(1/2\pi fCR)}$$

and using the frequency defined above,

$$A_v = \frac{1}{1 - j(f_1/f)} \qquad (11.21)$$

In the magnitude and phase form,

$$A_v = \frac{V_o}{V_i} = \underbrace{\frac{1}{\sqrt{1 + (f_1/f)^2}}}_{\text{magnitude of } A_v} \underbrace{\angle \tan^{-1}(f_1/f)}_{\substack{\text{phase} \angle \text{ by which} \\ V_o \text{ leads } V_i}} \qquad (11.22)$$

For the magnitude when $f = f_1$,

$$|A_v| = \frac{1}{\sqrt{1 + (1)^2}} = \frac{1}{\sqrt{2}} = 0.707 \rightarrow -3 \text{ dB}$$

In the logarithmic form, the gain in dB is

$$A_{v(\text{dB})} = 20 \log_{10} \frac{1}{\sqrt{1 + (f_1/f)^2}} = -20 \log_{10}\left[1 + \left(\frac{f_1}{f}\right)^2\right]^{1/2}$$

$$= -(\tfrac{1}{2})(20) \log_{10}\left[1 + \left(\frac{f_1}{f}\right)^2\right]$$

$$= -10 \log_{10}\left[1 + \left(\frac{f_1}{f}\right)^2\right]$$

For frequencies where $f \ll f_L$ or $(f_L/f)^2 \gg 1$, the equation above can be approximated by

$$A_{v(\text{dB})} = -10 \log_{10}\left(\frac{f_L}{f}\right)^2$$

and finally,

$$A_{v(\text{dB})} = -20 \log_{10} \frac{f_1}{f} \Bigg|_{f \ll f_1} \qquad (11.23)$$

$$\text{At } f = f_1: \frac{f_1}{f} = 1 \text{ and } -20 \log_{10} 1 = 0 \text{ dB}$$

$$\text{At } f = \tfrac{1}{2} f_1: \frac{f_1}{f} = 2 \text{ and } -20 \log_{10} 2 \cong -6 \text{ dB}$$

$$\text{At } f = \tfrac{1}{4} f_1: \frac{f_1}{f} = 4 \text{ and } -20 \log_{10} 4 \cong -12 \text{ dB}$$

$$\text{At } f = \tfrac{1}{10} f_1: \frac{f_1}{f} = 10 \text{ and } -20 \log_{10} 10 = -20 \text{ dB}$$

EXAMPLE 11.8 For the network of Fig. 11.13:

(a) Determine the break frequency.
(b) Sketch the asymptotes and locate the −3-dB point.
(c) Sketch the frequency response curve.

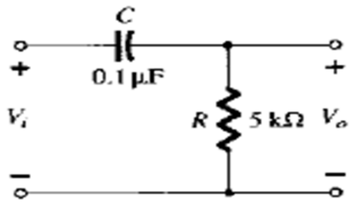


Figure 11.13 Example 11.8

Solution

$$\text{(a) } f_1 = \frac{1}{2\pi RC} = \frac{1}{(6.28)(5 \times 10^3\ \Omega)(0.1 \times 10^{-6}\ \text{F})}$$

$$\cong \mathbf{318.5\ Hz}$$

(b) and (c). See Fig. 11.14.

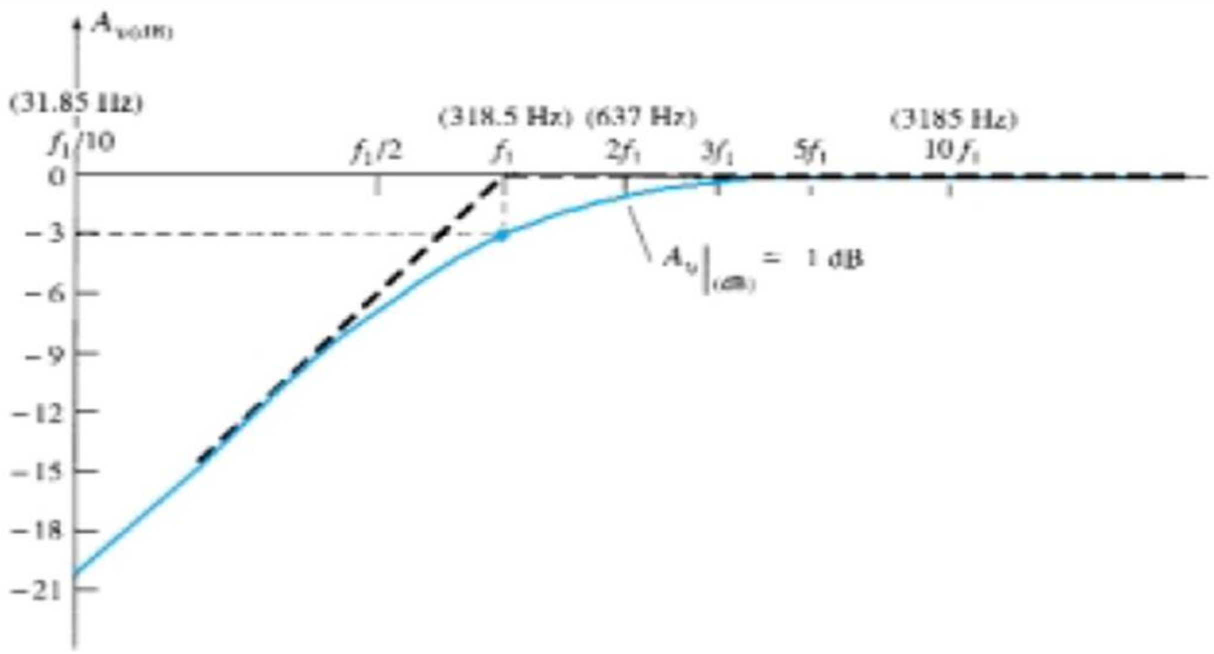


Figure 11.14 Frequency response for the R-C circuit of Figure 11.13.

$$A_{v(\text{dB})} = 20 \log_{10} \frac{V_o}{V_i}$$

but

$$\frac{A_{v(\text{dB})}}{20} = \log_{10} \frac{V_o}{V_i}$$

and

$$A_v = \frac{V_o}{V_i} = 10^{\left(\frac{A_{v(\text{dB})}}{20}\right)} \qquad (11.24)$$

For example, if $A_{v(\text{dB})} = -3$ dB,

$$A_v = \frac{V_o}{V_i} = 10^{(-3/20)} = 10^{(-0.15)} \cong 0.707 \qquad \text{as expected}$$

The phase angle of θ is determined from

$$\theta = \tan^{-1} \frac{f_1}{f} \qquad (11.25)$$

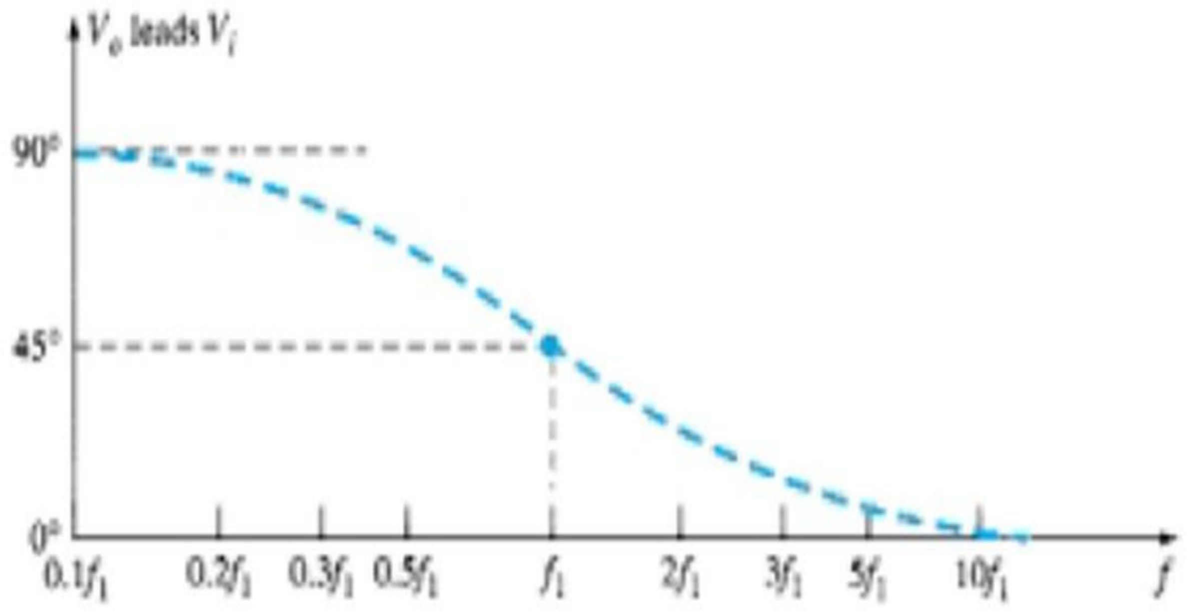


Figure 11.15 Phase response for the R-C circuit of Figure 11.8.

LOW-FREQUENCY RESPONSE - BJT AMPLIFIER

For the network of Fig. 11.16, the capacitors *Cs*, *CC*, and *CE* will determine the low-frequency response.

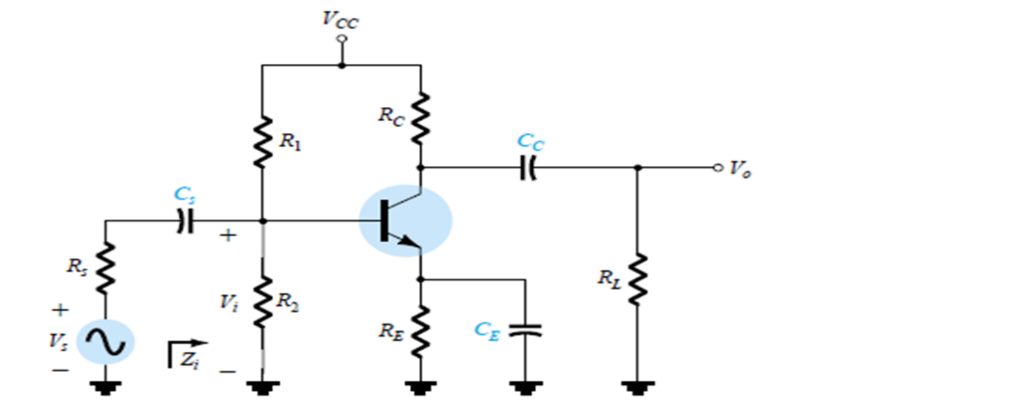


Figure 11.16 Loaded BJT amplifier with capacitors that affect the low-frequency response.

The effect of C_s

$$f_{Ls} = \frac{1}{2\pi(R_s + R_i)C_s}$$

$$V_i|_{mid} = \frac{R_i V_s}{R_i + R_s}$$

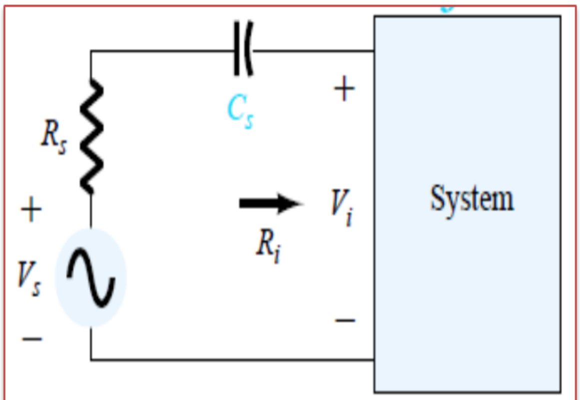
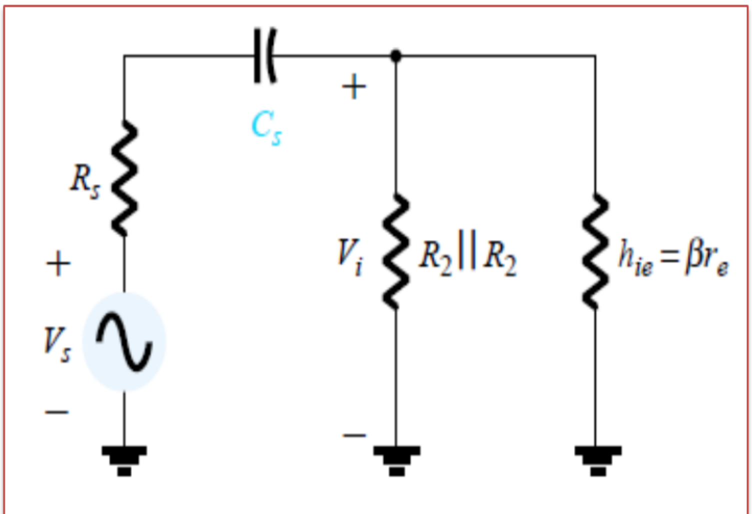


Figure 11.17 Determining the effect of C_s on the low frequency response.

$$R_i = R_1 || R_2 || \beta r_e$$



The voltage V_i applied to the input of the active device can be calculated using the voltage-divider rule:

$$V_i = \frac{R_i V_s}{R_s + R_i - jX_{cs}}$$

The effect of C_c

$$f_{Lc} = \frac{1}{2\pi(R_o + R_L)C_c}$$

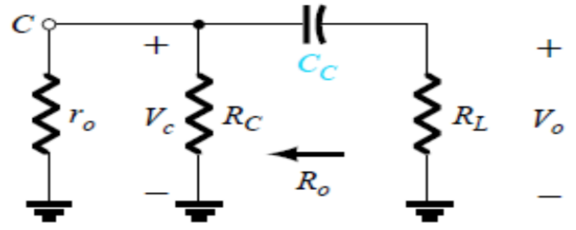


Figure 11.20 Localized ac equivalent for C_C with $V_i = 0$ V.

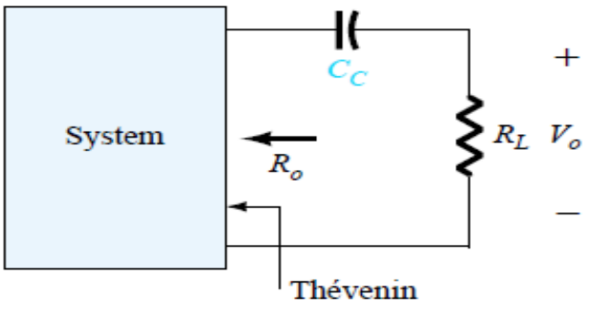


Figure 11.19 Determining the effect of C_C on the low-frequency response.

$$R_o = R_C \| r_o$$

The effect of C_E

$$f_{L_E} = \frac{1}{2\pi R_e C_E}$$

$$R_e = R_E \| \left(\frac{R_1 \| R_2}{\beta} + r_e \right)$$

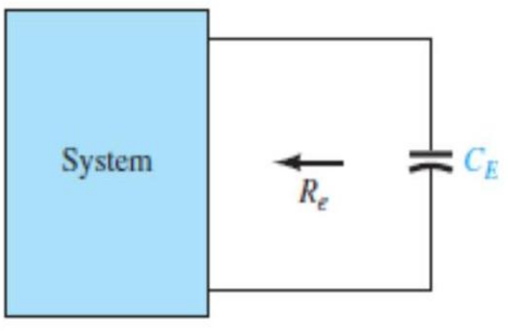


FIG. 9.29
Determining the effect of C_E on the low-frequency response.

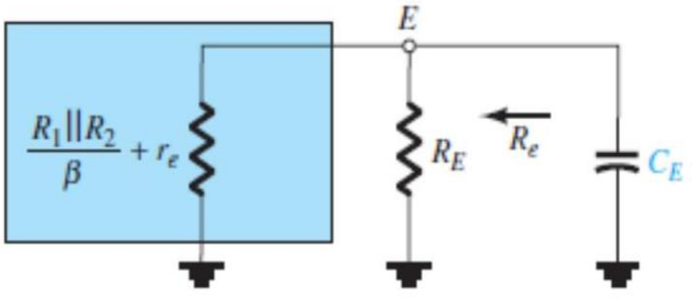
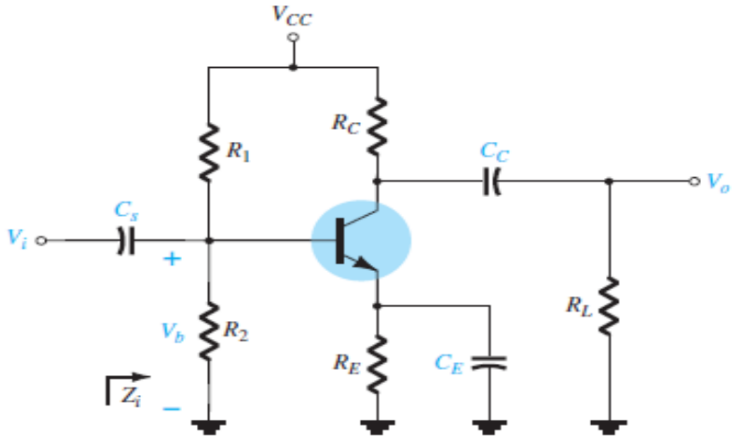


FIG. 9.30
Localized ac equivalent of C_E.

EXAMPLE

Determine the cutoff frequencies for the network of Fig. 9.25 using the following parameters:



$C_s = 10\ \mu F$, $C_E = 20\ \mu F$, $C_C = 1\ \mu F$

$R_1 = 40\ k\Omega$, $R_2 = 10\ k\Omega$, $R_E = 2\ k\Omega$, $R_C = 4\ k\Omega$,

$R_L = 2.2\ k\Omega$

$\beta = 100$, $r_o = \infty\ \Omega$, $V_{CC} = 20\ V$

(b) Sketch the frequency response using a Bode plot.

Solution

(a) Determining r_e for dc conditions:

$$\beta R_E = (100)(2\ k\Omega) = 200\ k\Omega \gg 10R_2 = 100\ k\Omega$$

The result is:

$$V_B \cong \frac{R_2 V_{CC}}{R_2 + R_1} = \frac{10\ k\Omega(20\ V)}{10\ k\Omega + 40\ k\Omega} = \frac{200\ V}{50} = 4\ V$$

with $$I_E = \frac{V_E}{R_E} = \frac{4\ V - 0.7\ V}{2\ k\Omega} = \frac{3.3\ V}{2\ k\Omega} = 1.65\ mA$$

so that $$r_e = \frac{26\ mV}{1.65\ mA} \cong \mathbf{15.76\ \Omega}$$

and $$\beta r_e = 100(15.76\ \Omega) = 1576\ \Omega = \mathbf{1.576\ k\Omega}$$

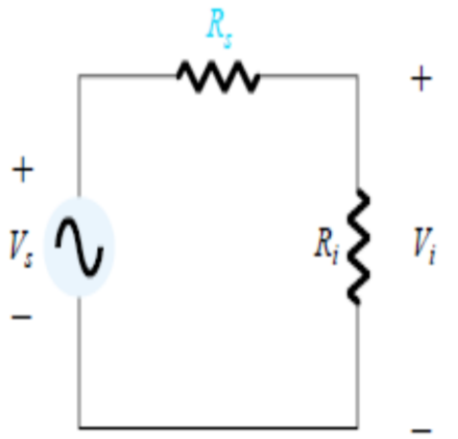
Midband Gain $A_v = \frac{V_o}{V_i} = \frac{-R_C \| R_L}{r_e} = -\frac{(4\,k\Omega)\|(2.2\,k\Omega)}{15.76\,\Omega} \cong -90$

$$V_i = \frac{R_i V_s}{R_i + R_s}$$

or $$\frac{V_i}{V_s} = \frac{R_i}{R_i + R_s} = \frac{1.32\,k\Omega}{1.32\,k\Omega + 1\,k\Omega} = 0.569$$

so that $$A_{v_s} = \frac{V_o}{V_s} = \frac{V_o}{V_i}\frac{V_i}{V_s} = (-90)(0.569)$$

$$= -51.21$$



C_s

$$R_i = R_1 \| R_2 \| \beta r_e = 40\,k\Omega \| 10\,k\Omega \| 1.576\,k\Omega \cong 1.32\,k\Omega$$

$$f_{L_S} = \frac{1}{2\pi R_i C_s} = \frac{1}{(6.28)(1.32\,k\Omega)(10\,\mu F)}$$

$$f_{L_S} \cong \mathbf{12.06\,Hz}$$

C_C

$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C} \quad \text{with} \quad R_o = R_C \| r_o \cong R_C$$

$$= \frac{1}{(6.28)(4\,k\Omega + 2.2\,k\Omega)(1\,\mu F)}$$

$$\cong \mathbf{25.68\,Hz}$$

C_E

$$R_e = R_E \| \left(\frac{R_1 \| R_2}{\beta} + r_e\right)$$
$$= 2\,\text{k}\Omega \| \left(\frac{40\,\text{k}\Omega \| 10\,\text{k}\Omega}{100} + 15.76\,\Omega\right)$$
$$= 2\,\text{k}\Omega \| \left(\frac{8\,\text{k}\Omega}{100} + 15.76\,\Omega\right)$$
$$= 2\,\text{k}\Omega \| (80\,\Omega + 15.76\,\Omega)$$
$$= 2\,\text{k}\Omega \| 95.76\,\Omega$$
$$= 91.38\,\Omega$$

$$f_{L_E} = \frac{1}{2\pi R_e C_E} = \frac{1}{(6.28)(91.38\,\Omega)(20\,\mu\text{F})} = \frac{10^6}{11{,}477.73} \cong \mathbf{87.13\ Hz}$$