The Ideal Transformer

1. Introduction

A transformer is a static electromagnetic device that transfers electrical energy from one circuit to another through mutual induction between two or more coupled windings.

An **ideal transformer** is a theoretical concept that assumes perfect conditions with no losses, infinite permeability of the core material, and perfect coupling between windings.

2. Construction and Basic Principle

An ideal transformer consists of as shown Fig.1:

- **Primary winding**: Connected to the input voltage source (N_1 turns)
- **Secondary winding**: Connected to the load (N_2 turns)
- **Magnetic core**: Provides a path for magnetic flux (assumed lossless and with infinite permeability)

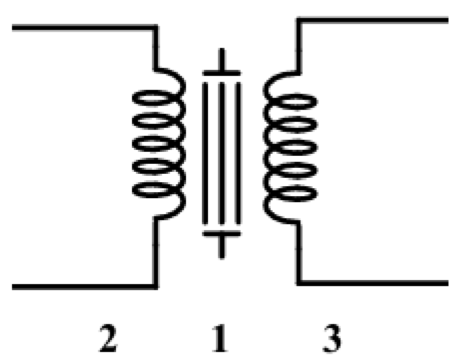

Fig. 1 Construction of the transformer

The transformer operates on **Faraday's Law of Electromagnetic Induction.** When an alternating voltage is applied to the primary winding, it creates a time-varying magnetic flux in the core. This flux links with both primary and secondary windings. The changing flux induces EMF in both windings according to Faraday's law.

3. Assumptions for an Ideal Transformer

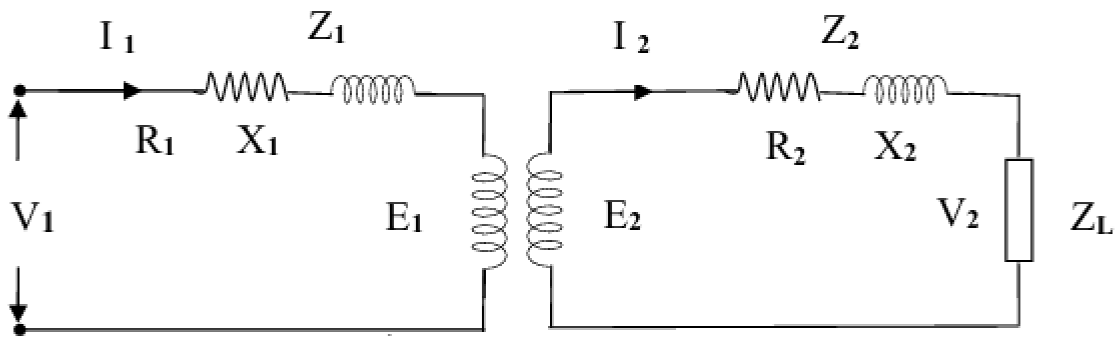

Fig 2. transformer circuit

Zero winding resistance: $R_1 = R_2 = 0$

Zero leakage flux: All flux links both windings perfectly

Infinite core permeability: No magnetizing current required

Zero core losses: No hysteresis or eddy current losses

Perfect coupling: k = 1 (coupling coefficient)

4. Mathematical Analysis

EMF Equations For the primary winding:

$$e_1 = -N_1 (d\varphi/dt)$$

For the secondary winding:

$$e_2 = -N_2 (d\varphi/dt)$$

Where:

e_1, e_2 = instantaneous EMF in primary and secondary windings, N_1, N_2 = number of turns in primary and secondary windings, φ = magnetic flux linking both windings, and $d\varphi/dt$ = rate of change of flux.

5. Voltage Transformation Ratio

Dividing the EMF equations:

$$e_1/e_2 = N_1/N_2 = a$$

For sinusoidal voltages:

$$V_1/V_2 = N_1/N_2 = a$$

Where **a** is the **transformation ratio** or **turns ratio**.

6. Current Transformation

From the principle of conservation of energy and zero losses:

Input Power = Output Power

$$V_1 I_1 = V_2 I_2$$

Therefore:

$$I_1/I_2 = V_2/V_1 = N_2/N_1 = 1/a$$

7. Impedance Transformation

The impedance referred from secondary to primary:

$$Z_1 = a^2 Z_2$$

Where:

Z_1 = impedance as seen from primary side,

Z_2 = actual secondary load impedance,

a = transformation ratio.

8. Types of Transformers

- **A. By Voltage Level:**

1. Step-up transformers - Secondary voltage > Primary voltage
2. Step-down transformers - Secondary voltage < Primary voltage

- **B. By Number of Phases:**

1. Single-phase transformers
2. Three-phase transformers

- **C. By Magnetic Circuit:**

1. Core-type - Windings surround the core
2. Shell-type - Core surrounds the windings

a) Step-Up Transformer

$$N_2 > N_1 \ (a < 1)$$

$$V_2 > V_1 \text{ (voltage is stepped up)}$$

$$I_2 < I_1 \text{ (current is stepped down)}$$

b) Step-Down Transformer

$$N_2 < N_1 \ (a > 1)$$

$$V_2 < V_1 \text{ (voltage is stepped down)}$$

$I_2 > I_1$ (current is stepped up)

c) Isolation Transformer

$N_2 = N_1$ (a = 1)

$V_2 = V_1$ (same voltage levels)

Provides electrical isolation between circuits

Ex.: An ideal transformer has 500 turns on the primary and 100 turns on the secondary. The primary is connected to a 240V, 50Hz AC supply. A load of 10Ω resistance is connected across the secondary. Find Secondary voltage, Secondary current, Primary current, Power consumed by the load

Equivalent resistance referred to primary

Solution

Given:

$N_1 = 500$ turns

$N_2 = 100$ turns

$V_1 = 240V$

$R_2 = 10\Omega$ (secondary load)

Step 1: Calculate transformation ratio

$$a = N_1/N_2 = 500/100 = 5$$

Step 2: Calculate secondary voltage

$$V_2 = V_1/a = 240/5 = 48V$$

Step 3: Calculate secondary current

$$I_2 = V_2/R_2 = 48/10 = 4.8A$$

Step 4: Calculate primary current

$$I_1 = I_2/a = 4.8/5 = 0.96A$$

Step 5: Calculate power consumed

$$P_2 = V_2 \times I_2 = 48 \times 4.8 = 230.4W$$

$$P_1 = V_1 \times I_1 = 240 \times 0.96 = 230.4W$$

(Verification: $P_1 = P_2$ ✓)

Step 6: Calculate equivalent resistance referred to primary

$$R_1 = a^2 \times R_2 = 5^2 \times 10 = 250\Omega$$

Verification:

$$I_1 = V_1/R_1 = 240/250 = 0.96A$$

9. Applications

Power transmission: Stepping up voltage for efficient long-distance transmission

Power distribution: Stepping down voltage for safe domestic use

Isolation: Electrical isolation between circuits

Impedance matching: Matching source and load impedances

In practical transformers:

1. Winding resistances cause I^2R losses
2. Leakage flux reduces coupling efficiency
3. Core losses occur due to hysteresis and eddy currents
4. Finite magnetizing current is required
5. Frequency response is limited