Lecture Four

Single Phase Transformer

1. Short Circuit

The short circuit is one of the two fundamental tests performed on transformers to determine their equivalent circuit parameters. While the open circuit test determines core losses and magnetizing parameters, the short circuit test determines copper losses and impedance parameters. The short circuit test serves several critical purposes:

- Copper Losses: Calculate **I^2R** losses in the windings at full load
- Find Equivalent Impedance: Determine the total impedance referred to one side
- Calculate Voltage Regulation
- Efficiency Calculations: Essential for determining transformer efficiency at various loads
- Negligible Core Losses: At such low voltages, the magnetic flux in the core is very small, making core losses negligible.

The circuit diagram of the equivalent circuit is shown in Figure 1.

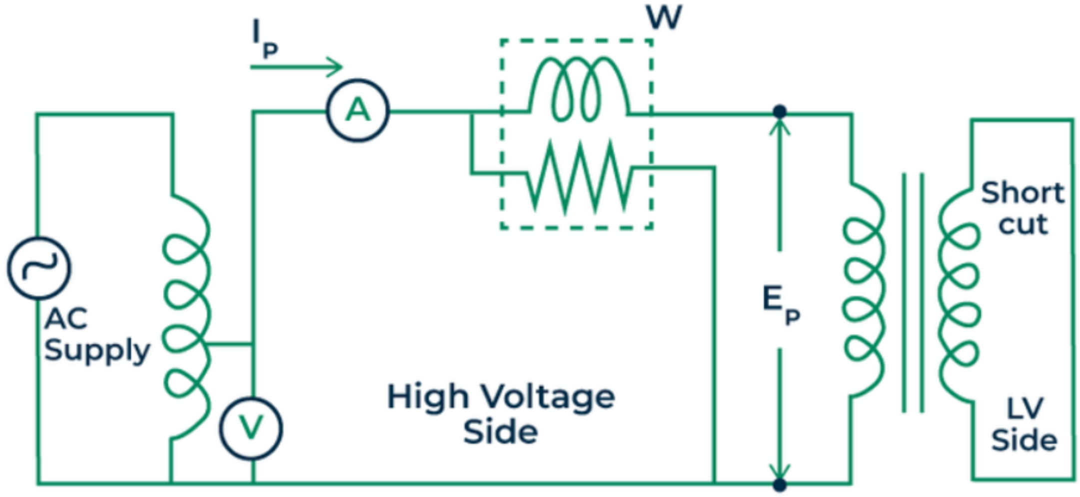
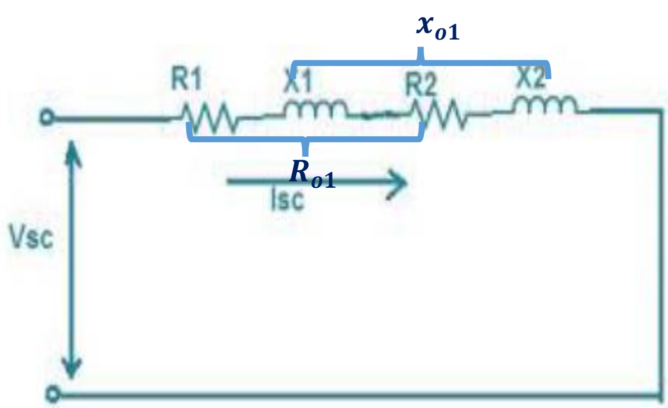


Figure 1. Equivalent circuit of short circuit transformer

2. Mathematical Analysis

Vsc = Short circuit voltage (volts)



Isc = Short circuit current = Rated current (amperes)

Psc = Short circuit power (watts)

Equivalent Impedance:

Z_{01} = Vsc / Isc (ohms)

Equivalent Resistance:

$R_{01} = Psc / Isc^2$ (ohms)

This represents the total copper losses: $R_{01} = R_1 + R_2$

Equivalent Reactance (referred to test side):

$X_{01} = \sqrt{(Z_{01}^2 - R_{01}^2)}$ (ohms)

This represents the total leakage reactance: **$X_{01} = X_1 + X_2$**

Copper Losses at Any Load:

Copper loss at load = $Psc \times (I/I\_rated)^2$

Where I = load current

Percentage Impedance

An important specification for transformers: **$\%Z = (Vsc / V\_rated) \times 100\%$**

Ex. A 10 kVA, 500/250 V, 50 Hz single-phase transformer gave the following test results: **Short Circuit Test** (LV side): Vsc = 25 V, Isc = 20 A (rated current), Psc = 120 W. Calculate: a) Equivalent impedance, resistance, and reactance referred to LV side b) Copper losses at half load c) Percentage impedance

Solution:

$Z_{01} = Vsc / Isc = 25 / 20 = 1.25\ \Omega$

$R_{01} = P_{sc} / I_{sc}^2 = 120 / (20)^2 = 120 / 400 = 0.3\ \Omega$

$X_{01} = \sqrt{(Z_{01}^2 - R_{01}^2)} = \sqrt{(1.25^2 - 0.3^2)} = \sqrt{(1.5625 - 0.09)} = \sqrt{1.4725} = 1.213\ \Omega$

- **Copper Losses at Half Load:**

At half load, current = 20/2 = 10 A

Copper loss = $P_{sc} \times (I/I_{rated})^2 = 120 \times (10/20)^2 = 120 \times 0.25 = 30$ W

- **Percentage Impedance:**

Rated voltage on LV side = 250 V

$\%Z = (V_{sc} / V_{rated}) \times 100\% = (25 / 250) \times 100\% = 10\%$

3. Energy Considerations

During the short circuit test:

Input Energy = Vsc × Isc (volt-amperes)

Active Power = Psc (watts) → Copper losses

Ex.: A 10 kVA, 500/250 V, 50 Hz single-phase transformer as the following test results:

O.C. Test (LV side):

$V_o = 250$ V, $I_o = 1.5$ A, $W_o = 80$ W

S.C. Test (LV side):

$V_{sc} = 20$ V, $I_{sc} = 40$ A, $W_{sc} = 120$ W

Calculate: a) All parameters of the equivalent circuit referred to LV side

b) Calculate efficiency at full load, 0.8 pf lagging

Solution:

a) From O.C. Test:

$I_w = P_o / V_o = 80 / 250 = 0.32$ A

$I_\mu = \sqrt{(I_o^2 - I_w^2)} = \sqrt{(1.5^2 - 0.32^2)} = \sqrt{(2.25 - 0.1024)} = 1.465$ A

$R_o = V_o / I_w = 250 / 0.32 = 781.25\ \Omega$

$X_o = V_o / I_\mu = 250 / 1.465 = 170.65\ \Omega$

From S.C. Test:

$Z_{01} = V_{sc} / I_{sc} = 20 / 40 = 0.5\ \Omega$

$R_{01} = W_{sc} / I_{sc}^2 = 120 / (40)^2 = 120 / 1600 = 0.075\ \Omega$

$X_{01} = \sqrt{(Z_{01}^2 - R_{01}^2)} = \sqrt{(0.5^2 - 0.075^2)} = \sqrt{(0.25 - 0.00562)} = 0.494\ \Omega$

b) Efficiency at Full Load, 0.8 pf:

Output power: Pout = $V_2 \times I_2 \times \cos\varphi$ = 10,000 × 0.8 = 8,000 W

Core losses = P_0 = 80 W

Copper losses at full load = Wsc = 120 W

Total losses = 80 + 120 = 200 W

Input power = Output + Losses = 8,000 + 200 = 8,200 W

Efficiency: η = (Pout / Pin) × 100% = (8,000 / 8,200) × 100% = 97.56%

Ex.: A 100 kVA, 6600/400 V, 50 Hz single-phase transformer has the following test data:

O.C. Test (LV side): P_0 = 500 W (core loss)

S.C. Test (LV side): Psc = 1800 W (copper loss at full load)

Calculate: a) The efficiency at full load, 0.8 pf lagging b) The efficiency at half load, 0.8 pf lagging

Solution: a) Efficiency at Full Load, 0.8 pf:

Output power = kVA × pf = 100 × 0.8 = 80 kW

Core loss = 500 W = 0.5 kW

Copper loss at full load = Pcu = 1800 W = 1.8 kW

Total loss = 0.5 + 1.8 = 2.3 kW

Input = Output + Losses = 80 + 2.3 = 82.3 kW

$\eta = (80 / 82.3) \times 100\% = 97.21\%$

b) Efficiency at Half Load, 0.8 pf:

Output = 100 × 0.5 × 0.8 = 40 kW

Core loss = 0.5 kW (constant)

Copper loss at half load = $P_{sc} \times (0.5)^2 = 1.8 \times 0.25 = 0.45$ kW

Total loss = 0.5 + 0.45 = 0.95 kW

Input = 40 + 0.95 = 40.95 kW

$\eta = (40 / 40.95) \times 100\% = 97.68\%$

4. Equivalent Circuit of a Single-Phase Transformer

The analysis of a transformer can be carried out by using an equivalent circuit which can be derived by considering the following: -

- The primary and secondary windings have finite resistances considered as lumped parameters.
- The leakage fluxes are modelled as leakage reactance in the equivalent circuit.
- The core-loss component of current is modelled using a shunt resistance.

- The magnetization of the core is modelled using a magnetizing reactance as a shunt branch.

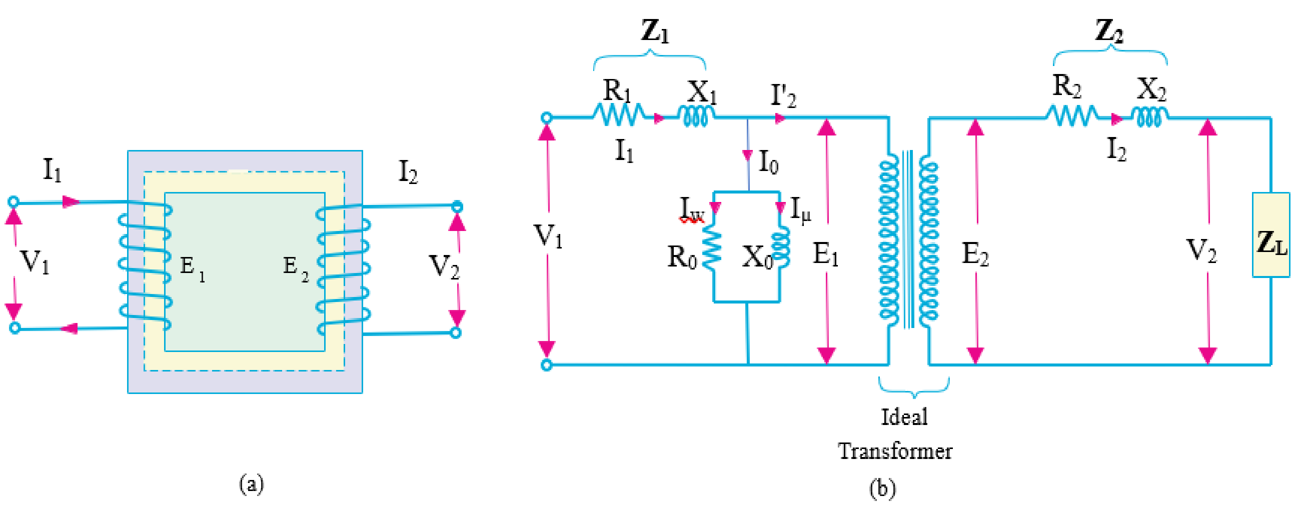


Fig.2 Diagram of equivalent circuit for the transformer with load

The transformer shown diagrammatically in Fig.1 (a) can be resolved into an equivalent circuit in which the resistance and leakage reactance of the transformer are imagined to be external to the winding whose only function then is to transform the voltage.

a. Exact Equivalent Circuit

The transformer circuit can be moved to the right or left by referring all quantities to the primary or secondary side, respectively. This is almost invariably done. The equivalent circuit moved to primary is shown in Fig. 2.

If we shift all the impedances from one winding to the other, the transformer core is eliminated and we get an equivalent electrical circuit. Various voltages and currents can be readily obtained by solving this electrical circuit.

b. Approximate Equivalent Circuit

The equivalent circuit can be simplified by assuming small voltage drop across the primary impedance and $V_1 = E_1$. If the applied voltage and the induced emf are the same then the shunt branch can be moved across the source voltage and the approximate equivalent circuit is drawn as shown in Figure 3.

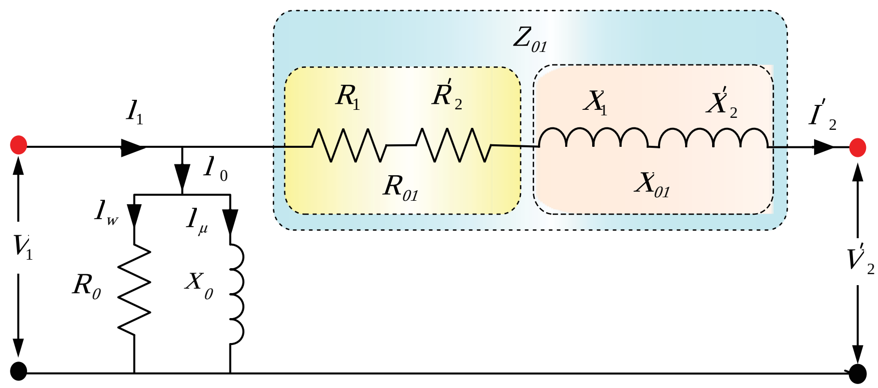


Fig 3. Approximate equivalent circuit

Referred to the primary	Referred to the secondary
$E_1 = E_2' = \frac{E_2}{a}$	$E_1' = E_2 = aE_1$
$V_2' = \frac{V_2}{a}$	$V_1' = aV_1$
$I_2' = aI_2$	$I_1' = \frac{I_1}{a}$
$X_2' = \frac{X_2}{a^2}$	$X_1' = a^2X_1$

$$R_2' = \frac{R_2}{a^2} \qquad R_1' = a^2 R_1$$

Ex.: A 30 kVA, 2400/120 V, 50-Hz transformer has a high voltage winding resistance of 0.1 Ω and a leakage reactance of 0.22 Ω. The low voltage winding resistance is 0.035 Ω and the leakage reactance is 0.012 Ω. Find the equivalent winding resistance, reactance and impedance (only magnitude) referred to:

1- High Voltage Side.

2- Low-Voltage Side.

Solution

$$k = \frac{120}{2400} = 1/20, R_1 = 0.1\ \Omega\ , X_1 = 0.22\ \Omega$$

$$R_2 = 0.035\ \Omega \text{ and } X_2 = 0.012\ \Omega$$

1- For high voltage side

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{k^2} = 0.1 + \frac{0.035}{\left(\frac{1}{20}\right)^2} = \mathbf{14.1\ \Omega}$$

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{k^2} = 0.22 + \frac{0.012}{\left(\frac{1}{20}\right)^2} = \mathbf{5.02\ \Omega}$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{14.1^2 + 5.02^2} = \mathbf{15\ \Omega}$$

2- For low voltage side

$$R_{02} = R_2 + R_1' = R_2 + k^2 R_1 = 0.035 + \left(\frac{1}{20}\right)^2 \times 0.1 = \mathbf{0.03525\ \Omega}$$

$$X_{02} = X_2 + X_1' = X_2 + k^2 X_1 = 0.012 + \left(\frac{1}{20}\right)^2 \times 0.22 = \mathbf{0.0125\ \Omega}$$

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{0.0325^2 + 0.01255^2} = \mathbf{0.0374\ \Omega}$$

Ex.: The equivalent circuit parameters of a single-phase 240/2400, 50 Hz, transformer are R_0 =600 Ω, X_0 =300 Ω, R_{01} =0.25 Ω, X_{01} =0.75 Ω. The transformer is supplying a load of 400+ j 200 Ω. Keeping the primary voltage of 240V, calculate the

1- The secondary terminal voltage

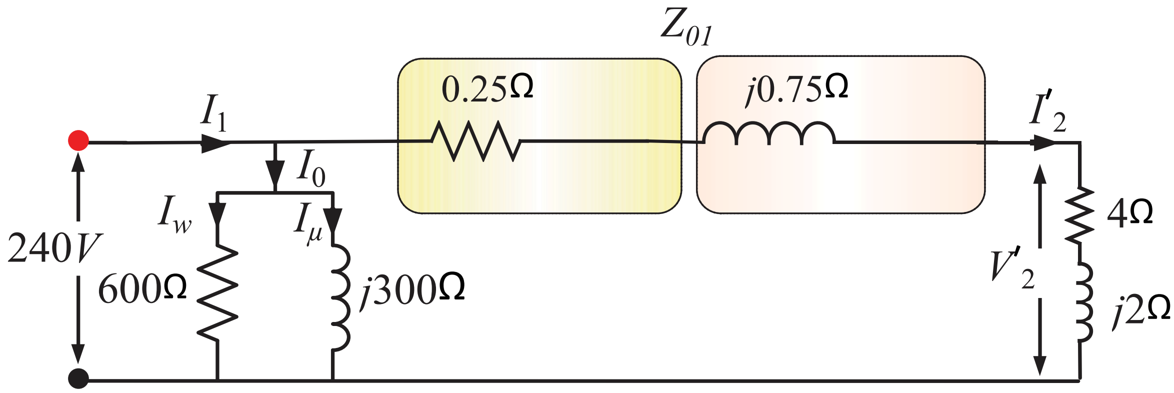
2- Current in the primary winding

3- Power factor of the primary side

4- Power output

5- Power Input

Solution



Since the equivalent circuit is referred to the low voltage (primary side), the load

impedance is also transformed to the low voltage side.

$k = 2400/240 = 10$

$$Z'_L = Z_L/k^2 = (400 + j200)/(0.1)^2 = \mathbf{4 + j2\ \Omega}$$

$$Z'_L + Z_{01} = 0.25 + j0.75 + 4 + j2 = 4.25 + j2.75 = \mathbf{5.062\angle 32.9\ \Omega}$$

$$I'_2 = V_1/(Z'_L + Z_{01}) = \frac{240\angle 0}{5.062\angle 32.9} = 47.412\angle -32.9 = \mathbf{39.8 - j25.753\ A}$$

1- Secondary terminal voltage (without phase)

$$V'_2 = I'_2 Z_L = 5.062 \times \sqrt{4^2 + 2^2} = \mathbf{212.03V}$$

2- Primary current:

The core loss component of current $I_w = \frac{V_1}{R_0} = \frac{240}{600} = \mathbf{0.4\ A}$

The magnetizing component of current $I_\mu = \frac{V_1}{X_0} = \frac{240}{300} = \mathbf{0.8\ A}$

The no-load current $I_0 = I_w + jI_\mu = \mathbf{0.4 - j0.8\ A}$

The primary current

$I_1 = I_0 + I_2' = 39.8 - j25.753 + 0.4 - j0.8 = 40.2 - j26.553 =$ **$48.178\angle -33.44$ A**

3- Power factor of the primary current pf $= \cos(33.44) =$ **0.834 lagging**

4- Power output = $I_2'^2 R_L' = 47.412^2 \times 4 =$ **8.99 kW**

5- Power Input = $V_1 I_1 \cos\phi_1 = 240 \times 48.178 \times \cos(33.44) =$ **9.65 kW**