Al-Mustaqbal University / College of Engineering & Technology
Class: first
Subject: integral Mathematics
Lecturer: M.Sc. Zahraa ibrahim
Lecture name: Method of integration
Lecture: 2

# Integration By Substitution

**The goal of this method is to transform the integral into a standard from**

**To evaluate the integral** $I = \int f[g(x)]\, g'(x)dx$ **carry out the following steps**

**1- substitute** $u = g(x)$ **the** $du = g'(x)dx$ **to obtain** $I = \int f(u)du$

**2- Evaluate** $I = \int f(u)du$ **by integrating w.r.t** $u$

**3- Replace** $u$ **by** $g(x)$ **in the final result**

**Evaluate** $I = \int \frac{dx}{\sqrt[3]{1-2x}}$

**Solution :-** $I = \int (1-2x)^{-\frac{1}{3}} dx$ **Let** $u = 1-2x \Rightarrow du = -2dx \Rightarrow dx = \frac{du}{-2}$

$$I = \int (1-2x)^{-\frac{1}{3}} dx \Rightarrow I = \int u^{-\frac{1}{3}} \frac{du}{-2} = \frac{-1}{2}\int u^{-\frac{1}{3}} du = \frac{-1}{2}\frac{u^{\frac{2}{3}}}{\frac{2}{3}} + c = \frac{-3}{4}(1-2x)^{\frac{2}{3}} + c$$

**Evaluate** $\int \frac{-\,e^{x^{-1}}}{x^2}\,dx$

$$= -\int x^{-2}\, e^{x^{-1}}\, dx$$

$$u = x^{-1} \quad , \quad du = -1.\, x^{-2}\, dx$$

$$= -\int x^{-2}\, e^{u} \,.\, \frac{du}{-\,x^{-2}}$$

$$= \int e^{u}\, du = e^{u} + c = e^{x^{-1}} + C$$

**Evaluate** $\int \frac{(\ln x)^2}{x}\, dx$

$$u = \ln x \quad , \quad du = \frac{1}{x} dx \quad , \quad dx = x\, du$$

$$\int \frac{u^2}{x} \,.\, x\ du = \int u^2 du$$

$$= \frac{u^3}{3} + C$$

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**Evaluate** $\int \sin(4x)dx$

**let** $u = 4x \rightarrow \frac{du}{dx} = 4 \qquad du = 4\,dx \qquad dx = \frac{du}{4}$

$$\int \sin(u)\,.\,\frac{du}{4} = \frac{1}{4}\int \sin(u)du$$

$$= \frac{1}{4}\left(-\cos u\right) + c$$

$$= -\frac{1}{4}\cos(4x) + c$$



**Evaluate** $I = \int \sin^2(5x)\cos(5x)\,dx$

**Solution :-** **Let** $u = \sin(5x) \Rightarrow du = 5\cos(5x)\,dx \Rightarrow dx = \frac{du}{5\cos(5x)}$

$$I = \int \sin^2(5x)\cos(5x)\,dx \Rightarrow I = \int u^2 \cos(5x)\frac{du}{5\cos(5x)} = \frac{1}{5}\int u^2 du = \frac{1}{5}\frac{u^3}{3} + c = \frac{1}{15}[\sin(5x]^3 + c$$

**Evaluate** $\int_0^1 (1+e^x)^2 e^x \, dx$

$$u = \mathbf{1} + \mathbf{e^x} \quad , \quad du = e^x dx \, , \quad dx = \frac{du}{e^x}$$

$$\int_0^1 (1+e^x)^2 e^x \, dx = \int_0^1 (u)^2 e^x . \ \frac{du}{e^x}$$

$$\int_0^1 (u)^2 . \ du = \left[\frac{u^3}{3}\right]_0^1 = \frac{1}{3}[u^3]_0^1$$

$$= \frac{1}{3}[1+e^x]_0^1 = \frac{1}{3} \ [1+e^1] - [1+e^0]$$

= ...

**Good Luck ..**