Al-Mustaqbal University
Department of Electrical Engineering Technique
Class 3rd
Subject: Advanced Power Engineering
Lecturer: Dr. Hasan Muwafaq Gheni

Generalised Circuit Constants of a Transmission Line

In any four terminal *network, the input voltage and input current can be expressed in terms of output voltage and output current. Incidentally, a transmission line is a 4-terminal network ; two input terminals where power enters the network and two output terminals where power leaves the network.

Therefore, the input voltage $\left(\vec{V_S}\right)$ and input current $\left(\vec{I_S}\right)$ of a 3-phase transmission line can be expressed as :

$$\vec{V_S} = \vec{A}\,\vec{V_R} + \vec{B}\,\vec{I_R}$$

$$\vec{I_S} = \vec{C}\,\vec{V_R} + \vec{D}\,\vec{I_R}$$

where $\vec{V_S}$ = sending end voltage per phase

$\vec{I_S}$ = sending end current

$\vec{V_R}$ = receiving end voltage per phase

$\vec{I_R}$ = receiving end current

and $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ (generally complex numbers) are the constants known as *generalised circuit constants* of the transmission line. The values of these constants depned upon the particular method adopted for solving a transmission line. Once the values of these constants are known, performance calculations of the line can be easily worked out. The following points may be kept in mind :

(*i*) The constants $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ are generally complex numbers.

(*ii*) The constants $\vec{A}$ and $\vec{D}$ are dimensionless whereas the dimensions of $\vec{B}$ and $\vec{C}$ are ohms and siemen respectively.

(*iii*) For a given transmisson line,

$$\vec{A} = \vec{D}$$

(*iv*) For a given transmission line,

$$\vec{A}\,\vec{D} - \vec{B}\,\vec{C} = 1$$

We shall establish the correctness of above characteristics of generalised circuit constants in the following discussion.

Email: Hasan.muwafaq.gheni@uomus.edu.iq

Determination of Generalised Constants for Transmission Lines

(i) **Short lines.** In short transmission lines, the effect of line capacitance is neglected. Therefore, the line is considered to have series impedance. Fig. 10.23 shows the circuit of a 3-phase transmission line on a single phase basis.

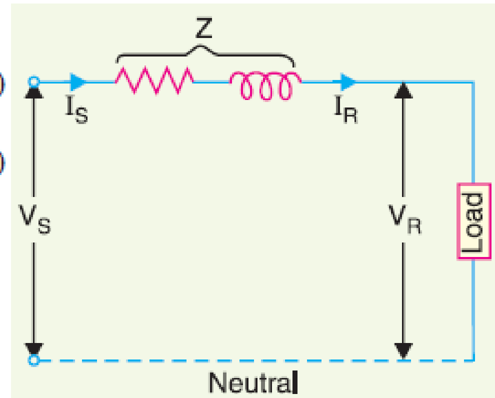

Fig. 10.23

Here, $$\vec{I_S} = \vec{I_R} \qquad ...(iii)$$

and $$\vec{V_S} = \vec{V_R} + \vec{I_R}\,\vec{Z} \qquad ...(iv)$$

Comparing these with eqs. (*i*) and (*ii*), we have,

$$\vec{A} = 1\,; \quad \vec{B} = \vec{Z}, \quad \vec{C} = 0 \quad \text{and} \quad \vec{D} = 1$$

Incidentally ; $\vec{A} = \vec{D}$

and $$\vec{A}\,\vec{D} - \vec{B}\,\vec{C} = 1 \times 1 - \vec{Z} \times 0 = 1$$

(ii) **Medium lines – Nominal T method.** In this method, the whole line to neutral capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on either side as shown in Fig. 10.24.

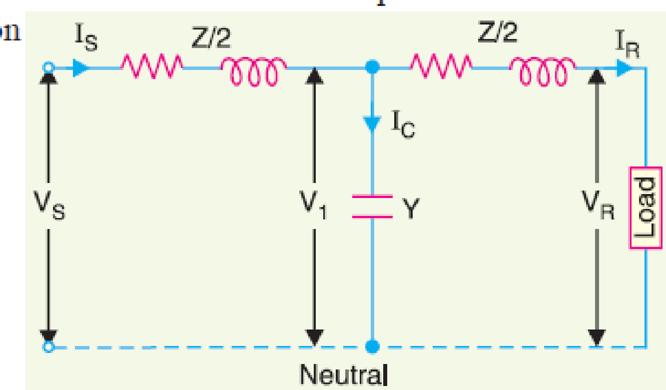

Fig. 10.24

Here, $$\vec{V_S} = \vec{V_1} + \vec{I_S}\,\vec{Z}/2 \qquad ...(v)$$

and $$\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2$$

Now, $$\vec{I_C} = \vec{I_S} - \vec{I_R}$$

$$= \vec{V_1}\,\vec{Y} \quad \text{where } Y = \text{shunt admittance}$$

$$= \vec{Y}\left(\vec{V_R} + \frac{\vec{I_R}\,\vec{Z}}{2}\right)$$

$$\therefore \quad \vec{I_S} = \vec{I_R} + \vec{Y}\,\vec{V_R} + \vec{Y}\,\frac{\vec{I_R}\,\vec{Z}}{2}$$

$$= \vec{Y}\,\vec{V_R} + \vec{I_R}\left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right) \qquad ...(vi)$$

Substituting the value of V_1 in eq. (*v*), we get,

$$\vec{V_S} = \vec{V_R} + \frac{\vec{I_R}\,\vec{Z}}{2} + \frac{\vec{I_S}\,\vec{Z}}{2}$$

Substituing the value of I_S, we get,

$$\vec{V_S} = \left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right)\vec{V_R} + \left(\vec{Z} + \frac{\vec{Y}\,\vec{Z}^2}{4}\right)\vec{I_R} \qquad \ldots(vii)$$

Comparing eqs. (*vii*) and (*vi*) with those of (*i*) and (*ii*), we have,

$$\vec{A} = \vec{D} = 1 + \frac{\vec{Y}\,\vec{Z}}{2}\ ; \quad \vec{B} = \vec{Z}\left(1 + \frac{\vec{Y}\,\vec{Z}}{4}\right)\ ; \ \vec{C} = \vec{Y}$$

Incidentally :

$$\vec{A}\,\vec{D} - \vec{B}\,\vec{C} = \left(1 + \frac{Y\,Z}{2}\right)^2 - Z\left(1 + \frac{Y\,Z}{4}\right)Y$$

$$= 1 + \frac{Y^2 Z^2}{4} + Y\,Z - Z\,Y - \frac{Z^2 Y^2}{4} = 1$$

(*iii*) Medium lines—Nominal π method. In this method, line-to-neutral capacitance is divided into two halves ; one half being concentrated at the load end and the other half at the sending end as shown in Fig. 10.25.

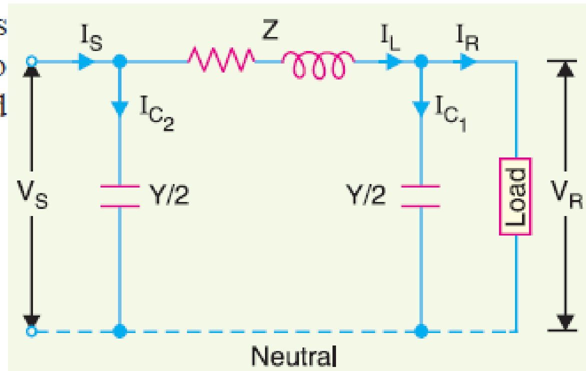

Fig. 10.25

Here, $\vec{Z} = R + jX_L$ = series impedenace/phase

$\vec{Y} = j\,\omega\,C$ = shunt admittance

$\vec{I_S} = \vec{I_L} + \vec{I_{C2}}$

or $\vec{I_S} = \vec{I_L} + \vec{V_S}\,\vec{Y}/2 \qquad \ldots(viii)$

Also $\quad \vec{I_L} = \vec{I_R} + \vec{I_{C1}} = \vec{I_R} + \vec{V_R}\,\vec{Y}/2 \qquad \ldots(ix)$

Now $\quad \vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \left(\vec{I_R} + \vec{V_R}\,\vec{Y}/2\right)\vec{Z}$ (Putting the value of $\vec{I_L}$)

$\therefore \quad \vec{V_S} = \vec{V_R}\left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right) + \vec{I_R}\,\vec{Z} \qquad \ldots(x)$

Also $\quad \vec{I_S} = \vec{I_L} + \vec{V_S}\,\vec{Y}/2 = \left(\vec{I_R} + \vec{V_R}\,\vec{Y}/2\right) + \vec{V_S}\,\vec{Y}/2$

(Putting the value of $\vec{I_L}$)

Putting the value of $\vec{V_S}$ from eq. (*x*), we get,

$$\vec{I_S} = \vec{I_R} + \vec{V_R}\frac{\vec{Y}}{2} + \frac{\vec{Y}}{2}\left\{\vec{V_R}\left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right) + \vec{I_R}\,\vec{Z}\right\}$$

Email: Hasan.muwafaq.gheni@uomus.edu.iq

$$= \vec{I_R} + \vec{V_R}\frac{\vec{Y}}{2} + \frac{\vec{V_R}\,\vec{Y}}{2} + \frac{\vec{V_R}\,\vec{Y}^2\,\vec{Z}}{4} + \frac{\vec{Y}\,\vec{I_R}\,\vec{Z}}{2}$$

$$= \vec{I_R}\left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right) + \vec{V_R}\,\vec{Y}\left(1 + \frac{\vec{Y}\,\vec{Z}}{4}\right) \quad \ldots(xi)$$

Comparing equations (*x*) and (*xi*) with those of (*i*) and (*ii*), we get,

$$\vec{A} = \vec{D} = \left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right)\ ;\quad \vec{B} = \vec{Z}\ ;\quad \vec{C} = \vec{Y}\left(1 + \frac{\vec{Y}\,\vec{Z}}{4}\right)$$

Also

$$\vec{A}\,\vec{D} - \vec{B}\,\vec{C} = \left(1 + \frac{Y\,Z}{2}\right)^2 - Z\,Y\left(1 + \frac{Y\,Z}{4}\right)$$

$$= 1 + \frac{Y^2 Z^2}{4} + Y\,Z - Z\,Y - \frac{Z^2 Y^2}{4} = 1$$

Example 10.16. *A balanced 3-phase load of 30 MW is supplied at 132 kV, 50 Hz and 0·85 p.f. lagging by means of a transmission line. The series impedance of a single conductor is (20 + j52) ohms and the total phase-neutral admittance is 315×10^{-6} siemen. Using nominal T method, determine: (i) the A, B, C and D constants of the line (ii) sending end voltage (iii) regulation of the line.*

Solution. Fig. 10.26 shows the representation of 3-phase line on the single phase basis.

Series line impedance/phase, $\vec{Z} = (20 + j\,52)\ \Omega$

Shunt admittance/phase, $\vec{Y} = j\,315 \times 10^{-6}$ S

(*i*) **Generalised constants of line.** For nominal *T* method, various constants have the values as under :

$$\vec{A} = \vec{D} = 1 + \vec{Z}\ \vec{Y}/2 = 1 + \frac{20 + j\,52}{2} \times j\,315 \times 10^{-6}$$

$$= 0{\cdot}992 + j\,0{\cdot}00315 = \mathbf{0{\cdot}992\ \angle 0{\cdot}18^\circ}$$

$$\vec{B} = \vec{Z}\left(1 + \frac{\vec{Z}\,\vec{Y}}{4}\right) = (20 + j\,52)\left[1 + \frac{(20 + j\,52)\,j\,315 \times 10^{-6}}{4}\right]$$

$$= 19{\cdot}84 + j\,51{\cdot}82 = \mathbf{55{\cdot}5\ \angle 69^\circ}$$

$$\vec{C} = \vec{Y} = \mathbf{0{\cdot}000315\ \angle 90^\circ}$$

Email: Hasan.muwafaq.gheni@uomus.edu.iq

Al-Mustaqbal University
Department of Electrical Engineering Technique
Class 3rd
Subject: Advanced Power Engineering
Lecturer: Dr. Hasan Muwafaq Gheni

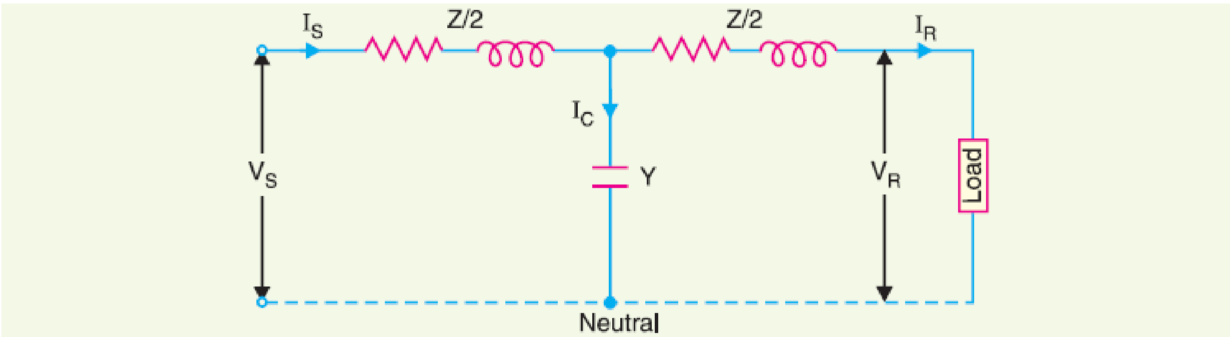

Fig. 10.26

(*ii*) Sending end voltage.

Receiving end voltage/phase, $V_R = 132 \times 10^3/\sqrt{3} = 76210$ V

Receiving end current, $I_R = \dfrac{30 \times 10^6}{\sqrt{3} \times 132 \times 10^3 \times 0{\cdot}85} = 154$ A

$\cos \phi_R = 0{\cdot}85\,; \quad \sin \phi_R = 0{\cdot}53$

Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j0 = 76210 \text{ V}$$

$$\vec{I_R} = I_R(\cos \phi_R - j \sin \phi_R) = 154\,(0{\cdot}85 - j\,0{\cdot}53) = 131 - j\,81{\cdot}62$$

Sending end voltage per phase is

$$\vec{V_S} = \vec{A}\ \vec{V_R} + \vec{B}\ \vec{I_R}$$
$$= (0{\cdot}992 + j\,0{\cdot}0032)\ 76210 + (19{\cdot}84 + j\,51{\cdot}82)\,(131 - j\,81{\cdot}62)$$
$$= 82{,}428 + j\,5413$$

∴ Magnitude of sending end voltage is

$$V_S = \sqrt{(82{,}428)^2 + (5413)^2} = 82{\cdot}6 \times 10^3 \text{V} = 82{\cdot}6 \text{ kV}$$

∴ Sending end line-to-line voltage

$$= 82{\cdot}6 \times \sqrt{3} = \mathbf{143\ kV}$$

(*iii*) Regulation. Regulation is defined as the change in voltage at the receiving end when full-load is thrown off.

Now, $\vec{V_S} = \vec{A}\ \vec{V_R} + \vec{B}\ \vec{I_R}$

At no load, $\vec{I_R} = 0$

∴ $\vec{V_S} = \vec{A}\ \vec{V_{R0}}$

where $\vec{V_{R0}}$ = voltage at receiving end at no load

or $\vec{V_{R0}} = \vec{V_S}/\vec{A}$

or $V_{R0} = V_S/A$ (in magnitude)

∴ % Regulation $= \dfrac{(V_S/A - V_R)}{V_R} \times 100 = \dfrac{(82 \cdot 6/0 \cdot 992) - 76 \cdot 21}{76 \cdot 21} \times 100 = \mathbf{9{\cdot}25\%}$

Email: Hasan.muwafaq.gheni@uomus.edu.iq