Ministry of Higher Education and Scientific Research

Al-Mustaqbal University

College of Engineering Technology

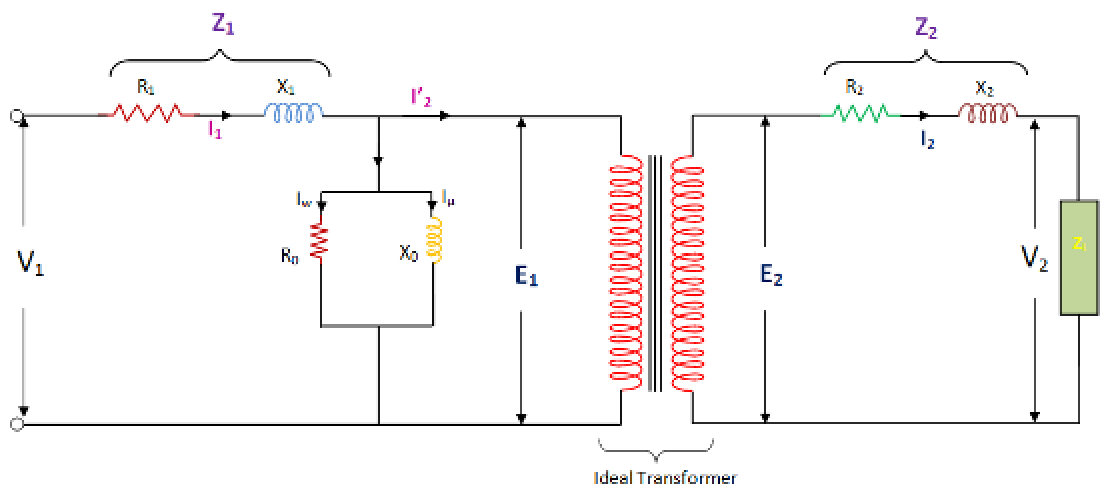
Medical Instrumentation Techniques Engineering Department

Electrical Machines

Second Class

Lectures 5 and 6

Equivalent Circuit of a Transformer



By Prof.Dr Bayan M Sabbar

1. Equivalent Circuit of a Single-Phase Transformer

The analysis of a transformer can be carried out by using an equivalent circuit. Equivalent circuit is derived considering the following:-

- The primary and secondary windings have finite resistances considered as lumped parameters.
- The leakage fluxes are modelled as leakage reactance in the equivalent circuit.
- The core-loss component of current is modelled using a shunt resistance.
- The magnetization of the core is modelled using a magnetizing reactance as a shunt branch.

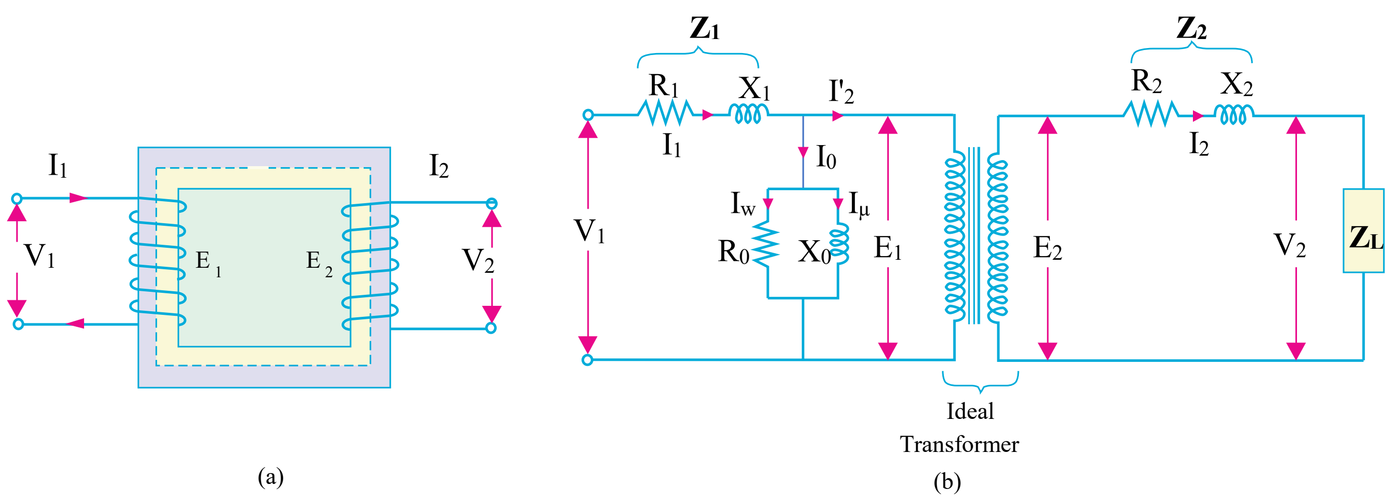


Fig.1 equivalent circuit diagram for the transformer with load

The transformer shown diagrammatically in Fig.1 (*a*) can be resolved into an equivalent circuit in which the resistance and leakage reactance of the transformer are imagined to be external to the winding whose only function then is to transform the voltage.

Generally there are two modes of operations in the transformers which are:

1- No-load operation.

2- On load operation.

The voltages, currents and elements of the circuit can be defined as in Table 1.

SYMBOL	DEFINITION
R_1, R_2	Primary and secondary resistances
X_1, X_2	primary and secondary leakage reactances
R_0	Exciting resistance
X_0	Exciting reactance
I_1	Primary side current
I_0	No-load/Excitation current component of primary current
I_w (or I_{c1})	Core-loss component of no-load current
I_μ (or I_{m1})	Magnetizing component of no-load current
I'_2	Load component of primary current
I_2	Secondary side current (load current)
V_1	Applied primary side voltage
V_2	Secondary side terminal voltage
E_1	Primary side induced emf
E_2	Secondary side induced emf

Table 1 Quantities of equivalent circuit.

1.1 Transformer at No-load

A transformer is said to be on no-load when its secondary winding is kept open and no-load is connected across it. As such, no current flows through the secondary i.e., $I_2 = 0$. Hence, the secondary winding is not causing any effect on the magnetic flux set-up in the core or on the current drawn by the primary. But the losses cannot be ignored. At no-load, a transformer draws a small current I_0 (usually 2 to 10% of the rated value). This current has to supply the iron losses (hysteresis and eddy current losses) in the core and a very small amount of copper loss in the primary (the primary copper losses are so small as compared to core losses that they are generally neglected moreover secondary copper losses are zero as I_2 is zero).

Therefore, current I_0 lags behind the voltage vector V_1 by an angle I_0 (called hysteresis angle of advance) which is less than 90°, as shown in Fig. 2. The angle of lag depends upon the losses in the transformer.

The basic equations in this mode can viewed as follows:-

Working component,	$I_w = I_0 \cos\phi_0$
Magnetizing component,	$I_\mu = I_0 \sin\phi_0$
No-load current,	$I_0 = \sqrt{{I_\mu}^2 + {I_w}^2}$ $\mathbf{I_0} = I_w + jI_\mu$
Primary p.f. at no-load,	$\cos\phi_0 = \frac{I_w}{I_0}$
No-load power input,	$P_0 = V_1 I_0 \cos\phi_0$
Exciting resistance,	$R_0 = \frac{V_1}{I_w}$
Exciting reactance	$X_0 = \frac{V_1}{I_\mu}$

Table 2. No-load parameters in transformer.

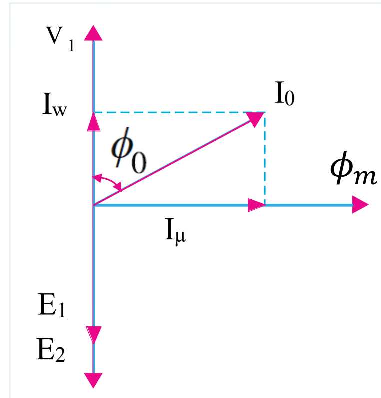


Fig.2 No-load current

Examples 1: A 230/110 V single-phase transformer has a core loss of 100 W. If the input under no-load condition is 400 VA, find core loss current, magnetizing current and no-load power factor angle.

Solution

No- load current $V_1 I_0 = 400$ VA, $I_0 = \frac{400}{230} = \mathbf{1.739\ A}$

Core loss current $I_w = \frac{P_i}{V_1} = \frac{100}{230} = \mathbf{0.4348\ A}$

Magnetizing current $I_\mu = \sqrt{{I_0}^2 - {I_w}^2} = \sqrt{(1.739)^2 - (0.4348)^2} = \mathbf{1.684\ A}$

No-load power factor, $\cos\phi_0 = \frac{I_w}{I_0} = \frac{0.4348}{1.739} = \mathbf{0.25}$ lag

No-load power factor angle $\phi_0 = \cos^{-1} 0.25 = \mathbf{75.52^o}$

Example 2: At open circuit, transformer of 10 kVA, 500/250 V, 50 Hz draws a power of 167 watt at 0.745 A, 500 V. Determine the magnetizing current, Ro current, no-load power factor, hysteresis angle of advance, equivalent resistance and reactance of exciting circuit referred to primary side.

Solution

$V_1 = 500\,V\,,\, I_0 = 0.745\,A,\, P_0 = 167\,W$

$$I_w = \frac{P_0}{V_1} = \frac{167}{500} = \mathbf{0.334\ A}$$

$$I_\mu = \sqrt{{I_0}^2 - {I_w}^2} = \sqrt{(0.745)^2 - (0.334)^2} = \mathbf{0.666\ A}$$

$$\cos\phi_0 = \frac{I_w}{I_0} = \frac{0.334}{0.745} = \mathbf{0.448\ lag}$$

$$\phi_0 = \cos^{-1} 0.448 = \mathbf{63.36° \ lag}$$

$$R_0 = \frac{V_1}{I_w} = \frac{500}{0.334} = \mathbf{1497\ \Omega}$$

$$X_0 = \frac{V_1}{I_\mu} = \frac{500}{0.666} = \mathbf{750\ \Omega}$$

1.2 Transformer on load

When the secondary is loaded, the secondary current I_2 is set up. The magnitude and phase of I_2 with respect to V_2 is determined by the characteristics of the load. Current I_2 is in phase with V_2 if load is non-inductive, it lags if load is inductive and it leads if load is capacitive. A set of equation can be drawn using basic theory of electric circuit analysis.

Applying KVL in the primary side of equivalent circuit:-

$$V_1 = E_1 + I_1(R_1 + jX_1)$$

Applying KVL in the secondary side of equivalent circuit:-

$$E_2 = V_2 + I_2(R_2 + jX_2)$$

Since $\frac{E_1}{E_2} = \frac{I_2}{I_2'} = a = \frac{1}{k}$ using $E_1 = aE_2$ we get:

$$V_1 = aE_2 + I_1(R_1 + jX_1)$$

Then we can rewrite the equation of V_1 by substituting the equation of E_2.

$$V_1 = a[V_2 + I_2(R_2 + jX_2)] + I_1(R_1 + jX_1)$$

Since $I_2 = aI_2' \Longrightarrow V_1 = [aV_2 + I_2'a^2(R_2 + jX_2)] + I_1(R_1 + jX_1)$

Let $R_2' = a^2R_2$ and $X_2' = a^2X_2$

Where R_2' is called secondary resistance referred to the primary side and X_2' is called secondary leakage reactance referred to the primary side.

Another modification to V_1 applying the same rules above then:

$$V_1 = V_2' + I_2'(R_2' + jX_2') + I_1(R_1 + jX_1)$$

Where

$V_2' = aV_2$ is called secondary voltage referred to the primary side, $I_2' = \frac{I_2}{a}$ is called secondary current referred to the primary side.

In Table 3 shown the quantities of the secondary side with their referred quantities.

Referred to the primary	Referred to the secondary
$E_1 = E_2' = \frac{E_2}{k}$	$E_1' = E_2 = kE_1$
$V_2' = \frac{V_2}{k}$	$V_1' = kV_1$
$I_2' = kI_2$	$I_1' = \frac{I_1}{k}$
$X_2' = \frac{X_2}{k^2}$	$X_1' = k^2 X_1$
$R_2' = \frac{R_2}{k^2}$	$R_1' = k^2 R_1$

Table 3. Referred quantities.

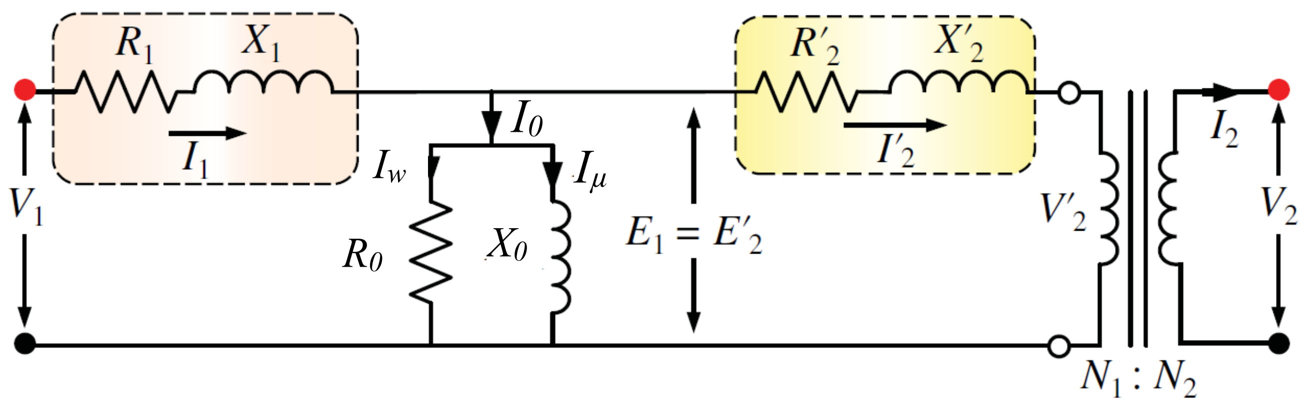


Fig 3. Modified equivalent circuit of a single-phase transformer.

1.3 Exact Equivalent Circuit

The transformer circuit can be moved to the right or left by referring all quantities to the primary or secondary side, respectively. This is almost invariably done. The equivalent circuit moved to primary is shown in Fig. 3.

If we shift all the impedances from one winding to the other, the transformer core is eliminated and we get an equivalent electrical circuit. Various voltages and currents can be readily obtained by solving this electrical circuit.

1.4 Approximate Equivalent Circuit

The equivalent circuit can be simplified by assuming small voltage drop across the primary impedance and $V_1 = E_1$. If the applied voltage and the induced emf are the same then the shunt branch can be moved across the source voltage and the approximate equivalent circuit is drawn as shown in Figure 4.

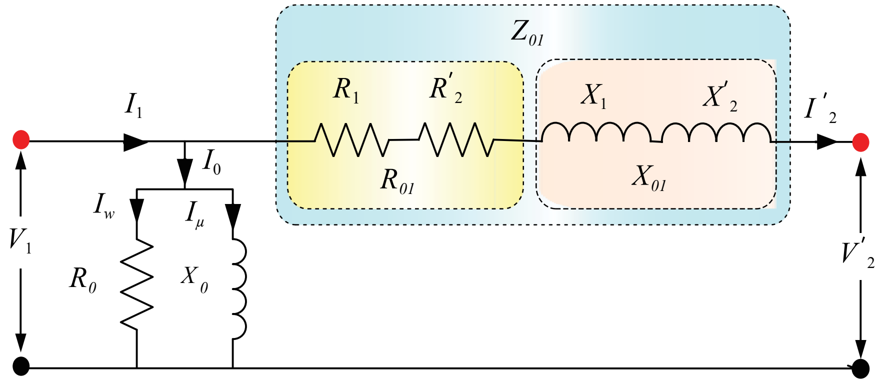


Fig 4. Approximate equivalent circuit

From the equivalent circuit the following relations are obtained

$$R_{01} = R_1 + R_2' \qquad , X_{01} = X_1 + X_2'$$

$$\boldsymbol{Z_{01}} = R_{01} + jX_{01}$$

Example 3: A 30 kVA, 2400/120 V, 50-Hz transformer has a high voltage winding resistance of 0.1 Ω and a leakage reactance of 0.22 Ω. The low voltage winding resistance is 0.035 Ω and the leakage reactance is 0.012 Ω. Find the equivalent winding resistance, reactance and impedance (only magnitude) referred to :

1- High Voltage Side.

2- Low-Voltage Side.

Solution

$$k = \frac{120}{2400} = 1/20, R_1 = 0.1\ \Omega\ , X_1 = 0.22\ \Omega$$

$R_2 = 0.035\ \Omega$ and $X_2 = 0.012\ \Omega$

1- For high voltage side

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{k^2} = 0.1 + \frac{0.035}{\left(\frac{1}{20}\right)^2} = \mathbf{14.1\ \Omega}$$

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{k^2} = 0.22 + \frac{0.012}{\left(\frac{1}{20}\right)^2} = \mathbf{5.02\ \Omega}$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{14.1^2 + 5.02^2} = \mathbf{15\ \Omega}$$

2- For low voltage side

$$R_{02} = R_2 + R_1' = R_2 + k^2 R_1 = 0.035 + \left(\frac{1}{20}\right)^2 \times 0.1 = \mathbf{0.03525\ \Omega}$$

$$X_{02} = X_2 + X_1' = X_2 + k^2 X_1 = 0.012 + \left(\frac{1}{20}\right)^2 \times 0.22 = \mathbf{0.0125\ \Omega}$$

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{0.0325^2 + 0.01255^2} = \mathbf{0.0374\ \Omega}$$

Example 4: The parameters of a 2300/230 V, 50-Hz transformer are given below:

$$R_1 = 0.286\ \Omega, \qquad R_2' = 0.319\ \Omega\ , \qquad R_0 = 250\ \Omega$$

$$X_1 = 0.73\ \Omega\ ,\ X_2' = 0.73\ \Omega\ , \qquad X_0 = 1250\ \Omega$$

The secondary load impedance $Z_L = 0.387 + j\,0.29\ \Omega$. Using the exact equivalent circuit Calculate the following.

1- Primary power factor.
2- Power input.
3- Power output.
4- Cu loss.
5- Efficiency.

Solution

$k = 230/2300 = 0.1$, $\boldsymbol{Z_L} = 0.387 + j\,0.29\ \Omega$

$Z'_L = {Z_L}/{k^2}$ =100 (0.387 + j 0.29) = 38.7 + j 29 = **48.4 ∠36.8° Ω**

$Z'_2 + Z'_L$ =(38.7 + 0.319) + j(29 + 0.73) = 39.02 + j29.73 = **49∠ 37.3° Ω**

$1/Z_m = 1/R_0 + 1/jX_0 \Rightarrow Z_m$ =240 + j48 = **245 ∠11.3° Ω**

$Z_m//(Z'_2 + Z'_L)$ = 245∠11.3° // 49 ∠ 37.3° = **41.4∠33° Ω**

$Z_{t1} = Z_1 + (Z_m//(Z'_2 + Z'_L))$=(0.286+j 0.73)+ 41.4 ∠33° = **42∠33.7° Ω**

$I_1 = V_1/Z_{t1}$=2300∠0 / 42∠33.7° = **54.8∠−33.7° A**

$Z_m + Z'_2 + Z'_L$=245 ∠11.3° + 49 ∠ 37.3° = **290∠15.6° Ω**

$I'_2 = I_1 \times \left(Z_m/Z'_2 + Z'_L + Z_m\right)$ = 54.8∠−33.7 × 0.845∠−4.3 = **46.2 ∠ − 38° A**

$I_0 = I_1 \times \left(Z'_2 + Z'_L/Z'_2 + Z'_L + Z_m\right)$ = 54.8∠−33.7 × $\frac{49 \angle 37.3}{290\angle 15.6}$ = **9.26∠−12° A**

1- Primary power factor = cos(33.7) = **0.832 lag**

2- Power input = $V_1 I_1 \cos \phi_1$ = 2300 × 54.8 ×0.832 = **105 kW**

3- Power output = $I_2'^2 R'_L$ = 46.2^2×38.7 = **82.7 kW**

4- Primary Cu loss = 54.8^2×0.286 = **860 W**

Secondary Cu loss = 46.2^2×0.319 = **680 W**

Core Cu loss = 9.26^2×240 = **20.6 kW**

5- Efficiency = $\frac{82.7}{105}$×100 = **78.8 %**

Example 5: The equivalent circuit parameters of a single-phase 240/2400, 50 Hz, transformer are R_0 =600 Ω, X_0 =300 Ω, R_{01} =0.25 Ω, X_{01} =0.75 Ω. The transformer is supplying a load of 400+ j 200 Ω. Keeping the primary voltage of 240V, calculate the

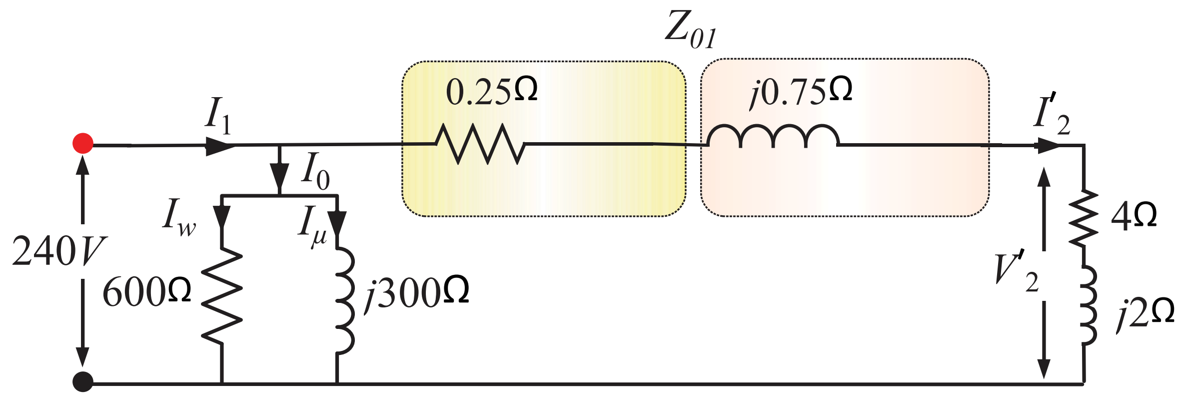
1- The secondary terminal voltage
2- Current in the primary winding

3- Power factor of the primary side

4- Power output

5- Power Input

Solution



Since the equivalent circuit is referred to the low voltage (primary side), the load impedance is also transformed to the low voltage side.

$k = 2400/240 = 10$

$\boldsymbol{Z'_L} = {\boldsymbol{Z_L}}/{k^2}$ =(400 + j200) $(0.1)^2$ = **4 + j2 Ω**

$\boldsymbol{Z'_L} + \boldsymbol{Z_{01}}$ =0.25 + j0.75 + 4 + j2 = 4.25 + j2.75 = **5.062∠32.9 Ω**

$$\boldsymbol{I'_2} = {\boldsymbol{V_1}}/{(\boldsymbol{Z'_L} + \boldsymbol{Z_{01}})} = \frac{240\angle 0}{5.062\angle 32.9} = 47.412\angle -32.9 = \mathbf{39.8 - j25.753\ A}$$

1- Secondary terminal voltage (without phase)

$V'_2 = I'_2 Z_L$ =5.062 × $\sqrt{4^2 + 2^2}$ = **212.03V**

2- Primary current:

The core loss component of current $I_w = \frac{V_1}{R_0} = \frac{240}{600}$ = **0.4 A**

The magnetizing component of current $I_\mu = \frac{V_1}{X_0} = \frac{240}{300}$ = **0.8 A**

The no-load current $I_0 = I_w + jI_\mu$ = **0.4 − j0.8 A**

The primary current

$$I_1 = I_0 + I_2' = 39.8 - j25.753 + 0.4 - j0.8 = 40.2 - j26.553 = \mathbf{48.178\angle{-33.44}\ A}$$

3- Power factor of the primary current pf = cos(33.44) = **0.834 lagging**

4- Power output = $I_2'^2 R_L' = 47.412^2 \times 4 =$ **8.99 kW**

5- Power Input = $V_1 I_1 \cos\phi_1 = 240 \times 48.178 \times \cos(33.44) =$ **9.65 kW**

2. Auto-transformer

It is a transformer with one winding only, part of this being common to both primary and secondary. Obviously, in this transformer the primary and secondary are not electrically *isolated* from each other as is the case with a 2-winding transformer. But its theory and operation are similar to those of a two-winding transformer. Because of one winding, it uses less copper and hence is cheaper. It is used where transformation ratio differs little from unity. Fig. 5 shows both step down and step-up auto-transformers.

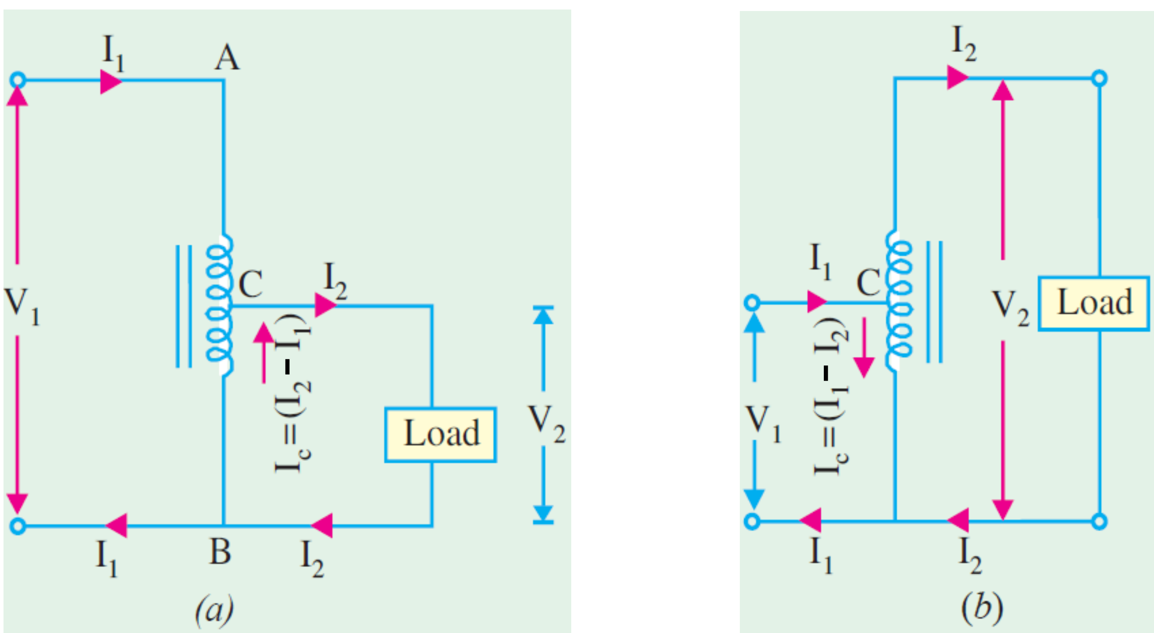


Fig 5. Auto-transformer

As shown in Fig. 5, *AB*, is primary winding having N_1 turns and *BC* is secondary winding having N_2 turns. Neglecting iron losses and no-load current.

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$

2.1 Saving of Cu

Volume and hence weight of Cu, is proportional to the length and area of the cross-section of the conductors. Now, length of conductors is proportional to the number of turns and cross-section depends on current. Hence, weight is proportional to the product of the current and number of turns.

Wt. of Cu in section AC is $\propto (N_1 - N_2)\, I_1$.

Wt. of Cu in section BC is $\propto N_2\, (I_2 - I_1)$.

$\therefore$ Total Wt. of Cu in auto-transformer $\propto (N_1 - N_2)\, I_1 + N_2\, (I_2 - I_1)$

If a two-winding transformer were to perform the same duty, then

Wt. of Cu on its primary $\propto N_1 I_1$; Wt. of Cu on secondary $\propto N_2 I_2$

Total Wt. of Cu $\propto N_1 I_1 + N_2 I_2$

$$\frac{\text{Wt. of Cu in auto} - \text{transformer}}{\text{Wt. of Cu in ordinary transformer}} = \frac{(N_1 - N_2)\, I_1 + N_2\, (I_2 - I_1)}{N_2\, I_1 + N_2\, I_2} = 1 - K$$

Wt. of Cu in auto-transformer

$(W_a) = (1 - K) \times$ (Wt. of Cu in ordinary transformer W_0)

$\therefore$ Saving = $W_0 - W_a = W_0 - (1 - K)\, W_0 = KW_0$

$\therefore$ Saving percent = $K \times 100$ %

Hence, saving will increase as K approaches unity.

2.2 Advantages of autotransformers

1- An autotransformer requires less Cu than a two-winding transformer of similar rating.

2- An autotransformer operates at a higher efficiency than a two-winding transformer of similar rating.

3- An autotransformer has better voltage regulation than a two-winding transformer of the same rating.

4- An autotransformer has smaller size than a two-winding transformer of the same rating.

5- An autotransformer requires smaller exciting current than a two-winding transformer of the same rating.

2.3 Applications of Auto transformer

The various applications of an auto-transformer are,

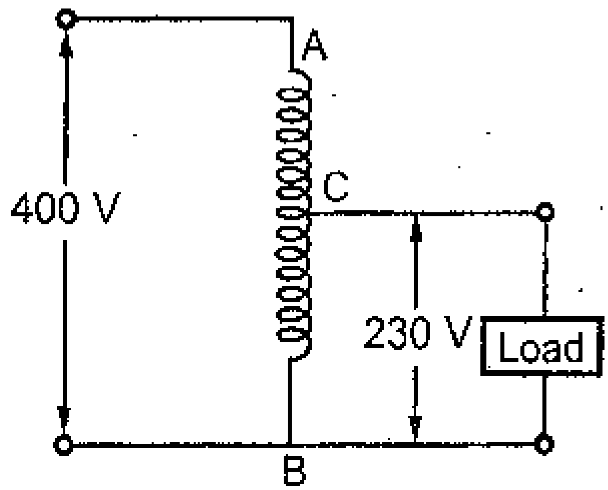
1- For safely starting the machines like induction motors, synchronous motors i.e. as a starter.

2- To give a small boost to a distribution cable to compensate for a voltage drop i.e. as a booster.

3- As a furnace transformer to supply power to the furnaces at the required supply voltage.

4- For interconnecting the systems which are operating roughly at same voltage level.

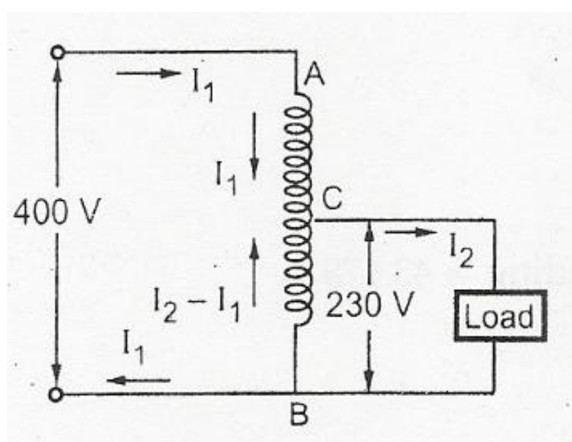
Example 5: In Figure shown an auto transformer used to supply a load of 2 kW at 230 V from a 400 V a.c. supply. Find the currents in parts AC and BC, neglecting losses and no load current . Also find the copper saving percent due to the use of autotransformer instead of using two winding transformer: Assume purely resistive load.



As the load is resistive, $\cos\phi_L = 1$

$P_{out} = V_2 I_2 \cos\phi_2 = 230 \times I_2 \times 1 = 2 \times 10^3 \Rightarrow I_2 = \mathbf{8.6956\ A}$

$$\frac{V_2}{V_1} = K = \frac{230}{400} = \mathbf{0.575}$$

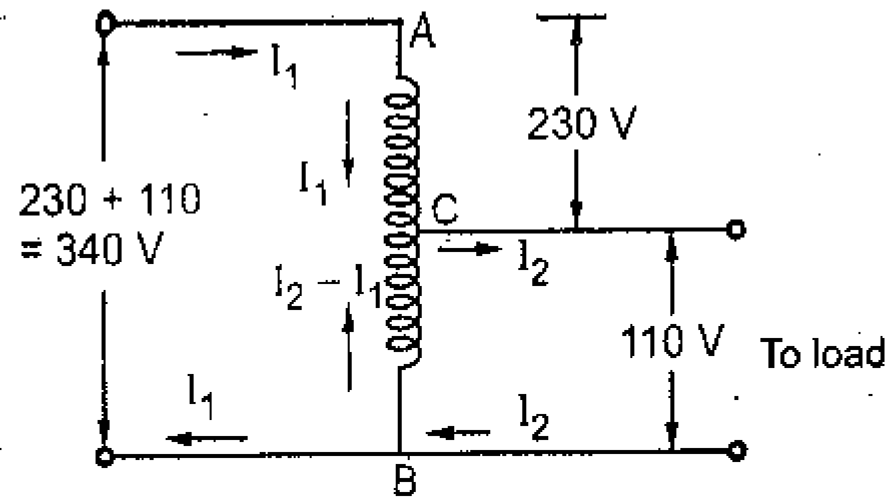


$\frac{I_1}{I_2} = K \Rightarrow I_1 = \mathbf{5\ A}$ which is the current in AC part

Current in BC = $I_2 - I_1 = \mathbf{3.6956\ A}$

Copper saving = $K \times 100\% = \mathbf{57.5\%}$

Example 6: A 10 kVA, 230/110 V transformer is to be used as an autotransformer. What will be the voltage ratio and output rating of an autotransformer.



Solution

$V_1 = 230\ V\ , V_2 = 110\ V\ , kVA = 10\ kVA$

Current through 230 V = $\frac{VA}{230} = \frac{10\times10^3}{230} = \mathbf{43.478\ A}$

Current through 110 V = $\frac{VA}{110} = \frac{10\times10^3}{110} = \mathbf{90.909\ A}$

Now as secondary voltage of two winding transformer is 110 V, let us assume. that autotransformer output voltage required is 110 V. So it can be connected as an autotransformer as shown in the Figure The part AC is primary of two winding while BC is secondary of two winding transformer.

$V_1 = 230 + 110 = \mathbf{340\ V},\quad V_2 = \mathbf{110\ V}$

$K = \frac{V_2}{V_1} = \frac{110}{340} = \mathbf{0.3235}$

$$K = \frac{I_1}{I_2} \Longrightarrow I_2 = \frac{43.478}{0.3235} = \mathbf{134.386\ A}$$

Output rating = $V_2 \times I_2$ = 110 x 134.386 = **14.782 kVA**