Ministry of Higher Education and Scientific Research
Al-Mustaqbal University
College of Engineering Technologies
Medical Instrumentation Techniques Engineering Department
Electrical Circuits
First year

Fundamentals of Electric Circuits

Fundamentals of
Electric Circuits

Charles K. Alexander

Department of Electrical and
Computer Engineering

Cleveland State University

Matthew N. O. Sadiku

Department of
Electrical Engineering

Prairie View A&M University

Charge and Current Relationship

Mathematically, the relationship between current i, charge q, and time t is

$$i \triangleq \frac{dq}{dt} \tag{1.1}$$

where current is measured in amperes (A), and

$$1 \text{ ampere} = 1 \text{ coulomb/second}$$

The charge transferred between time t_0 and t is obtained by integrating both sides of Eq. (1.1). We obtain

$$Q \triangleq \int_{t_0}^{t} i \, dt \tag{1.2}$$

The way we define current as i in Eq. (1.1) suggests that current need not be a constant-valued function. As many of the examples and problems in this chapter and subsequent chapters suggest, there can be several types of current; that is, charge can vary with time in several ways.

If the current does not change with time, but remains constant, we call it a *direct current* (dc).

> A direct current (dc) is a current that remains constant with time.

By convention the symbol I is used to represent such a constant current.

A time-varying current is represented by the symbol i. A common form of time-varying current is the sinusoidal current or *alternating current* (ac).

> An alternating current (ac) is a current that varies sinusoidally with time.

Such current is used in your household, to run the air conditioner, refrigerator, washing machine, and other electric appliances. Figure 1.4

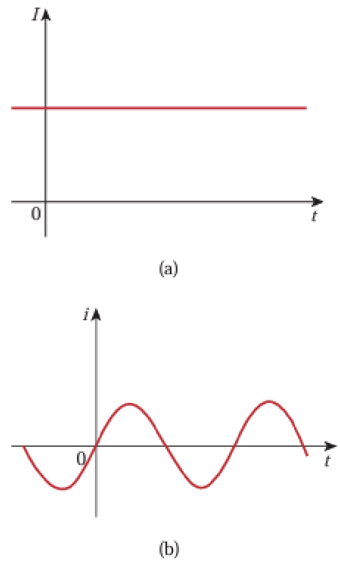

Figure 1.4
Two common types of current: (a) direct current (dc), (b) alternating current (ac).

shows direct current and alternating current; these are the two most common types of current. We will consider other types later in the book.

Once we define current as the movement of charge, we expect current to have an associated direction of flow. As mentioned earlier, the direction of current flow is conventionally taken as the direction of positive charge movement. Based on this convention, a current of 5 A may be represented positively or negatively as shown in Fig. 1.5. In other words, a negative current of −5 A flowing in one direction as shown in Fig. 1.5(b) is the same as a current of +5 A flowing in the opposite direction.

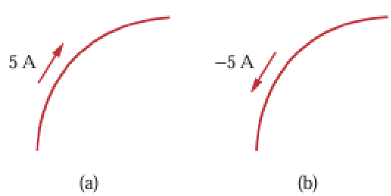

Figure 1.5
Conventional current flow: (a) positive current flow, (b) negative current flow.

Example 1.1

How much charge is represented by 4,600 electrons?

Solution:

Each electron has -1.602×10^{-19} C. Hence 4,600 electrons will have

$$-1.602 \times 10^{-19} \text{ C/electron} \times 4{,}600 \text{ electrons} = -7.369 \times 10^{-16} \text{ C}$$

Practice Problem 1.1

Calculate the amount of charge represented by four million protons.

Answer: $+6.408 \times 10^{-13}$ C.

Example 1.2

The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

At $t = 0.5$,

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$

Practice Problem 1.2

If in Example 1.2, $q = (10 - 10e^{-2t})$ mC, find the current at $t = 0.5$ s.

Answer: 7.36 mA.

Example 1.3

Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution:

$$Q = \int_{t=1}^{2} i\,dt = \int_{1}^{2} (3t^2 - t)\,dt$$

$$= \left(t^3 - \frac{t^2}{2}\right)\Bigg|_1^2 = (8 - 2) - \left(1 - \frac{1}{2}\right) = 5.5 \text{ C}$$

Practice Problem 1.3

The current flowing through an element is

$$i = \begin{cases} 2 \text{ A}, & 0 < t < 1 \\ 2t^2 \text{ A}, & t > 1 \end{cases}$$

Calculate the charge entering the element from $t = 0$ to $t = 2$ s.

Answer: 6.667 C.

1.4 Voltage

As explained briefly in the previous section, to move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery in Fig. 1.3. This emf is also known as *voltage* or *potential difference*. The voltage v_{ab} between two points a and b in an electric circuit is the energy (or work) needed to move a unit charge from a to b; mathematically,

$$v_{ab} \triangleq \frac{dw}{dq} \tag{1.3}$$

where w is energy in joules (J) and q is charge in coulombs (C). The voltage v_{ab} or simply v is measured in volts (V), named in honor of the Italian physicist Alessandro Antonio Volta (1745–1827), who invented the first voltaic battery. From Eq. (1.3), it is evident that

$$1 \text{ volt} = 1 \text{ joule/coulomb} = 1 \text{ newton-meter/coulomb}$$

Thus,

> **Voltage** (or **potential difference**) is the energy required to move a unit charge through an element, measured in volts (V).

Figure 1.6 shows the voltage across an element (represented by a rectangular block) connected to points a and b. The plus (+) and minus (−) signs are used to define reference direction or voltage polarity. The v_{ab} can be interpreted in two ways: (1) point a is at a potential of v_{ab} volts higher than point b, or (2) the potential at point a with respect to point b is v_{ab}. It follows logically that in general

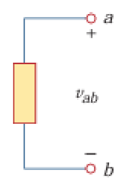

Figure 1.6
Polarity of voltage v_{ab}.

$$v_{ab} = -v_{ba} \tag{1.4}$$

For example, in Fig. 1.7, we have two representations of the same voltage. In Fig. 1.7(a), point a is +9 V above point b; in Fig. 1.7(b), point b is −9 V above point a. We may say that in Fig. 1.7(a), there is a 9-V *voltage drop* from a to b or equivalently a 9-V *voltage rise* from b to a. In other words, a voltage drop from a to b is equivalent to a voltage rise from b to a.

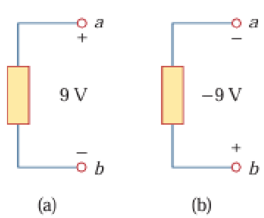

Figure 1.7
Two equivalent representations of the same voltage v_{ab}: (a) point a is 9 V above point b, (b) point b is −9 V above point a.

> Keep in mind that electric current is always *through* an element and that electric voltage is always *across* the element or between two points.

Current and voltage are the two basic variables in electric circuits. The common term *signal* is used for an electric quantity such as a current or a voltage (or even electromagnetic wave) when it is used for conveying information. Engineers prefer to call such variables signals rather than mathematical functions of time because of their importance in communications and other disciplines. Like electric current, a constant voltage is called a *dc voltage* and is represented by V, whereas a sinusoidally time-varying voltage is called an *ac voltage* and is represented by v. A dc voltage is commonly produced by a battery; ac voltage is produced by an electric generator.

1.5 Power and Energy

Although current and voltage are the two basic variables in an electric circuit, they are not sufficient by themselves. For practical purposes, we need to know how much *power* an electric device can handle. We all know from experience that a 100-watt bulb gives more light than a 60-watt bulb. We also know that when we pay our bills to the electric utility companies, we are paying for the electric *energy* consumed over a certain period of time. Thus, power and energy calculations are important in circuit analysis.

To relate power and energy to voltage and current, we recall from physics that:

Power is the time rate of expending or absorbing energy, measured in watts (W).

We write this relationship as

$$p \triangleq \frac{dw}{dt} \tag{1.5}$$

where p is power in watts (W), w is energy in joules (J), and t is time in seconds (s). From Eqs. (1.1), (1.3), and (1.5), it follows that

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi \tag{1.6}$$

or

$$p = vi \tag{1.7}$$

The power p in Eq. (1.7) is a time-varying quantity and is called the *instantaneous power*. Thus, the power absorbed or supplied by an element is the product of the voltage across the element and the current through it. If the power has a + sign, power is being delivered to or absorbed by the element. If, on the other hand, the power has a − sign, power is being supplied by the element. But how do we know when the power has a negative or a positive sign?

Current direction and voltage polarity play a major role in determining the sign of power. It is therefore important that we pay attention to the relationship between current i and voltage v in Fig. 1.8(a). The voltage polarity and current direction must conform with those shown in Fig. 1.8(a) in order for the power to have a positive sign. This is known as the *passive sign convention*. By the passive sign convention, current enters through the positive polarity of the voltage. In this case, $p = +vi$ or $vi > 0$ implies that the element is absorbing power. However, if $p = -vi$ or $vi < 0$, as in Fig. 1.8(b), the element is releasing or supplying power.

Passive sign convention is satisfied when the current enters through the positive terminal of an element and $p = +vi$. If the current enters through the negative terminal, $p = -vi$.

Unless otherwise stated, we will follow the passive sign convention throughout this text. For example, the element in both circuits of Fig. 1.9 has an absorbing power of +12 W because a positive current enters the positive terminal in both cases. In Fig. 1.10, however, the element is supplying power of +12 W because a positive current enters the negative terminal. Of course, an absorbing power of −12 W is equivalent to a supplying power of +12 W. In general,

$$+\text{Power absorbed} = -\text{Power supplied}$$

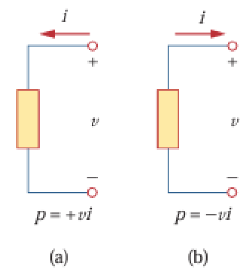

Figure 1.8
Reference polarities for power using the passive sign convention: (a) absorbing power, (b) supplying power.

When the voltage and current directions conform to Fig. 1.8(b), we have the *active sign convention* and $p = +vi$.

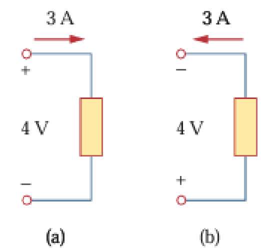

Figure 1.9
Two cases of an element with an absorbing power of 12 W: (a) $p = 4 \times 3 = 12$ W, (b) $p = 4 \times 3 = 12$ W.

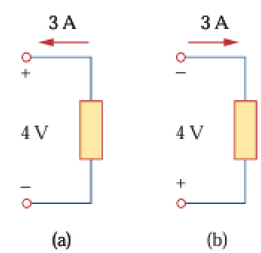

Figure 1.10
Two cases of an element with a supplying power of 12 W: (a) $p = -4 \times 3 = -12$W, (b) $p = -4 \times 3 = -12$ W.