Al-Mustaqbal University
Department of Electrical Engineering Technique
Class 1st
Subject: AC Electrical Circuits
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Lecture One
Introduction

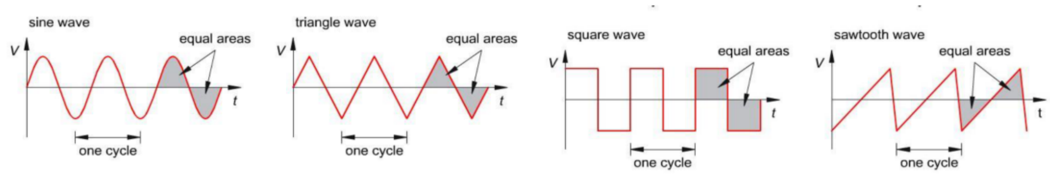
2.1 Introduction:

In **Electrical Engineering**, there are basically two types of voltage or current source which defines the kind of circuit and they are; **Alternating Current (or voltage) and Direct Current**.

"AC" stands for **A**lternating **C**urrent, which can refer to either voltage or current that alternates in polarity or direction, respectively.

An electrical circuit is a complete conductive path through which electrons flow from the source to the load and back to the source. The direction and magnitude of the electrons flow however depend on the kind of source. An alternating quantity is on that periodically reverses its direction. Sinusoidal wave is one of the periodic wave forms. In this sinusoidal wave magnitude is varies at every instant of time.

The following are different AC quantities



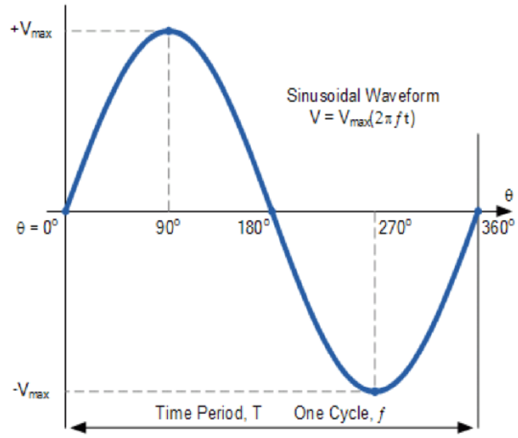
2.2 Alternating Quantity :

The shape obtained by plotting the instantaneous ordinate values of either voltage or current against time is called an **AC Waveform**. An AC waveform is constantly changing its polarity every half cycle alternating between a positive maximum value and a negative maximum value respectively with regards to time.

Waveforms are basically a visual representation of the variation of a voltage or current plotted to a base of time". Generally, for AC waveforms this horizontal base line represents a zero condition of either voltage or current. Any part of an AC type waveform which lies above the horizontal zero axis represents a voltage or current flowing in one direction.

An **Alternating Current or Voltage**, is one in which the value of either the voltage or the current varies about a particular mean value and reverses direction periodically.

AC CIRCUITS



1) **The Amplitude (A)** is the magnitude or intensity of the signal waveform measured in volts or amps.
2) **Cycle:** In an **AC Waveform** to complete one full pattern from its positive half to its negative half and back to its zero baseline again is called a **Cycle**.
   one complete cycle contains both a positive half-cycle and a negative half-cycle.
3) **Time Period:** The time taken by the waveform to complete one full cycle is called the **Periodic Time** of the waveform, and is given the symbol "T". The Period, (T) is the length of time in seconds that the waveform takes to repeat itself from start to finish.
   Time period can be calculated by considering
   1. Zero Crossing point to the next successive zero crossing point.
   2. From +ve peak to Next +ve peak
   3. From -ve peak to next -ve peak.

   One cycle of sine wave repeats the number of times mathematically represented as $f(t) = f(t+T)$ for all time.

4) **The Frequency, (*f*)** is the number of times the waveform repeats itself within a one second time period. Frequency is measured in **Hertz**, (Hz) named after the German physicist Heinrich Hertz. Frequency is the reciprocal of the time period.

$$frequency\ (f) = \frac{1}{T}\ Hz$$

One Hz is defined as one cycle per one second. (The number of complete cycles that are produced within one second (cycles/second) is called the **Frequency**)

5) **Peak Value:**
   The maximum value of +ve or -ve attained by a waveform is called peak value.

6) **Peak to Peak value;**
   The peak to peak value of sine wave form is the value from +ve peak to -ve peak.

2.3 Sinusoidal waveform

2.3.1. Basic terms

As mentioned in sinusoid wave generation at time t=0 , magnitude is zero , as time increases magnitude reaches to maximum value and then decrease and reaches to zero at T/2. Later polarity changes i.e decreases to -ve maximum and again increases to zero at time 'T'. The complete +ve and -Ve portion is called one cycle. This is repeated for t=infinity.

The most commonly used waveform is the sinusoidal one. Suppose V (t) is a sinusoidal waveform:

$$V(t)=V_m \sin(\omega t + \varphi)$$

where V_m is the amplitude – it is the minimal and maximal value of the waveform;

$\omega=2.\pi.f$ is the angular frequency of the waveform. It is measured in rad. s−1.

Φ is the phase shift in degrees or radians that the waveform has shifted left or right from the reference point (t=0).

When φ=0 we say that the waveform is in phase. When φ>0 or φ<0 the phase is positive or negative respectively.

i) Instantaneous value:

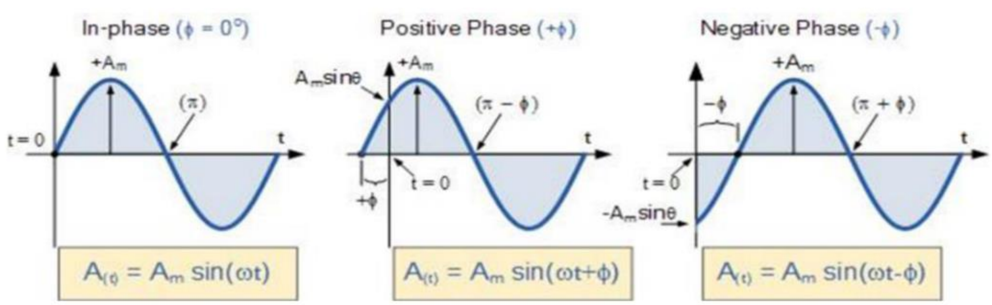
The value of an alternating quantity at a particular instant is called its **instantaneous value**. Each repetition of a set of positive and negative instantaneous values of an alternating quantity is called a cycle (The value of wave form magnitude at each instant time is called instantaneous value)

ii)Phase: Phase of a sine wave is an angular measurement that specifies the position of the sine wave relative to reference wave. This is of two types

1. In phase: Any wave which is starts from the starting position of reference wave.
2. Out of phase: A wave form starts not from the position of reference wave is out of phase this may be lead or lag. Consider the above figure w.r.t wave A , Wave B is lagging by phase ɵ. W.r.t wave A Wave C is stats earlier thus it lead by phase ɵ. This difference is called phase difference.

Mathematically represent by as follows. $v(t) = V_m Sin(\omega t \mp \emptyset)$

Where ω is angular frequency



iii) <u>Average value:</u>

The average value or mean value of an alternating current is the average of all instantaneous values during a half cycle. It is the ratio of all instantaneous values to the number of instantaneous values selected during a half cycle.

The average value of an AC waveform is giving by the equation;

$$Vavg = \frac{V1 + V2 + V3 + \ldots + Vn}{n}$$

Where V1...Vn is the instantaneous value of voltage during the half cycle.

Average value of any curve is the total area under the complete curve divided by the distance of the curve.

$$\boldsymbol{V_{av} = \frac{1}{T}\int_0^T v(t)dt}$$

Note:

1. Average value is calculated for half cycles for symmetrical waves and Non symmetrical waves calculated for full cycles.

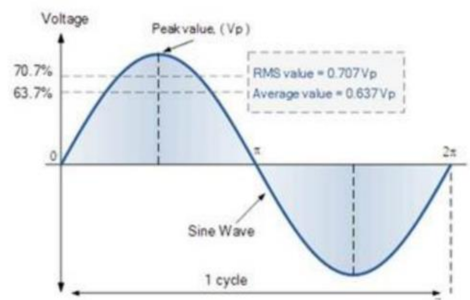
Average Value

- In a pure sine wave if the average value is calculated over the full cycle, the average value would be equal to zero as the positive and negative halves will cancel each other out.
  - Then the average value is obtained by adding the instantaneous values of voltage over one half cycle only.

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} V_p \sin\theta \; d\theta$$

$$V_{av} = \frac{V_p}{\pi}[-\cos\theta]_0^{\pi}$$

$$V_{av} = \frac{2V_p}{\pi} = 0.637V_p$$

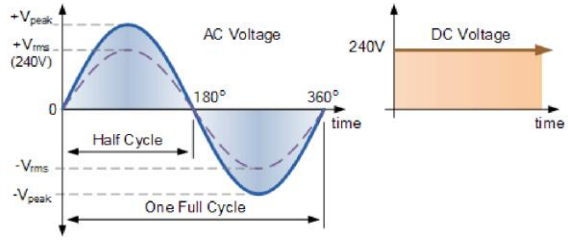


iv) RMS Value:

The term "RMS" stands for "Root-Mean-Squared". Most books define this as the "amount of AC power that produces the same heating effect as an equivalent DC power", or something similar along these lines, but an RMS value is more than just that. The RMS value is the square root of the mean (average) value of the squared function of the instantaneous values. The symbols used for defining an RMS value are V_{RMS} or I_{RMS}.

$$V_{RMS} = \sqrt{(\frac{V_m^2}{2\pi}\int_0^{2\pi} Sin^2(\omega t)d(\omega t)}$$

Similarly, $I_{RMS} = \sqrt{(\frac{I_m^2}{2\pi}\int_0^{2\pi} Sin^2(\omega t)d(\omega t)}$.



For example, the domestic mains supply in the United Kingdom is 240Vac. This value is assumed to indicate an effective value of "240 Volts rms". This means then that the sinusoidal rms voltage from the wall sockets of a UK home is capable of producing the same average positive power as 240 volts of steady DC voltage as shown below.

So how do we calculate the **RMS Voltage** of a sinusoidal waveform. The RMS voltage of a sinusoid or complex waveform can be determined by two basic methods.

- **Graphical Method** – which can be used to find the RMS value of any non-sinusoidal time-varying waveform by drawing a number of mid-ordinates onto the waveform.
- **Analytical Method** – is a mathematical procedure for finding the effective or RMS value of any periodic voltage or current using calculus.

The positive half of the waveform is divided up into any number of "n" equal portions or *mid-ordinates* and the more mid-ordinates that are drawn along the waveform, the more accurate will be the final result. The width of each mid-ordinate will therefore be n° degrees and the height of each mid-ordinate will be equal to the instantaneous value of the waveform at that time along the x-axis of the waveform.

Then we can define the term used to describe a rms voltage (V_{RMS}) as being "the square root of the mean of the square of the mid-ordinates of the voltage waveform" and this is given as:

$$V_{RMS} = \sqrt{\frac{\text{sum of mid-ordinate (voltages)}^2}{\text{number of mid-ordinates}}}$$

and for our simple example above, the RMS voltage will be calculated as:

$$i = I_{max} \sin \omega t$$

$$I_{rms}^2 = \frac{\text{Area of first half cycle of } i^2}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} i^2 \, d(\omega t)$$

$$= \frac{1}{\pi}\int_0^{\pi} I_{max}^2 \sin^2 \omega t \, d(\omega t)$$

$$= \frac{I_{max}^2}{2\pi}\int_0^{\pi}(1-\cos 2\omega t)\, d(\omega t)$$

$$= \frac{I_{max}^2}{2\pi}\left[\omega t - \frac{1}{2}\sin 2\omega t\right]_0^{\pi}$$

$$= \frac{I_{max}^2}{2\pi} \times \pi = \frac{I_{max}^2}{2}$$

$$\text{or } I_{rms} = \sqrt{\frac{I_{max}^2}{2}} = \frac{I_{max}}{\sqrt{2}}$$

$$\text{Similarly, } E_{rms} = \frac{E_{max}}{\sqrt{2}}$$

v) Form Factor:

One other quantity associated with an Alternating current that we need to look at is the form factor. The form factor is a parameter used in describing AC waveforms and is giving by the **ratio between the RMS value** of the alternating quantity and the **Average Value**.

$$\textbf{Form Factor} = \frac{V_{rms}}{V_{avg}} = \frac{V_{m/\sqrt{2}}}{2V_{m/\pi}} = 1.11$$

Where V_m is the peak or maximum voltage.

One other application of form factors is found in digital multi meters used in measuring Alternating current or voltage. Most of these meters are generally scaled to display the RMS value of sine waves which they are designed to obtain by calculating the average value and multiplying by the form factor of a sinusoid (1.11) since it can be a little bit difficult to digitally calculate the rms values. Thus, at times, for AC waveforms which are not pure sinusoidal, the reading from a multi meter may be a little bit inaccurate.

vi) Crest Factor:

The crest factor is the ratio of the **peak value** of an alternating current or voltage to the **root mean square (RMS)** of the waveform. Mathematically, it is given by the equation;

$$\textbf{Crest Factor (or)Peakfactor} = \frac{V_m}{V_{rms}} = \frac{V_m}{V_m/\sqrt{2}} = 1.414$$

Where V_m is the maximum amplitude of the waveform. **For a pure sine wave**, similar to the form factor, the **crest factor is always fixed at 1.414**. The crest factor is majorly an indication of how high the peaks of an alternating quantity are. In direct current, for instance, the crest factor is always equal to 1 which is an indication of the lack of peaks in the waveform of a Direct current.

2.4 Phasor

Basically, a rotating vector, simply called a "**Phasor**" is a scaled line whose length represents an AC quantity that has both magnitude ("peak amplitude") and direction ("phase") which is "frozen" at some point in time.

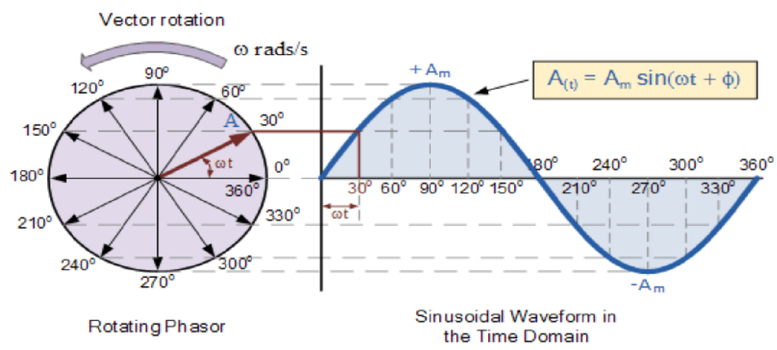
A phasor is a vector that has an arrow head at one end which signifies partly the maximum value of the vector quantity (V or I) and partly the end of the vector that rotates.

Generally, vectors are assumed to pivot at one end around a fixed zero point known as the "point of origin" while the arrowed end representing the quantity, freely rotates in an **anti-clockwise** direction at an angular velocity, (ω) of one full revolution for every cycle. This anti-clockwise rotation of the vector is considered to be a positive rotation. Likewise, a clockwise rotation is considered to be a negative rotation.

The phase of an alternating quantity at any instant in time can be represented by a phasor diagram, so phasor diagrams can be thought of as "functions of time". A complete sine wave can be constructed by a single vector rotating at an angular velocity of $\omega = 2\pi f$, where f is the frequency of the waveform. Then a **Phasor** is a quantity that has both "Magnitude" and "Direction".

Generally, when constructing a phasor diagram, angular velocity of a sine wave is always assumed to be: ω in rad/sec. Consider the phasor diagram below.

Phasor Diagram of a Sinusoidal Waveform



As the single vector rotates in an anti-clockwise direction, its tip at point A will rotate one complete revolution of 360° or 2π representing one complete cycle. If the length of its moving tip is transferred at different angular intervals in time to a graph as shown above, a sinusoidal waveform would be drawn starting at the left with zero time. Each position along the horizontal axis indicates the time that has elapsed since zero-time, $t = 0$. When the vector is horizontal the tip of the vector represents the angles at 0°, 180° and at 360°.

Likewise, when the tip of the vector is vertical it represents the positive peak value, (+Am) at 90° or π/2 and the negative peak value, (-Am) at 270° or 3π/2. Then the time axis of the waveform represents the angle either in degrees or radians through which the phasor has moved.

Sometimes when we are analysing alternating waveforms, we may need to know the position of the phasor, representing the Alternating Quantity at some particular instant in time especially when we want to compare two different waveforms on the same axis. For example, voltage and current. We have assumed in the waveform above that the waveform starts at time t = 0 with a corresponding phase angle in either degrees or radians.

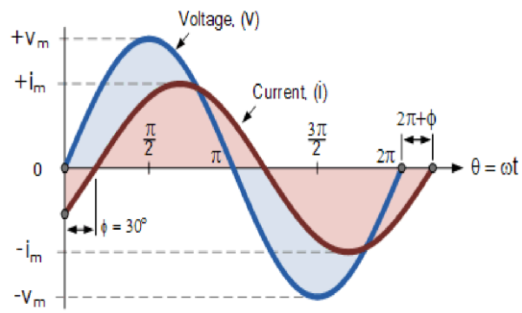
But if a second waveform starts to the left or to the right of these zero points or we want to represent in phasor notation the relationship between the two waveforms then we will need to take into account this phase difference, Φ of the waveform. Consider the diagram below for **Phase Difference**.

Phase Difference of a Sinusoidal Waveform

The generalised mathematical expression to define these two sinusoidal quantities will be written as:

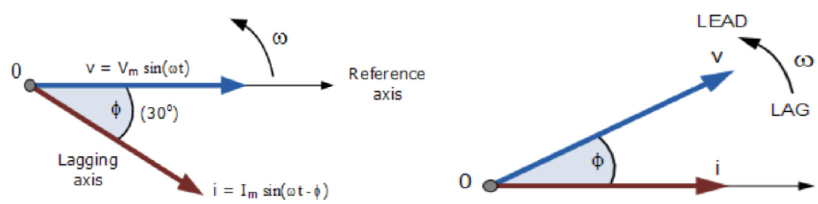
$$v_{(t)} = V_m \sin(\omega t)$$

$$i_{(t)} = I_m \sin(\omega t - \phi)$$



The current, i is lagging the voltage, v by angle Φ and in our example above this is 30°. So the difference between the two phasors representing the two sinusoidal quantities is angle Φ and the resulting phasor diagram will be.

Phasor Diagram of a Sinusoidal Waveform



The phasor diagram is drawn corresponding to time zero (t = 0) on the horizontal axis. The lengths of the phasors are proportional to the values of the voltage, (V) and the current, (I) at the instant in time that the phasor diagram is drawn. The current phasor lags the voltage phasor by the angle, Φ, as the two phasors rotate in an *anticlockwise* direction as stated earlier, therefore the angle, Φ is also measured in the same anticlockwise direction.

If however, the waveforms are frozen at time, t = 30º, the corresponding phasor diagram would look like the one shown on the right. Once again, the current phasor lags behind the voltage phasor as the two waveforms are of the same frequency.

However, as the current waveform is now crossing the horizontal zero axis line at this instant in time we can use the current phasor as our new reference and correctly say that the voltage phasor is "leading" the current phasor by angle, Φ. Either way, one phasor is designated as the *reference* phasor and all the other phasors will be either leading or lagging with respect to this reference.

2.5 Impedance:

Pure resistance within an AC circuit produces a relationship between its voltage and current phasors in exactly the same way as it would relate the same resistors voltage and current relationship within a DC circuit. However, in a DC circuit this relationship is commonly called **Resistance**, as defined by Ohm's Law but in a sinusoidal AC circuit this voltage-current relationship is now called **Impedance**.

In other words, in an AC circuit electrical resistance is called "Impedance".

In both cases this voltage-current (V-I) relationship is always linear in a pure resistance. So when using resistors in AC circuits the term **Impedance**, symbol **Z** is the generally used to mean its resistance. Therefore, we can correctly say that for a resistor, DC resistance = AC impedance, or R = Z.

The impedance vector is represented by the letter, (Z) for an AC resistance value with the units of Ohm's (Ω) the same as for DC. Then Impedance (or AC resistance) can be defined as:

AC Impedance

$$Z = \frac{V}{I}\ \Omega\text{'s}$$

Impedance can also be represented by a complex number as it depends upon the frequency of the circuit, ω when reactive components are present. But in the case of a purely resistive circuit this reactive component will always be zero and the general expression for impedance in a purely resistive circuit given as a complex number will be:

The impedance Z in ac circuits is defined as the ratio of voltage function to current function. Hence, the impedance is a complex number and can be expressed in the rectangular form as

$$Z = \frac{V_{rms}\angle 0°}{I_{rms}\angle \theta°} = R \pm jX$$

$$Z = R + jX, \qquad Z = Z\angle\theta, \qquad Z = |Z|e^{j\theta}, \qquad Z = |Z|(\cos\theta + j\sin\theta)$$

The real component of the impedance is called the resistance *R* and the imaginary component is called the reactance *X*, both of which are in ohms. The reactance is a function of *ω* in L and C loads, and for an inductive load, *X* is positive, whereas for a capacitive load, *X* is negative.

Resistor R	**Capacitor** C	**Inductor** L
Resistance	Capacitive reactance	Inductive reactance
$V_R/I = R$	$V_C/I = X_C = \frac{1}{\omega C}$	$V_L/I = X_L = \omega L$
V and I in phase	V lags I by π/2	V leads I by π/2