Chapter One

Vectors Analysis

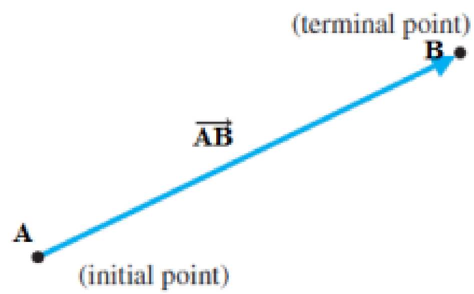
Many physical quantities such as area, length, mass and temperature are completely described once the magnitude of the quantity is give, such quantities are called ***scalars***. Other physical quantities called ***vectors*** are not completely determined until both a magnitude and a direction are specified such as force, velocity, and acceleration.

Component Form

A quantity such as force, displacement, or velocity is called a **vector** and is represented by a **directed line segment**.

DEFINITIONS:

The vector represented by the directed line segment $\overrightarrow{AB}$ has **initial point** A and **terminal point** B.



Vectors in 2-dimensional coordinates

If A represent by (x_1, y_1) and

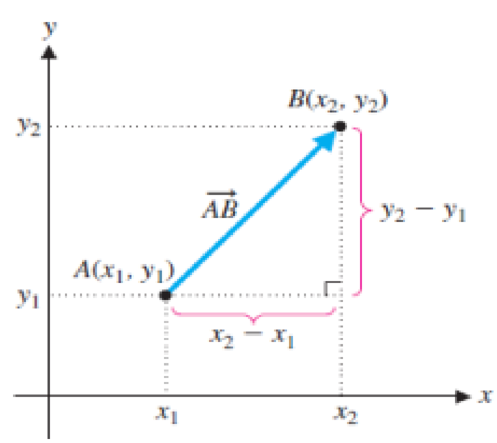
B represent by (x_2, y_2)

Vector is represent by

$$\overrightarrow{AB} = \langle v_1, v_2 \rangle$$

Where $v_1 = x_2 - x_1$

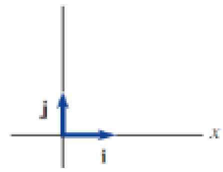
and $v_2 = y_2 - y_1$



or another method to represent the vector

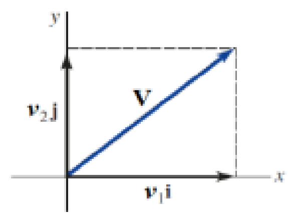
If $\boldsymbol{i}$, $\boldsymbol{j}$ are <u>*standard basis vectors*</u>, then

$$\mathbf{i} = \langle 1, 0 \rangle \quad \text{and} \quad \mathbf{j} = \langle 0, 1 \rangle,$$



(a)

$$\overrightarrow{AB} = \mathbf{V} = v_1\mathbf{i} + v_2\mathbf{j}.$$

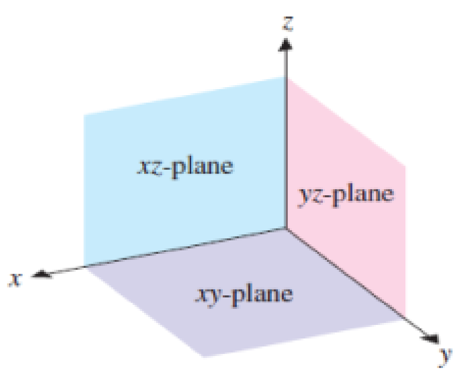


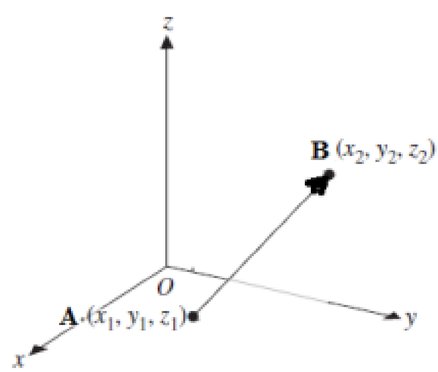
(b)

<u>Note</u>: <u>Zero vector</u>

$$V = \langle 0, 0 \rangle$$

<u>Vectors in 3-dimensional coordinates (vector in space)</u>





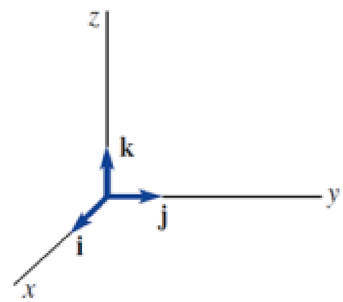
If A represent by (x_1, y_1, z_1) and B represent by (x_2, y_2, z_2)
Vector is represent by

$$\overrightarrow{AB} = \langle v_1, v_2, v_3 \rangle$$

Where $v_1 = x_2 - x_1$, $v_2 = y_2 - y_1$ *and* $v_3 = z_2 - z_1$

or another method to represent the vector

If $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ are *standard basis vectors*, then

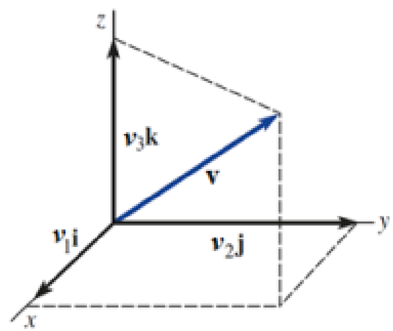


$$\mathbf{i} = \langle 1, 0, 0 \rangle,$$

$$\mathbf{j} = \langle 0, 1, 0 \rangle,$$

$$\mathbf{k} = \langle 0, 0, 1 \rangle.$$

$$\overrightarrow{AB} = \mathbf{V} = v_1 i + v_2 j + v_3 k$$



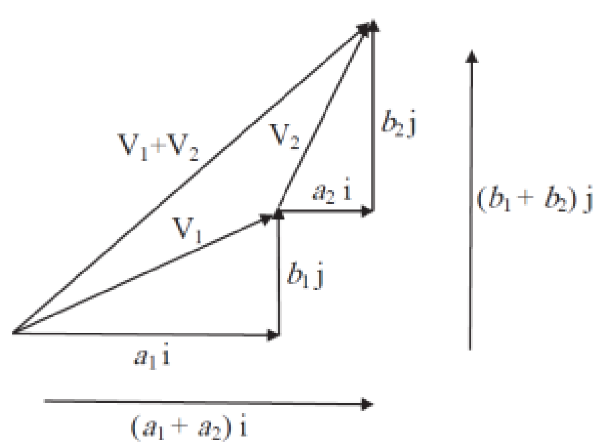
Note: ***Zero vector*** $\quad V = \langle \mathbf{0}, \mathbf{0}, \mathbf{0} \rangle$

Algebra of vector:

Algebraic addition:

Let $V_1 = a_1 i + b_1 j$

$V_2 = a_2 i + b_2 j$



Two vector may be added algebraically by adding their corresponding scalar components:

$$V_1 + V_2 = (a_1 + a_2)i + (b_1 + b_2)j$$

Example: If $V_1 = 2i - 5j$ and $V_2 = 4i + 2j$, find V_1+V_2

solution:

$$V_1 + V_2 = (2i - 5j) + (4i + 2j)$$
$$= (2 + 4)i + (-5 + 2)j$$
$$= 6i - 3j$$

H.W: The vector $u = 4i + 3j$ and $v = 5i + 6j$, find $u + v$

Subtraction:

Let $V_1 = a_1 i + b_1 j$

$V_2 = a_2 i + b_2 j$

$$V_1 - V_2 = (a_1 - a_2)i + (b_1 - b_2)j$$

Example: If $V_1 = 7i + 3j$ and $V_2 = 2i - 6j$, find $V_1 - V_2$

solution:

$$V_1 - V_2 = (7i + 3j) - (2i - 6j)$$
$$= (7 - 2)i + (3 - (-6))j$$
$$= 5i + 9j$$

H.W: The vector $u = 9i + 6j$ and $v = 5i + 2j$, find $u - v$

***Length of the vector (magnitude)*:**

The length of the vector is $V = ai + bj$ usually denoted by $|V|$, which may be read (the magnitude of V):

$$|V| = |ai + bj| = \sqrt{a^2 + b^2}$$

Example: find length of vector $V = 3i - 5j$

solution:

$$|V| = |3i - 5j| = \sqrt{(3)^2 + (-5)^2} = \sqrt{9 + 25} = \sqrt{34}$$

H.W: The vector $u = 5i + 2j$, find the magnitude (length) of the vector u

Unit vector

Any vector whose length is equal to the unit of length used along the coordinate axes is called a unit vector.

Direction:

$$\text{Direction of the vector } V = \frac{V}{|V|}$$

Example: find the direction of $A = 3i - 4j$

solution:

$$\text{Direction of } A = \frac{A}{|A|} = \frac{3i - 4j}{\sqrt{(3)^2 + (-4)^2}} = \frac{3i - 4j}{\sqrt{25}} = \frac{3}{5}i - \frac{4}{5}j$$

Properties of vectors

Let $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{w}$ be vector and $\boldsymbol{a}$, $\boldsymbol{b}$ be scalars

1. $u+v=v+u$ **2.** $(u+v)+w=u+(v+w)$

3. $u+0=u$ **4.** $u+(-u)=0$

5. $0u=0$ **6.** $1u=u$

7. $a(bu)=(ab)u$ **8.** $a(u+v)=au+av$

9. $(a+b)u=au+bu$

Note : **Zero vector** $\langle 0,0\rangle$ **or** $\langle 0,0,0\rangle$

Example: For vectors $a=\langle \mathbf{2},\mathbf{1}\rangle$ and $b=\langle \mathbf{3},-\mathbf{2}\rangle$, compute (a) a + b, (b) 2a, (c) 2a + 3b, (d) 2a − 3b and (e) $|\mathbf{2a}-\mathbf{3b}|$.

Solution:

(a) $\mathbf{a}+\mathbf{b}=\langle 2,1\rangle+\langle 3,-2\rangle=\langle 2+3,1-2\rangle=\langle 5,-1\rangle.$

(b) $2\mathbf{a}=2\langle 2,1\rangle=\langle 2\cdot 2,2\cdot 1\rangle=\langle 4,2\rangle.$

(c) $2\mathbf{a}+3\mathbf{b}=2\langle 2,1\rangle+3\langle 3,-2\rangle=\langle 4,2\rangle+\langle 9,-6\rangle=\langle 13,-4\rangle.$

(d) $2\mathbf{a}-3\mathbf{b}=2\langle 2,1\rangle-3\langle 3,-2\rangle=\langle 4,2\rangle-\langle 9,-6\rangle=\langle -5,8\rangle.$

(e) $|2\mathbf{a}-3\mathbf{b}|=|\langle -5,8\rangle|=\sqrt{25+64}=\sqrt{89}.$

Vector Function and Motion

The most important vector function is the radius vector

$$\bar{R}(t) = X(t)i + y(t)j + Z(t)k$$

Where $\vec{R}(t)$:- is the position vector from the origion to the point ***P[x(t),y(t),z(t)]***. That gives the position at time ***t*** of a particle moving through space.

The Derivative of a Vector Function

If $x(t)$, $y(t)$ & $z(t)$ are functions of variable ,

$$\bar{F}(t) = X(t)i + y(t)j + Z(t)k$$

Is a vector function of t , $\bar{F}$(t) may be position vector of a moving body.

And $\dot{\bar{F}}(t) = \frac{d\bar{F}}{dt} = \frac{dx}{dt}i + \frac{dy}{dt}j + \frac{dz}{dt}k$

Is called the derivative of $\bar{F}$ with respect to t.

Position, Velocity, Speed, and Acceleration:-

If $R(t) = x(t)i + y(t)j + z(t)k$,is the position vector of a body in space,

Definition :- Position $R(t) = x(t)i + y(t)j + z(t)k$

Velocity $\bar{v}(t) = dR/dt$

Speed $|\bar{v}|$

Direction $\frac{\bar{v}}{|\bar{v}|}$

Acceleration $\frac{d^2R}{dt^2}$

Example:

Find $\frac{d\bar{F}}{dt}$ at $t = \pi/3$ if $\bar{F}(t) = (\sin t)i + (\ln t)j + (\tan t)\,k$

Solution:

$$\frac{d\bar{F}}{dt} = (\cos t)i + (\tfrac{1}{t})j + (sec^2 t)\,k$$

$$\therefore\ F'\left(\frac{\pi}{3}\right) = \frac{1}{2}i + \frac{3}{\pi}j + 4k$$

Example:

If $\bar{R}(t) = (250\,t)i + (250\sqrt{3}\,t\ - 4.9\,t^2\,)j$, is the position vector of the projectile, find the velocity vector of projectile.

Solution:

$d\bar{R}/dt\ = \bar{v}(t)$ = velocity vector of projectile

$d\bar{R}/dt = 250i + (250\sqrt{3}t - 9.8t)j$

Example:

If $\vec{\mathbf{R}}(t) = (3\cos t)i+(3\sin t)j + t^2\,k$, is the position vector , find the speed and direction when t=2.

Solution:

Direction: $\frac{\bar{v}(2)}{|\bar{v}(2)|} = -\ (\frac{3}{5}sin2)i +\ (\frac{3}{5}\,cos\,2)j\ + \frac{4}{5}\,k$