Al-Mustaqbal University / College of Engineering & Technology
Departmentof Electrical Engineering Techniques
Class (2)
Subject : Applied Mathematics
Lecturer :Zahraa Ibrahim
Lecture Name (6) : Ordinary Differential Equation

3 – First order : linear

It is found in two forms

1- $\dfrac{dy}{dx} + P(x). y = Q(x)$

The $P(x)$ and $Q(x)$ are function for (x)

The solution is

$\rho = e^{\int P(x)dx}$

$\rho . y = \int \rho\, Q(x)dx + c$

2- $\dfrac{dx}{dy} + P(y) . x = Q(y)$

The $P(y)$ and $Q(y)$ are function for (y)

$\rho = e^{\int P(y)dy}$

$\rho . x = \int \rho\, Q(y)dy + c$

Ex. 1.: Solve the differential equation:-

$$x\dfrac{dy}{dx} + 3y = x^2$$

Solution

$$\dfrac{x}{x}\dfrac{dy}{dx} + \dfrac{3y}{x} = \dfrac{x^2}{x}$$

$$\dfrac{dy}{dx} + \dfrac{3}{x}y = x$$

Let $P(x) = \dfrac{3}{x}$, $Q(x) = x$

$$\rho = e^{\int P(x)dx}$$

$$\rho = e^{\int \frac{3}{x}dx} = e^{3\ln x} = x^3$$

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$$\rho \cdot y = \int \rho\, Q(x)dx + c$$

$$x^3 \cdot y = \int x^3 \cdot x\, dx + c$$

$$x^3 \cdot y = \int x^4\, dx + c$$

$$x^3 \cdot y = \frac{x^5}{5} + c$$

$$y = \frac{x^5}{5x^3} + \frac{c}{x^3}$$

$$y = \frac{x^2}{5} + cx^{-3}$$

Ex. 2.: Solve the differential equation:-

$$x\frac{dy}{dx} - 3y = x^2$$

Solution

$$\frac{x}{x}\frac{dy}{dx} - \frac{3y}{x} = \frac{x^2}{x}$$

$$\frac{dy}{dx} - \frac{3y}{x} = x$$

$$\text{Let } P(x) = -\frac{3}{x} \quad , \quad Q(x) = x$$

$$\rho = e^{\int P(x)dx}$$

$$\rho = e^{\int \frac{-3}{x}dx} = e^{-3\ln x} = x^{-3}$$

$$\rho \cdot y = \int \rho\, Q(x)dx + c$$

$$x^{-3} \cdot y = \int x^{-3} \cdot x\, dx + c$$

$$x^{-3} \cdot y = \int x^{-2}\, dx + c$$

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$$x^{-3} \cdot y = \frac{x^{-1}}{-1} + c$$

$$\frac{x^{-3}}{x^{-3}} y = \frac{-x^{-1}}{x^{-3}} + \frac{c}{x^{-3}}$$

$$y = -x^2 + cx^3$$

<u>_H.W.:_</u> **Find a general solution of each the following equations:-**

1) $\dfrac{dy}{dx} + 2y = e^{-x}$

2) $2\dfrac{dy}{dx} - y = e^{x/2}$

3) $x\dfrac{dy}{dx} + 3y = \dfrac{\sin x}{x^2}$

4) $(x - 2y)\, dy + y\, dx = 0$

4 - First order: Exact

The general formula of the equation $M(x,y)\, dx + N(x,y)\, dy = 0$

If $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ then the equation is **Exact**

If the equation was Exact the solve:

$$df(x,y) = 0$$

$$\int df(x,y) = \int 0$$

f(x,y) = c

$$M(x,y)\, dx + N(x,y)\, dy = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = df$$

Ex. 1.: Show that differential equation:-

$$(x^2 + y^2)dx + (2xy + \cos y)dy = 0 \quad \text{is exact and solve it?}$$

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Solution

$$\frac{\partial M}{\partial y} = 2y \ , \quad \frac{\partial N}{\partial x} = 2y$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \textbf{Exact D.E.}$$

To find f(x,y) such that

$$\frac{\partial f}{\partial x} = x^2 + y^2 \ , \quad \frac{\partial f}{\partial y} = 2xy + \cos y$$

$$f(x,y) = \int M(x,y)dx + K(y)$$

$$f(x,y) = \int (x^2 + y^2)dx + K(y)$$

$$f(x,y) = \frac{x^3}{3} + xy^2 + K(y)$$

To find K(y)

$$\frac{\partial f}{\partial y} = 2xy + K'y = 2xy + \cos y$$

$$K'(y) = \cos y \quad تكامل$$

$$K(y) = \text{siny}$$

$$f(x,y) = \frac{x^3}{3} + xy^2 + siny$$

f(x,y) = c

$$\frac{x^3}{3} + xy^2 + siny = c$$

Ex. 2.: State the type of D.E. (y sec²x + secx tanx) dx + (tanx + 2y) dy = 0

Solution

$$M = ysec^2x + secx \ tanx$$

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$$\frac{\partial M}{\partial y} = sec^2 x$$

N = tanx +2y

$$\frac{\partial N}{\partial x} = sec^2 x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \qquad \textbf{Exact D.E.}$$

$$\frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = df = (ysec^2 x + secxtanx)dx + (tanx + 2y)dy$$

$$\frac{\partial f}{\partial x} = ysec^2 x + secxtanx$$

df = $\int (ysec^2 x + secxtanx)\,dx$

f(x,y) = y tanx + secx + K(y)

$$\frac{\partial f}{\partial y} = tanx + 0 + K'(y) = tanx + 2y$$

$$K'(y) = 2y$$

K(y) = $\int 2ydy$

K(y) = $\frac{2y^2}{2}$

f(x,y) = y tanx + secx + y^2

f(x,y) = c

y tanx + secx + y^2 = c

H.W.: Solve equation $\frac{dx}{x^2 y} + \frac{dy}{xy^2} = 0$