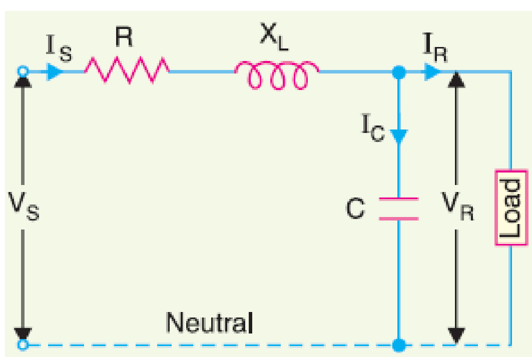
Medium Transmission Lines

The most commonly used methods (known as localized capacitance methods) for the solution of medium transmissions lines are:

(i)End condenser method (ii) Nominal T method (iii) Nominal π method. Although the above methods are used for obtaining the performance calculations of medium lines, they can also be used for short lines if their line capacitance is given in a particular problem.

1. End Condenser Method

In Figure, one phase of the 3-phase transmission line is shown as it is more convenient to work in phase instead of line-to-line values.



Let I_R = load current per phase

R = resistance per phase

X_L = inductive reactance per phase

C = capacitance per phase

$\cos \phi_R$ = receiving end power factor (*lagging*)

V_S = sending end voltage per phase

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V_S = sending end voltage per phase

The *phasor diagram for the circuit is shown in Fig 10.9. Taking the receiving end voltage $\vec{V_R}$ as the reference phasor, we have, $\vec{V_R} = V_R + j\,0$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

Capacitive current, $\vec{I_C} = j\,\vec{V_R}\,\omega C = j\,2\pi f C\,\vec{V_R}$

The sending end current $\vec{I_S}$ is the phasor sum of load current $\vec{I_R}$ and capacitive current $\vec{I_C}$ *i.e.*,

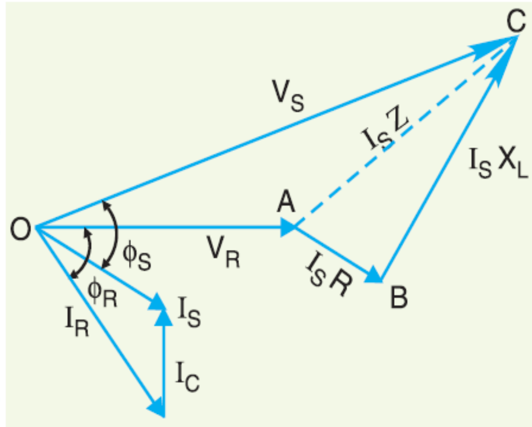

Fig. 10.9

$$\vec{I_S} = \vec{I_R} + \vec{I_C}$$
$$= I_R(\cos\phi_R - j\sin\phi_R) + j\,2\pi f C\,V_R$$
$$= I_R\cos\phi_R + j(-I_R\sin\phi_R + 2\pi f C V_R)$$

Voltage drop/phase $= \vec{I_S}\,\vec{Z} = \vec{I_S}(R + jX_L)$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_S}\,\vec{Z} = \vec{V_R} + \vec{I_S}(R + jX_L)$

Thus, the magnitude of sending end voltage V_S can be calculated.

$$\%\ \text{Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$\%\ \text{Voltage transmission efficiency} = \frac{\text{Power delivered / phase}}{\text{Power delivered / phase + losses / phase}} \times 100$$
$$= \frac{V_R I_R \cos\phi_R}{V_R I_R \cos\phi_R + I_S^2 R} \times 100$$

Limitations. Although end condenser method for the solution of medium lines is simple to work out calculations, yet it has the following drawbacks :

(*i*) There is a considerable error (about 10%) in calculations because the distributed capacitance has been assumed to be lumped or concentrated.

(*ii*) This method overestimates the effects of line capacitance.

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Example 10.10. *A (medium) single phase transmission line 100 km long has the following constants :*

Resistance/km = 0·25 Ω ; *Reactance/km = 0·8 Ω*

Susceptance/km = 14 × 10^{-6} siemen ; *Receiving end line voltage = 66,000 V*

Assuming that the total capacitance of the line is localised at the receiving end alone, determine (i) the sending end current (ii) the sending end voltage (iii) regulation and (iv) supply power factor. The line is delivering 15,000 kW at 0.8 power factor lagging. Draw the phasor diagram to illustrate your calculations.

Solution: Figure show the circuit diagram and phasor diagram of the line respectively.

Total resistance, $R = 0{\cdot}25 \times 100 = 25\ \Omega$

Total reactance, $X_L = 0{\cdot}8 \times 100 = 80\ \Omega$

Total susceptance, $Y = 14 \times 10^{-6} \times 100 = 14 \times 10^{-4}\ S$

Receiving end voltage, $V_R = 66{,}000$ V

$\therefore$ Load current, $I_R = \dfrac{15{,}000 \times 10^3}{66{,}000 \times 0\cdot 8} = 284$ A

$\cos\phi_R = 0{\cdot}8$; $\sin\phi_R = 0{\cdot}6$

Taking receiving end voltage as the reference phasor [see Fig.10.10 (*ii*)], we have,

$$\vec{V_R} = V_R + j\,0 = 66{,}000 V$$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 284\,(0{\cdot}8 - j\,0{\cdot}6) = 227 - j\,170$

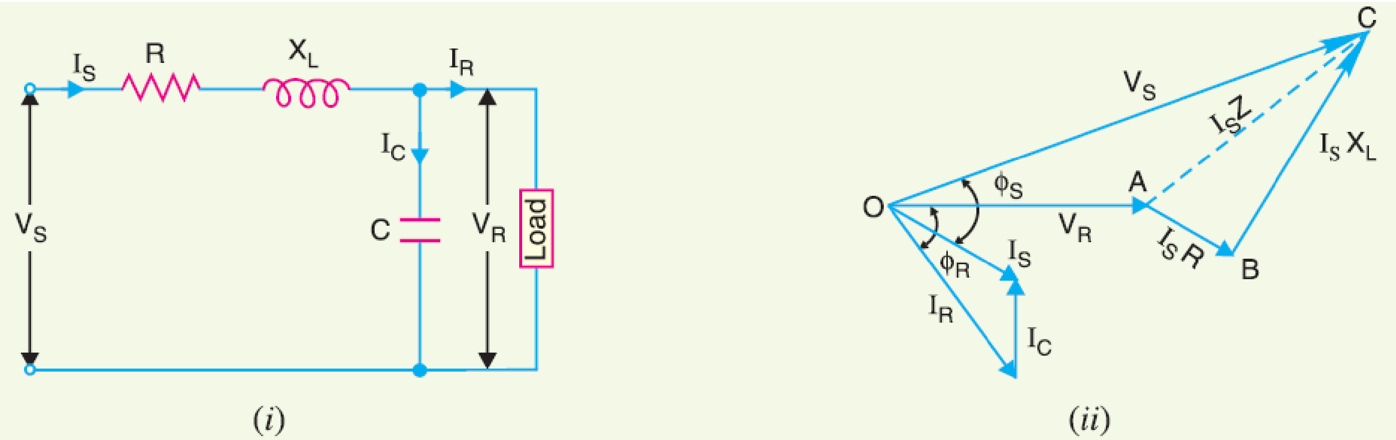

Fig. 10.10

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Capacitive current, $\vec{I_C} = jY \times V_R = j\,14 \times 10^{-4} \times 66000 = j\,92$

(i) Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (227 - j\,170) + j\,92$

$= 227 - j\,78$ ... (*i*)

Magnitude of $I_S = \sqrt{(227)^2 + (78)^2} =$ **240 A**

(ii) Voltage drop $= \vec{I_S}\,\vec{Z} = \vec{I_S}(R + j\,X_L) = (227 - j\,78)\,(25 + j\,80)$

$= 5{,}675 + j\,18,\,160 - j\,1950 + 6240$

$= 11{,}915 + j\,16{,}210$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_S}\,\vec{Z} = 66{,}000 + 11{,}915 + j\,16{,}210$

$= 77{,}915 + j\,16{,}210$ ...(*ii*)

Magnitude of $V_S = \sqrt{(77915)^2 + (16210)^2} =$ **79583V**

(iii) % Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100 = \dfrac{79{,}583 - 66{,}000}{66{,}000} \times 100 =$ **20·58%**

(iv) Referring to exp. (*i*), phase angle between $\vec{V_R}$ and $\vec{I_R}$ is :

$\theta_1 = \tan^{-1} - 78/227 = \tan^{-1}(-0{\cdot}3436) = -18{\cdot}96°$

Referring to exp. (*ii*), phase angle between $\vec{V_R}$ and $\vec{V_S}$ is :

$\theta_2 = \tan^{-1}\dfrac{16210}{77915} = \tan^{-1}(0\cdot 2036) = 11\cdot 50°$

∴ Supply power factor angle, $\phi_S = 18{\cdot}96° + 11{\cdot}50° = 30{\cdot}46°$

∴ Supply p.f. $= \cos\phi_S = \cos 30{\cdot}46° =$ **0·86 lag**

2. Nominal T Method

In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Figure. Therefore, in this arrangement, full charging current flows over half the line. In Figure, one phase of 3-phase transmission line is shown as it is advantageous to work in phase instead of line-to-line values.

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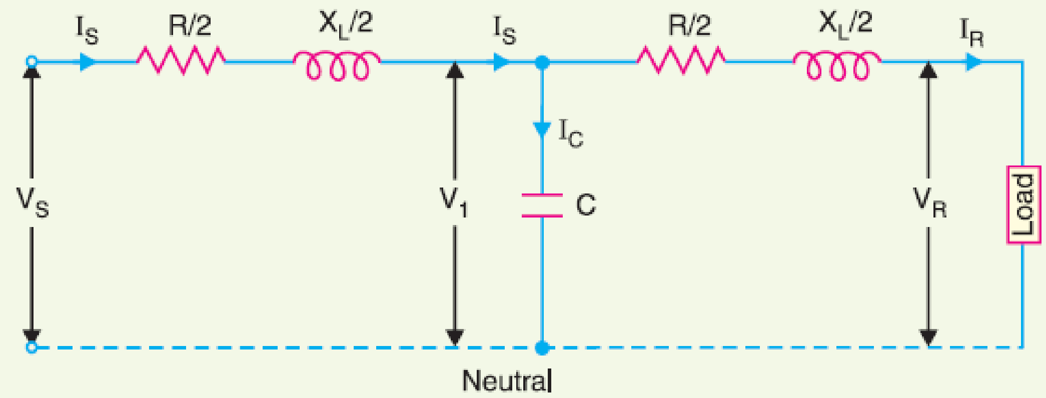

Fig. 10.11

Let I_R = load current per phase ; R = resistance per phase
X_L = inductive reactance per phase ; C = capacitance per phase
$\cos\phi_R$ = receiving end power factor (*lagging*) ; V_S = sending end voltage/phase
V_1 = voltage across capacitor C

The *phasor diagram for the circuit is shown in Fig. 10.12. Taking the receiving end voltage $\vec{V_R}$ as the reference phasor, we have,

Receiving end voltage, $\vec{V_R} = V_R + j\,0$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

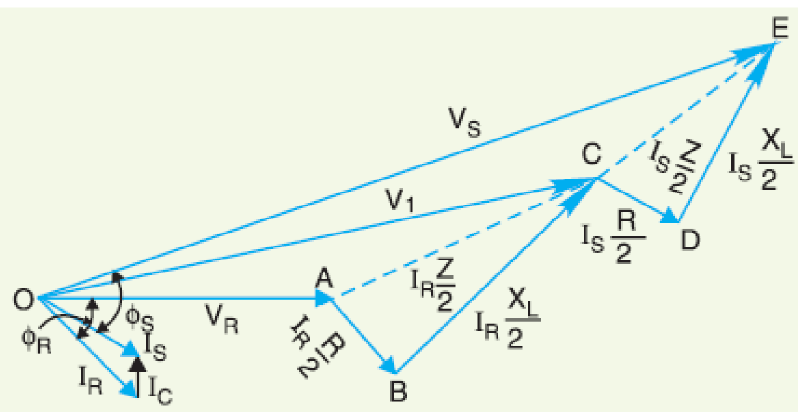

Fig. 10.12

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2$

$$= V_R + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\frac{X_L}{2}\right)$$

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Capacitive current, $\vec{I_C} = j\,\omega\, C\,\vec{V_1} = j\,2\pi f\,C\,\vec{V_1}$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C}$

Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\dfrac{\vec{Z}}{2} = \vec{V_1} + \vec{I_S}\left(\dfrac{R}{2} + j\,\dfrac{X_L}{2}\right)$

Example 10.11. *A 3-phase, 50-Hz overhead transmission line 100 km long has the following constants :*

Resistance/km/phase = 0.1 Ω

Inductive reactance/km/phase = 0·2 Ω

Capacitive susceptance/km/phase = 0·04 × 10^{-4} siemen

Determine (i) the sending end current (ii) sending end voltage (iii) sending end power factor and (iv) transmission efficiency when supplying a balanced load of 10,000 kW at 66 kV, p.f. 0·8 lagging. Use nominal T method.

Solution. Figs. 10.13 (*i*) and 10.13 (*ii*) show the circuit diagram and phasor diagram of the line respectively.

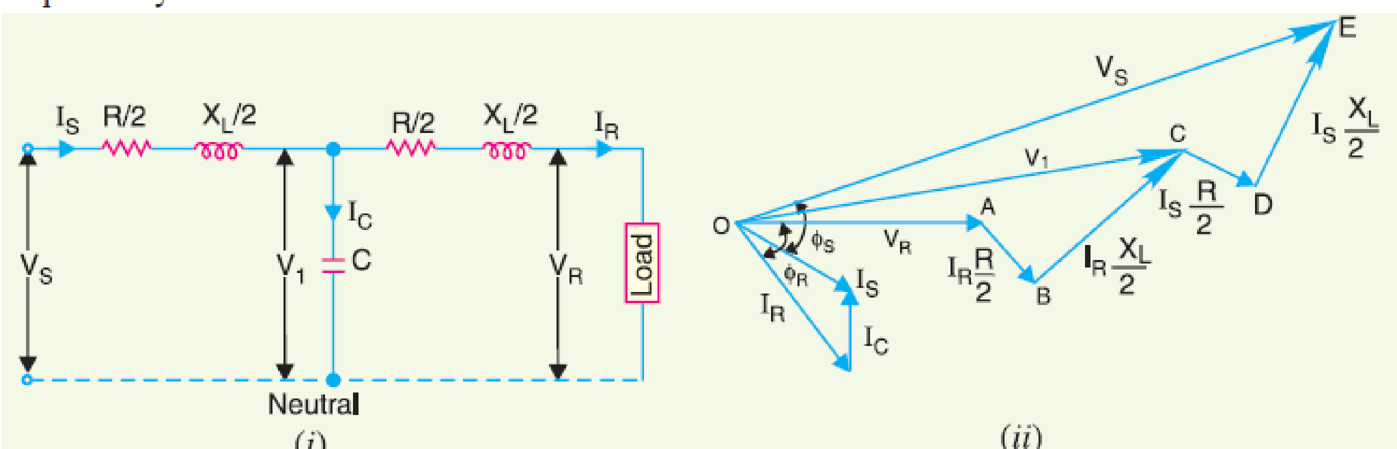

Fig. 10.13

Total resistance/phase, $R = 0{\cdot}1 \times 100 = 10\ \Omega$

Total reactance/phase. $X_L = 0{\cdot}2 \times 100 = 20\ \Omega$

Capacitive susceptance, $Y = 0{\cdot}04 \times 10^{-4} \times 100 = 4 \times 10^{-4}$ S

Receiving end voltage/phase, $V_R = 66{,}000/\sqrt{3} = 38105\ V$

Load current, $I_R = \dfrac{10{,}000 \times 10^3}{\sqrt{3} \times 66 \times 10^3 \times 0{\cdot}8} = 109$ A

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$$\cos\phi_R = 0{\cdot}8\ ;\quad \sin\phi_R = 0{\cdot}6$$

Impedance per phase, $\vec{Z} = R + jX_L = 10 + j\,20$

(*i*) Taking receiving end voltage as the reference phasor [see Fig. 10.13 (*ii*)], we have,

Receiving end voltage, $\vec{V_R} = V_R + j\,0 = 38{,}105\ V$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 109\,(0{\cdot}8 - j\,0{\cdot}6) = 87{\cdot}2 - j\,65{\cdot}4$

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2 = 38,105 + (87{\cdot}2 - j\,65{\cdot}4)(5 + j\,10)$

$= 38{,}105 + 436 + j\,872 - j\,327 + 654 = 39{,}195 + j\,545$

Charging current, $\vec{I_C} = j\,Y\,\vec{V_1} = j\,4\times10^{-4}(39{,}195 + j\,545) = -\,0\cdot218 + j\,15\cdot6$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (87{\cdot}2 - j\,65{\cdot}4) + (-\,0{\cdot}218 + j\,15{\cdot}6)$

$= 87{\cdot}0 - j\,49{\cdot}8 = 100\ \angle\ -29°47'$ A

∴ Sending end current $=$ **100 A**

(*ii*) Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\vec{Z}/2 = (39{,}195 + j\,545) + (87{\cdot}0 - j\,49{\cdot}8)(5 + j\,10)$

$= 39{,}195 + j\,545 + 434{\cdot}9 + j\,870 - j\,249 + 498$

$= 40128 + j\,1170 = 40145\ \angle 1°40'$ V

∴ Line value of sending end voltage

$= 40145 \times \sqrt{3} = 69\ 533$ V $=$ **69·533 kV**

(*iii*) Referring to phasor diagram in Fig. 10.14,

θ_1 = angle between $\vec{V_R}$ and $\vec{V_S}$ = 1°40′

θ_2 = angle between $\vec{V_R}$ and $\vec{I_S}$ = 29° 47′

∴ ϕ_S = angle between $\vec{V_S}$ and $\vec{I_S}$

$= \theta_1 + \theta_2 = 1°40' + 29°47' = 31°27'$

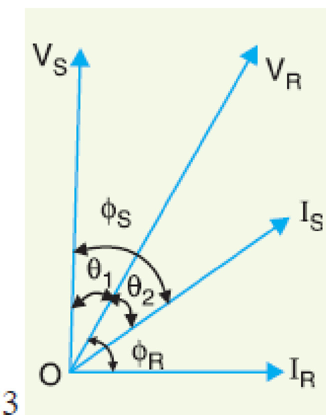

Fig. 10.14

∴ Sending end power factor, $\cos\phi_S = \cos 31°27' =$ **0·853 lag**

(*iv*) Sending end power $= 3\,V_S I_S\cos\phi_S = 3\times40{,}145\times100\times0{\cdot}853$

$= 10273105$ W $= 10273{\cdot}105$ kW

Power delivered $= 10{,}000$ kW

∴ Transmission efficiency $= \dfrac{10{,}000}{10273{\cdot}105}\times100 =$ **97·34%**

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