Ministry of Higher Education and Scientific Research

Al-Mustaqbal University

College of Engineering Technologies

Medical Instrumentation Techniques Engineering Department

Electrical Circuits

First year

2.4 Kirchhoff's Laws

Ohm's law by itself is not sufficient to analyze circuits. However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits. Kirchhoff's laws were first introduced in 1847 by the German physicist Gustav Robert Kirchhoff (1824–1887). These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.

> **Kirchhoff's current law (KCL)** states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

Mathematically, KCL implies that

$$\sum_{n=1}^{N} i_n = 0 \tag{2.13}$$

where N is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node. By this law, currents

> The sum of the currents entering a node is equal to the sum of the currents leaving the node.

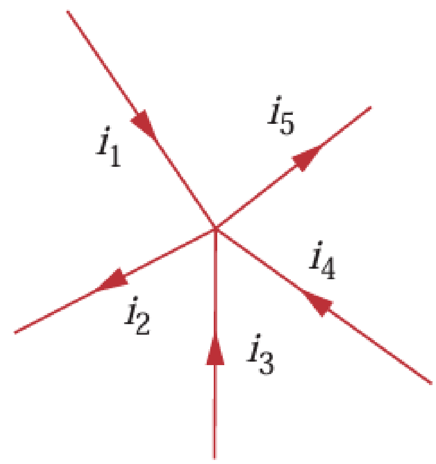


Figure 2.16
Currents at a node illustrating KCL.

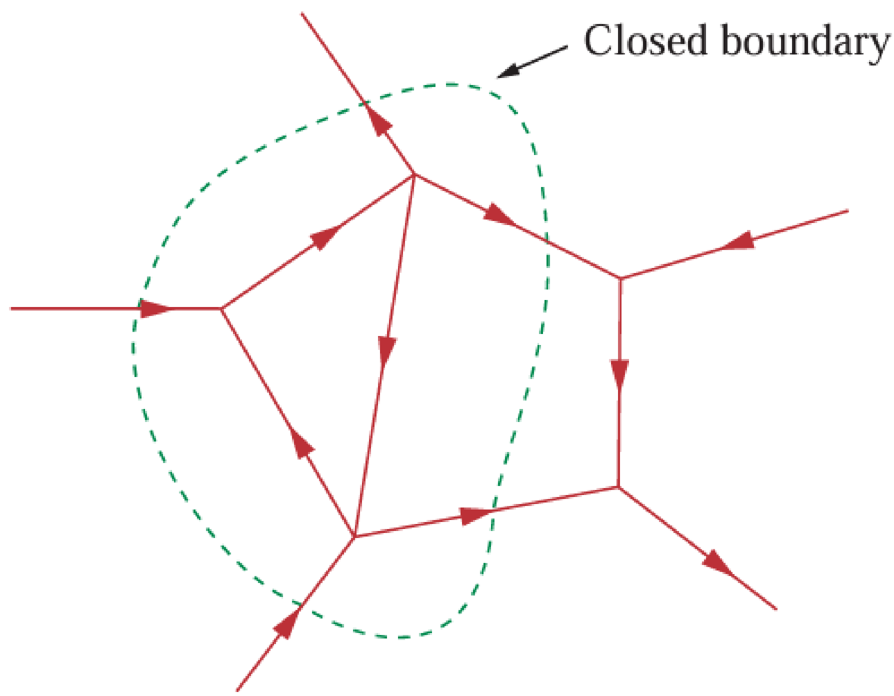
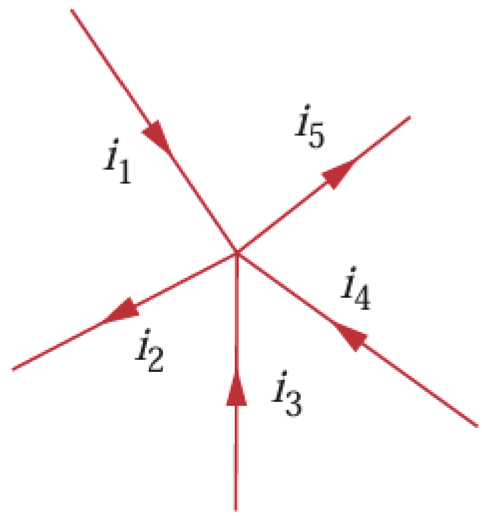


Figure 2.17
Applying KCL to a closed boundary.



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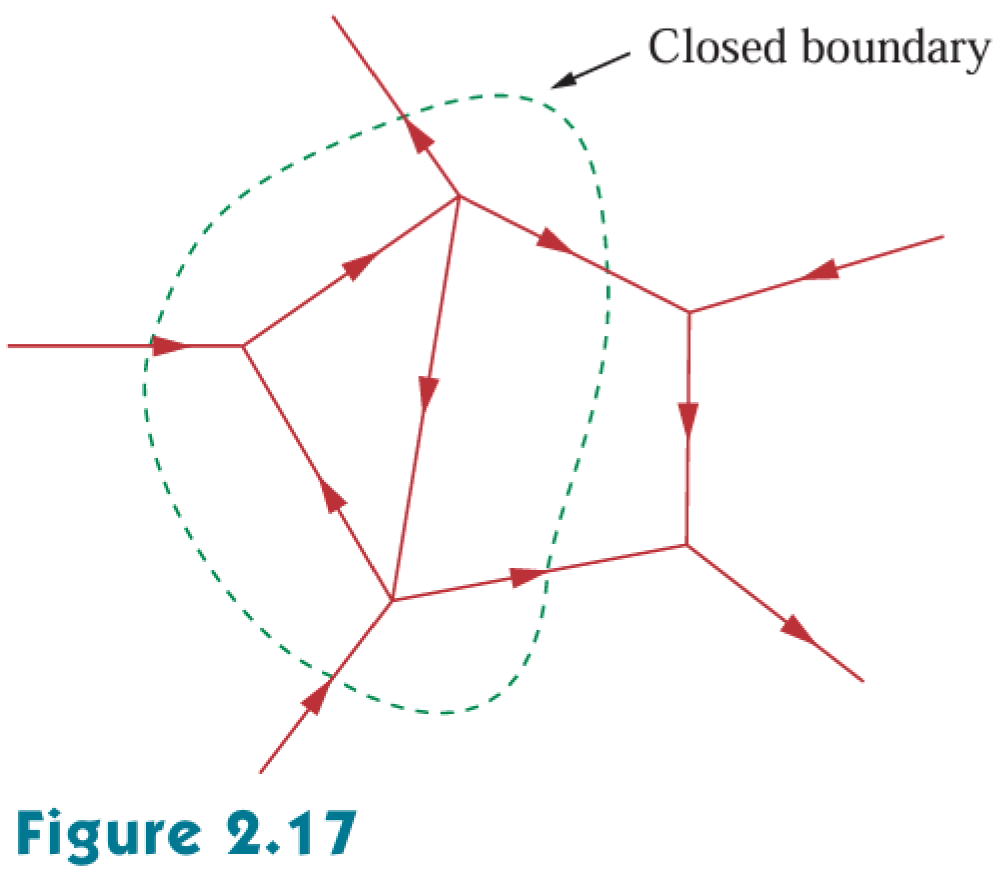


Figure 2.17
Applying KCL to a closed boundary.

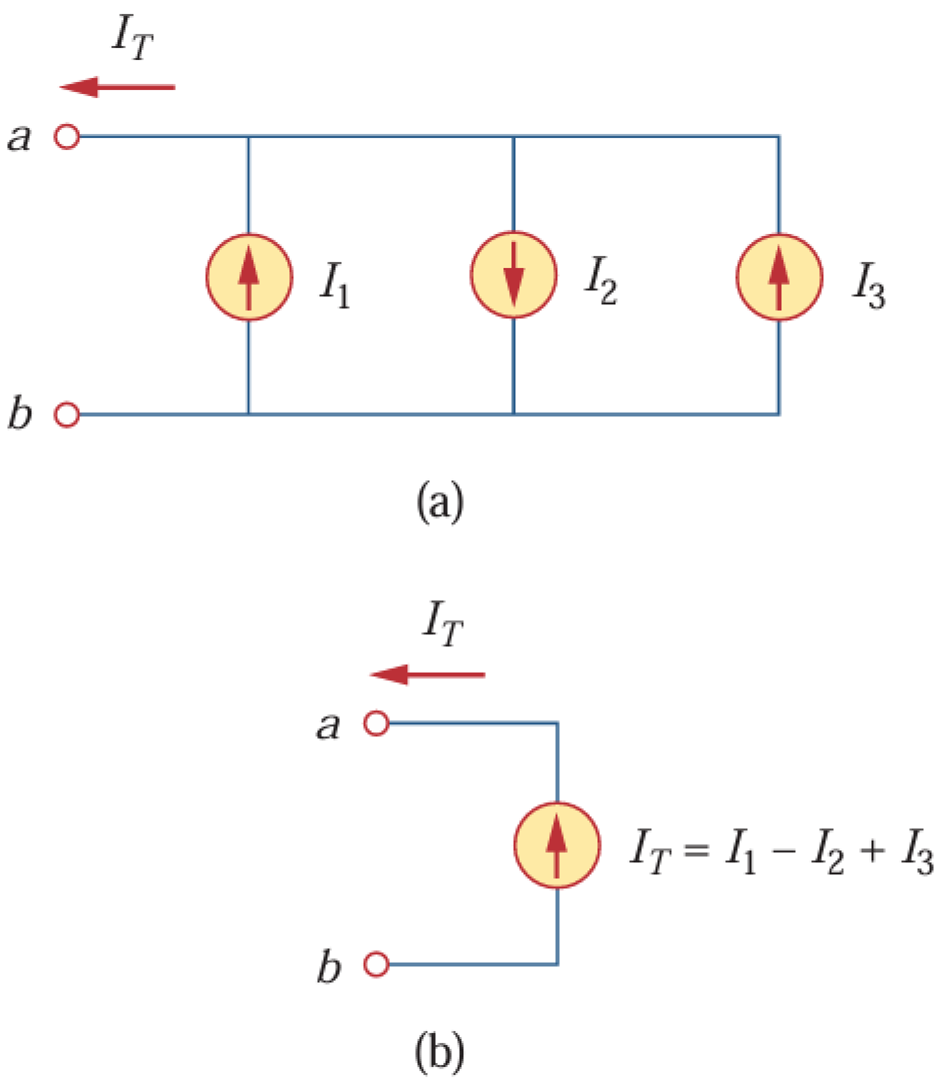


Figure 2.18
Current sources in parallel: (a) original circuit, (b) equivalent circuit.

Fig. 2.18(a) can be combined as in Fig. 2.18(b). The combined or equivalent current source can be found by applying KCL to node *a*.

$$I_T + I_2 = I_1 + I_3$$

or

$$I_T = I_1 - I_2 + I_3 \tag{2.18}$$

A circuit cannot contain two different currents, I_1 and I_2, in series, unless $I_1 = I_2$; otherwise KCL will be violated.

Kirchhoff's second law is based on the principle of conservation of energy:

> **Kirchhoff's voltage law (KVL)** states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that

$$\boxed{\sum_{m=1}^{M} v_m = 0} \tag{2.19}$$

where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the mth voltage.

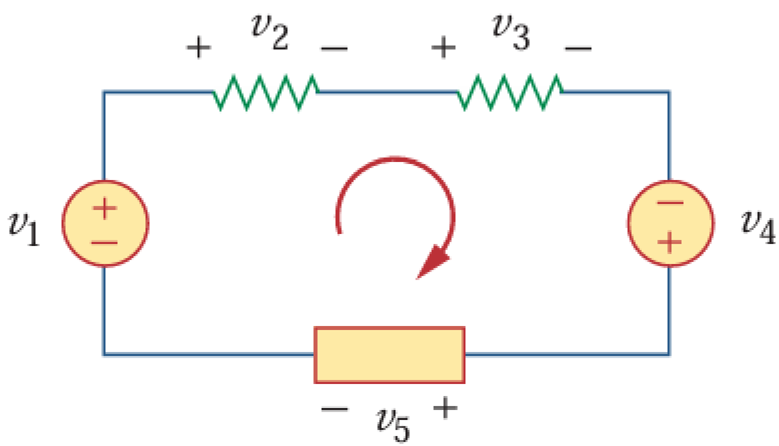


Figure 2.19
A single-loop circuit illustrating KVL.

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0 \tag{2.20}$$

Rearranging terms gives

$$v_2 + v_3 + v_5 = v_1 + v_4 \tag{2.21}$$

which may be interpreted as

$$\text{Sum of voltage drops} = \text{Sum of voltage rises} \tag{2.22}$$

Example 2.5

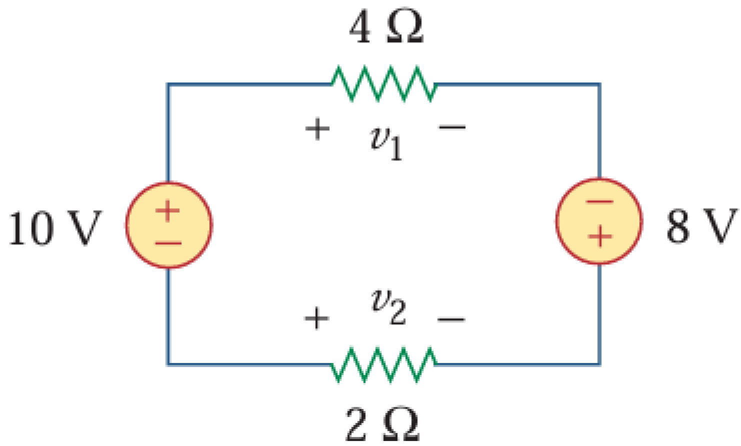


Figure 2.22
For Practice Prob. 2.5.

Determine v_o and i in the circuit shown in Fig. 2.23(a).

For the circuit in Fig. 2.21(a), find voltages v_1 and v_2.

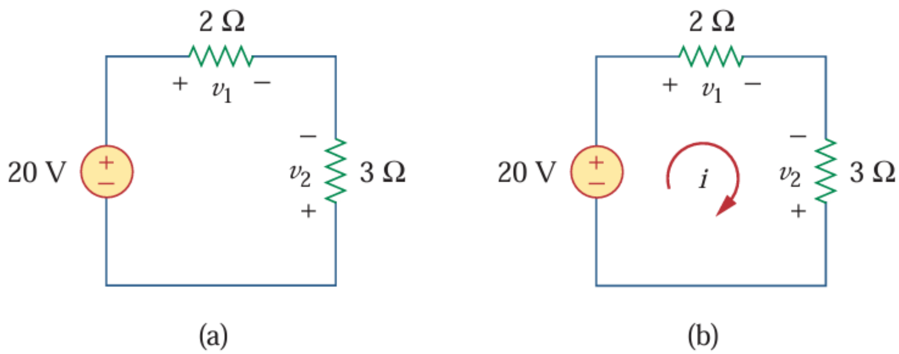


Figure 2.21
For Example 2.5.

Solution:

To find v_1 and v_2, we apply Ohm's law and Kirchhoff's voltage law. Assume that current i flows through the loop as shown in Fig. 2.21(b). From Ohm's law,

$$v_1 = 2i, \qquad v_2 = -3i \tag{2.5.1}$$

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0 \tag{2.5.2}$$

Substituting Eq. (2.5.1) into Eq. (2.5.2), we obtain

$$-20 + 2i + 3i = 0 \qquad \text{or} \qquad 5i = 20 \qquad \Rightarrow \qquad i = 4 \text{ A}$$

Substituting i in Eq. (2.5.1) finally gives

$$v_1 = 8 \text{ V}, \qquad v_2 = -12 \text{ V}$$

Find v_1 and v_2 in the circuit of Fig. 2.22.

Determine v_o and i in the circuit shown in Fig. 2.23(a).

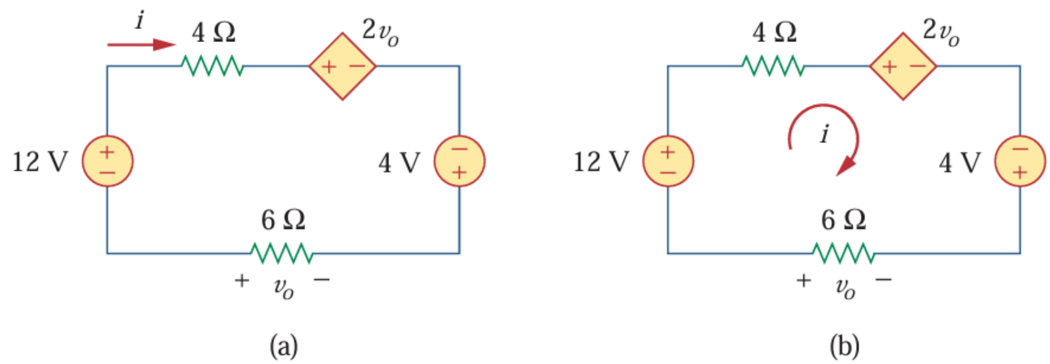


Figure 2.23

Solution:

We apply KVL around the loop as shown in Fig. 2.23(b). The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \qquad \textbf{(2.6.1)}$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i \qquad \textbf{(2.6.2)}$$

Substituting Eq. (2.6.2) into Eq. (2.6.1) yields

$$-16 + 10i - 12i = 0 \qquad \Rightarrow \qquad i = -8 \text{ A}$$

and $v_o = 48$ V.

Find v_x and v_o in the circuit of Fig. 2.24.

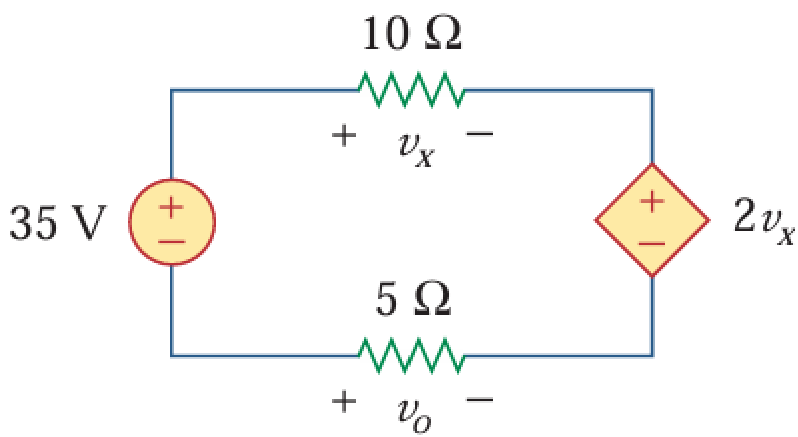


Figure 2.24
For Practice Prob. 2.6.

Find current i_o and voltage v_o in the circuit shown in Fig. 2.25.

Solution:

Applying KCL to node *a*, we obtain

$$3 + 0.5i_o = i_o \qquad \Rightarrow \qquad i_o = 6 \text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$

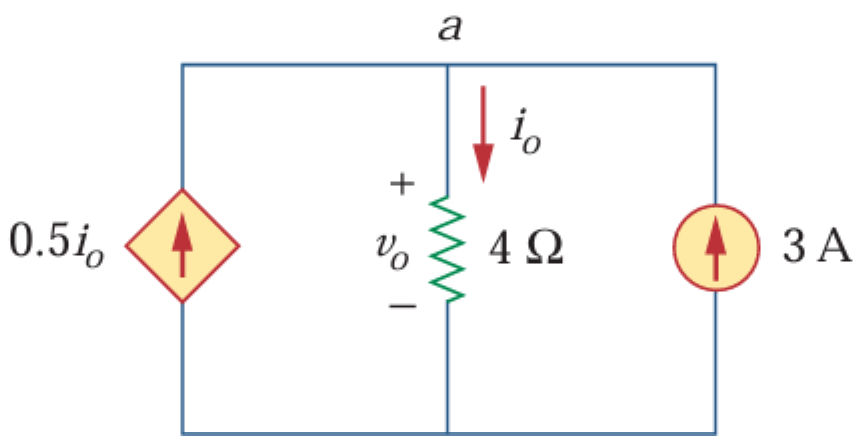


Figure 2.25
For Example 2.7.

Find currents and voltages in the circuit shown in Fig. 2.27(a).

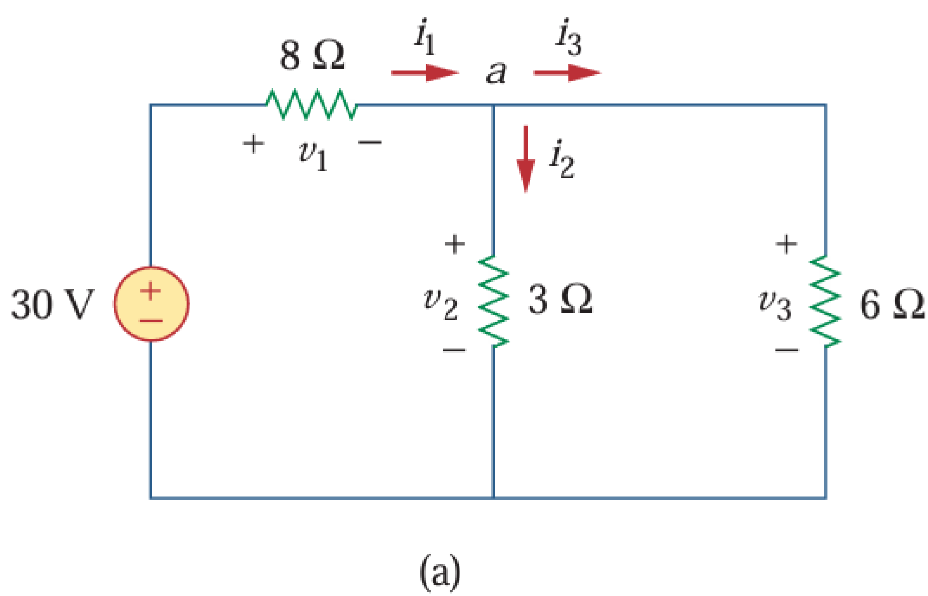
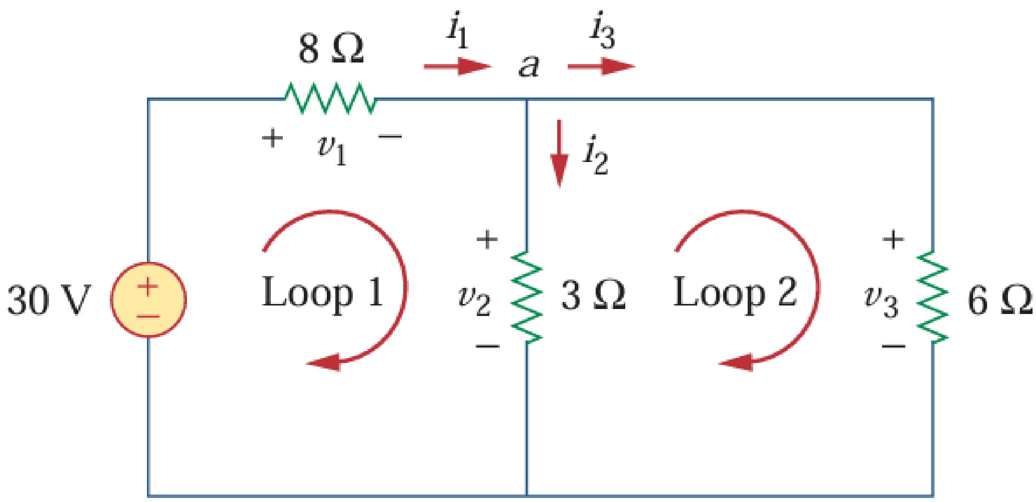


Figure 2.27
For Example 2.8.



(b)

Solution:

We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$$v_1 = 8i_1, \qquad v_2 = 3i_2, \qquad v_3 = 6i_3 \tag{2.8.1}$$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1, v_2, v_3) or (i_1, i_2, i_3). At node *a*, KCL gives

$$i_1 - i_2 - i_3 = 0 \tag{2.8.2}$$

Applying KVL to loop 1 as in Fig. 2.27(b),

$$-30 + v_1 + v_2 = 0$$

We express this in terms of i_1 and i_2 as in Eq. (2.8.1) to obtain

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8} \tag{2.8.3}$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \qquad \Rightarrow \qquad v_3 = v_2 \tag{2.8.4}$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2 as in Eq. (2.8.1). Equation (2.8.4) becomes

$$6i_3 = 3i_2 \qquad \Rightarrow \qquad i_3 = \frac{i_2}{2} \tag{2.8.5}$$

Substituting Eqs. (2.8.3) and (2.8.5) into (2.8.2) gives

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

or $i_2 = 2$ A. From the value of i_2, we now use Eqs. (2.8.1) to (2.8.5) to obtain

$$i_1 = 3\text{ A}, \qquad i_3 = 1\text{ A}, \qquad v_1 = 24\text{ V}, \qquad v_2 = 6\text{ V}, \qquad v_3 = 6\text{ V}$$