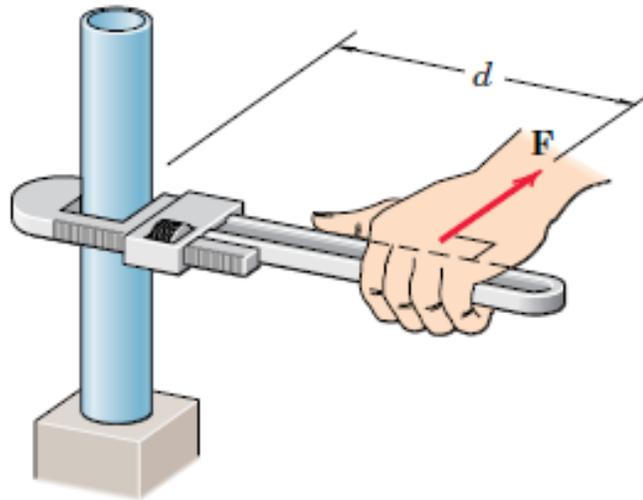|	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

Moment of the Force

- **The moment M**: is a force that tends to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action

- **Moment is also referred to as torque.**

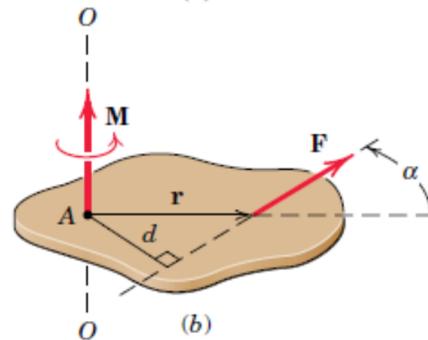|	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

Moment about a Point

▪ The magnitude of the ***moment*** or ***tendency*** of the force to rotate the body about the axis *O-O* perpendicular to the plane of the body is proportional both to the magnitude of the ***force*** and to the ***moment arm d***, which is the perpendicular distance from the axis to the line of action of the force.

$$M=F.d$$

M = the moment (N.M)
F = applied force (N).
d = moment arm (m).

▪ The moment is a vector **M** perpendicular to the plane of the body.

▪ The sense of **M** depends on the direction in which **F** tends to rotate the body.

▪ The right-hand rule is used to identify this sense.

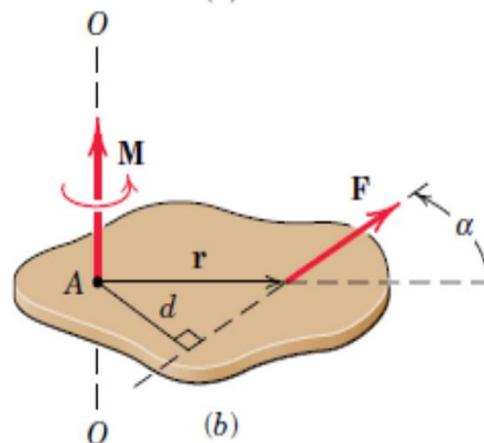|	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

(b)
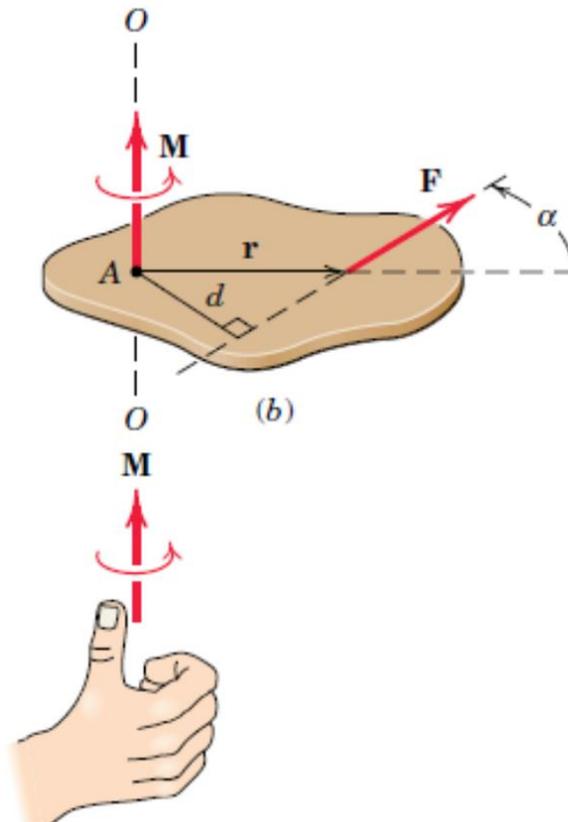
- The moment of force **F** about point *A* in **Fig *d*** has the magnitude *M = Fd* and is counterclockwise.

- **Plus sign (+)** for counterclockwise moments and a minus **sign (-)** for clockwise moments.

Email (mariam.ghassan.ghaffar@uomus.edu.iq)

	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

(d)
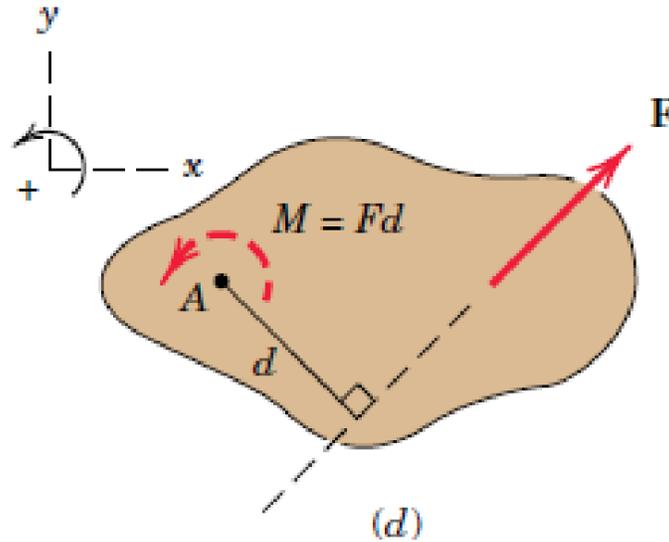
The Cross Product

▪   The moment of **F** about point *A* of **Fig b** may be represented by the cross-product expression:

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

(b)

	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

■   Where **r** is a position vector which runs from the moment reference point *A* to *any* point on the line of action of **F**.

Varignon's Thermos

■   One of the most useful principles of mechanics is ***Varignon's theorem*, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.**

$$M_\circ = r \times R$$

$$\text{Because, } R = P + Q$$

$$r \times R = r \times (P + Q)$$

$$M_\circ = r \times R = r \times P + r \times Q$$

(a)

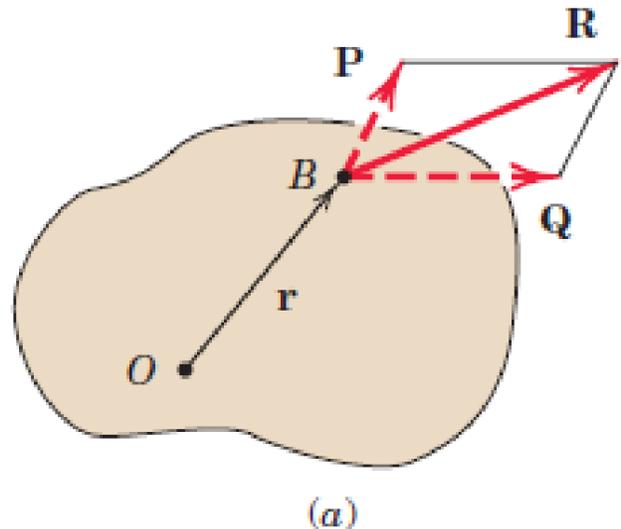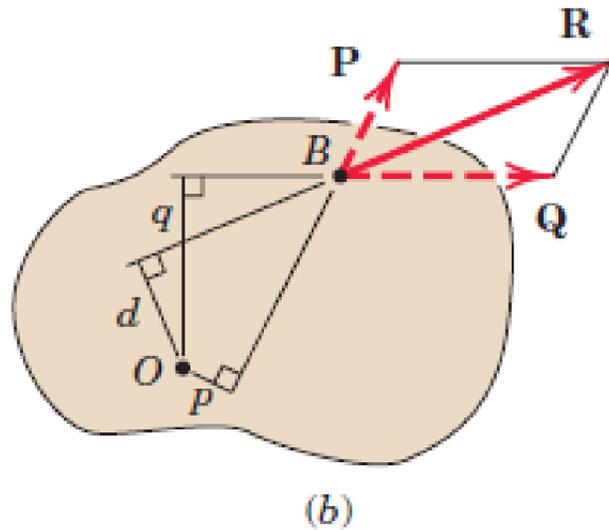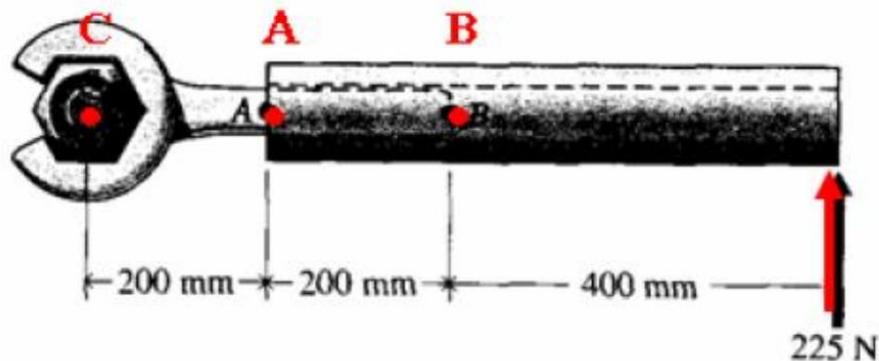|	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1ˢᵗ term – Lecture 3	

Figure **b** illustrates the usefulness of Varignon's theorem. The moment of **R** about point **O** is **Rd.**

$$M_\circ = R . d = P . p + Q . q$$

(b)

Ex1: Determine the moment of the force 225 N about the Points A , B , and C.

	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

MA =|F| dA = 225 × 0.6 = 135 Nm

MB =|F| dB = 225 × 0.4 = 90 Nm

MC =|F| dC = 225 × 0.8 = 180 Nm

Example2. *A force of **15 N** is applied perpendicular to the edge of a door **0.8 m** wide as shown in Fig. (a). Find the moment of the force about the hinge.*

15 N

O · ————————————— A

|← 0.8 m →|

(a)
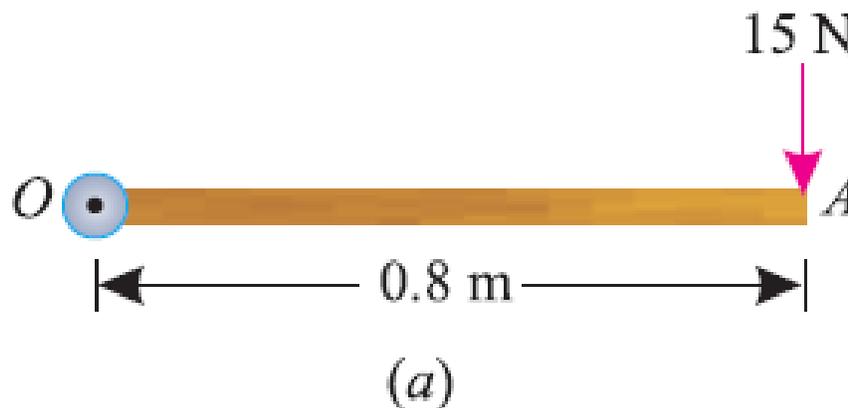
Solution. given : Force applied (**P**) = **15 N** and width of the door (**l**) = **0.8 m** Moment when the force acts perpendicular to the door.
We know that the moment of the force about the hinge,

$$M = P * l = 15 * 0.8 = 12.0 \text{ N.m} \quad \textbf{Ans.}$$

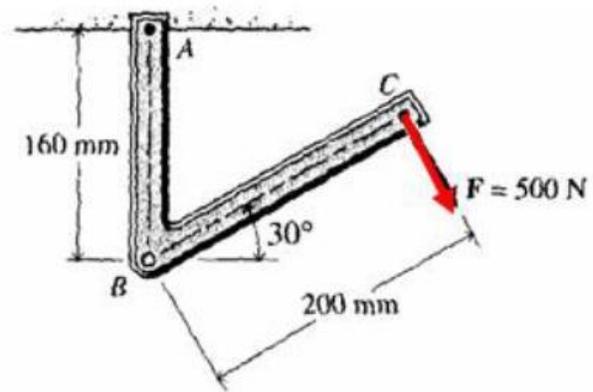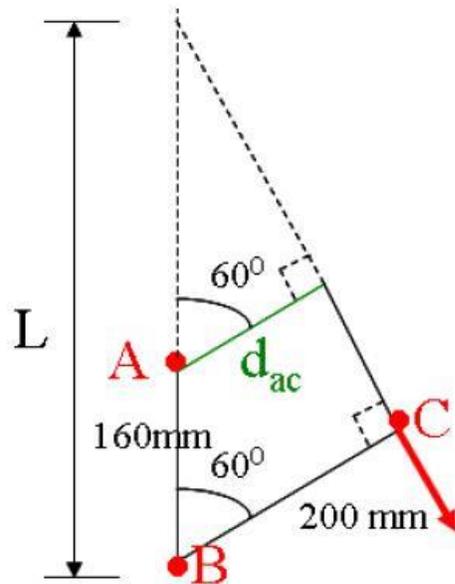|	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1ˢᵗ term – Lecture 3	

*Ex3: Determine the moment of the force **500 N** about the point A and B.*

Solution:

$$\text{Cos}(60) = 200/L$$
$$\text{Cos}(60) = dac/(L-160)$$
$$L = 200/\text{Cos}(60)$$
$$L = 160 + dac/\text{Cos}(60)$$
$$dac = 200 - 160\,\text{Cos}(60) = 120 \text{ mm}$$
$$dac = 120 \text{ mm}$$
$$MA = |F|dAC = 500*0.12 = 60 \text{ Nm}$$
$$MB = |F|\ dB = 500*0.2 = 100 \text{ Nm}$$

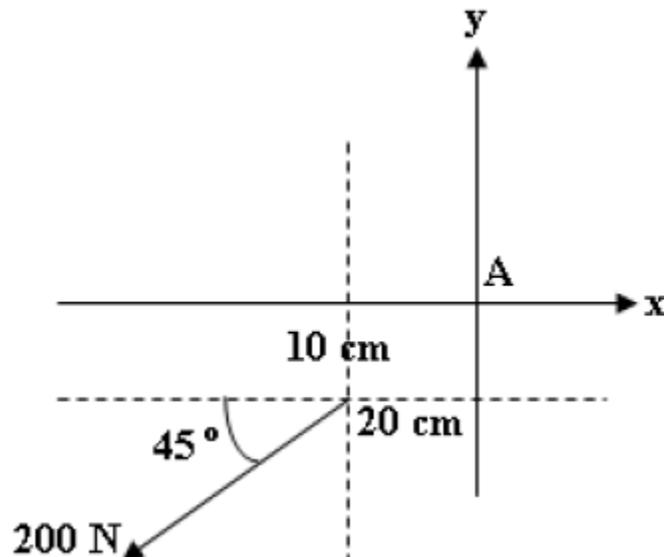|	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1ˢᵗ term – Lecture 3	

Ex4: Find the moment of the force 200 N About the point(A) shown in fig.

Solution:

Fx = F . cos θ = 200 cos 45
   = 200 * 0.707 = 141.42 N
Fy = F . sin θ = 200 sin 45
   = 200 * 0.707 = 141.42 N
M1 = Fx * d = 141.42 * 10 = 1414.2 N . cm
M2 = Fy * d = 141.42 * 20 = 2828.4 N .cm
M (A) = M1- M2 = - 1414.2 N .cm

Email (mariam.ghassan.ghaffar@uomus.edu.iq)

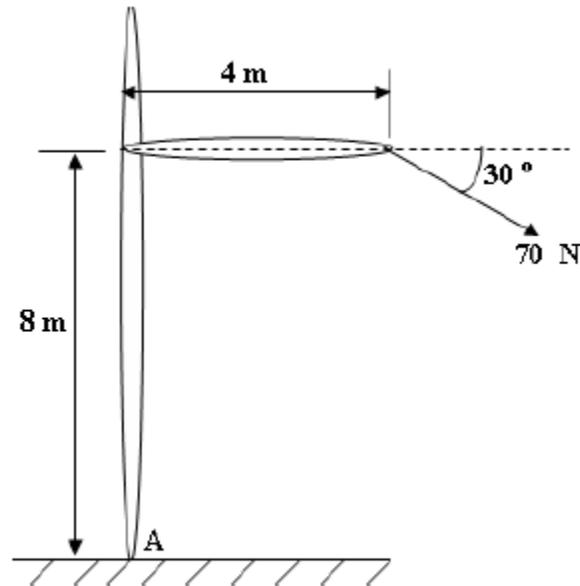|	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

Ex5:Determine the moment of the force(70 N) shown in fig. about the Point (A).

Solution:

Fx = F . cos θ = 70 cos 30
    = 70 * 0.866 = 60.62 N
Fy = F . sin θ = 70 sin 30
    = 70 * 0.5 = 35 N
M1 = Fx * d = 60.62 * 8 = 484.97 N . m
M2 = Fy * d = 35 * 4 = 140 N . m
M (A) = M1 + M2 = 484.97 N + 140 = 624.97 N . m

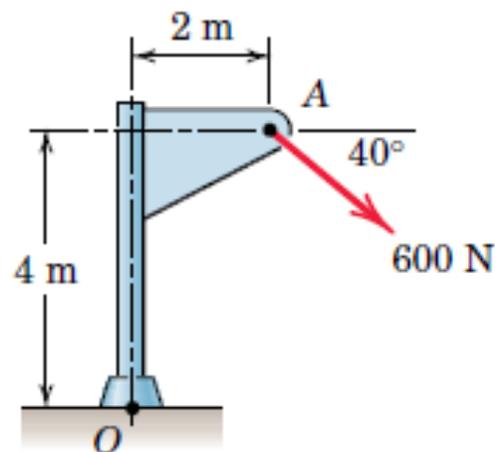|	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1ˢᵗ term – Lecture 3	

Sample problem 2/5

Calculate the magnitude of the moment about the base point O of the 600 N force.

Solution. *(I)* The moment arm to the 600-N force is

$$d = 4 \cos 40° + 2 \sin 40° = 4.35 \text{ m}$$

① By $M = Fd$ the moment is clockwise and has the magnitude

$$M_O = 600(4.35) = 2610 \text{ N} \cdot \text{m}$$

	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	
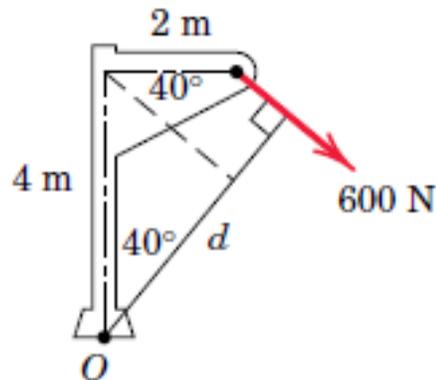
(II)   Replace the force by its rectangular components at A

$$F_1 = 600 \cos 40° = 460 \text{ N}, \qquad F_2 = 600 \sin 40° = 386 \text{ N}$$

By Varignon's theorem, the moment becomes

② $$M_O = 460(4) + 386(2) = 2610 \text{ N} \cdot \text{m}$$

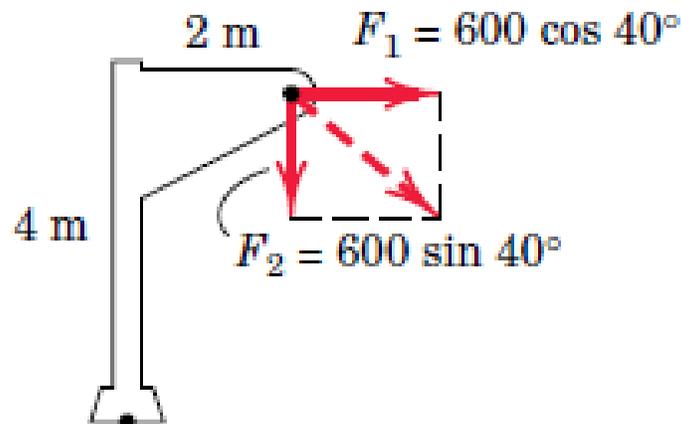|	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

(V) By the vector expression for a moment, and by using the coordinate system indicated on the figure together with the procedures for evaluating cross products, we have

④

$$M_O = \mathbf{r} \times \mathbf{F} = (2\mathbf{i} + 4\mathbf{j}) \times 600(\mathbf{i}\cos 40° - \mathbf{j}\sin 40°)$$

$$= -2610\mathbf{k} \text{ N·m}$$

The minus sign indicates that the vector is in the negative z-direction. The magnitude of the vector expression is

$$M_O = 2610 \text{ N·m} \qquad \textit{Ans.}$$

2/29 The 10-kN force is applied at point A. Determine the moment of **F** about point O. Determine the points on the x- and y-axes about which the moment of **F** is zero.

$$\textit{Ans. } M_O = 16 \text{ kN·m CW}$$
$$(x, y) = (2.67, 0) \text{ m and } (0, 2) \text{ m}$$
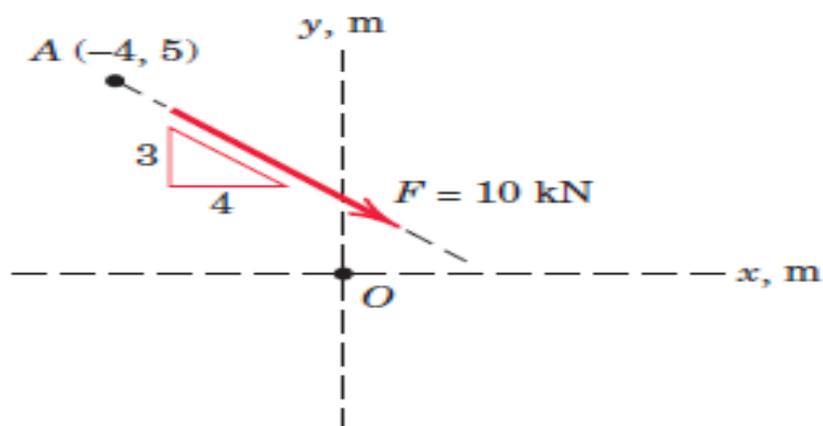
Problem 2/29

	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

2/29

$$F_x = F \cos \theta$$

$$F_x = 10 \left(\frac{4}{5} \right) = 8 \text{ KN}$$

$$|F_y| = F \sin \theta$$

$$= 10 \left(\frac{3}{5} \right) = 6 \text{ KN}$$
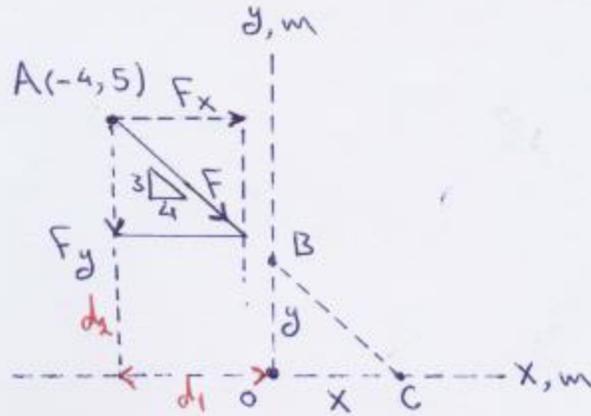
$$+\circlearrowleft \quad M_o = F_x (d_2) - F_y (d_1)$$

$$= 8(5) - 6(4) = 16 \text{ KN.m} \quad CW$$

$$\tan \theta = \frac{3}{4} = \frac{5}{4+x} \implies x = 2.67 \text{ m}$$

$$\tan \theta = \frac{y}{x} = \frac{3}{4}$$

$$y = \frac{3x}{4} = \frac{3(2.67)}{4} = 2 \text{ m}$$

∴ The intercept points are :

(2.67 , 0) m and (0, 2) m

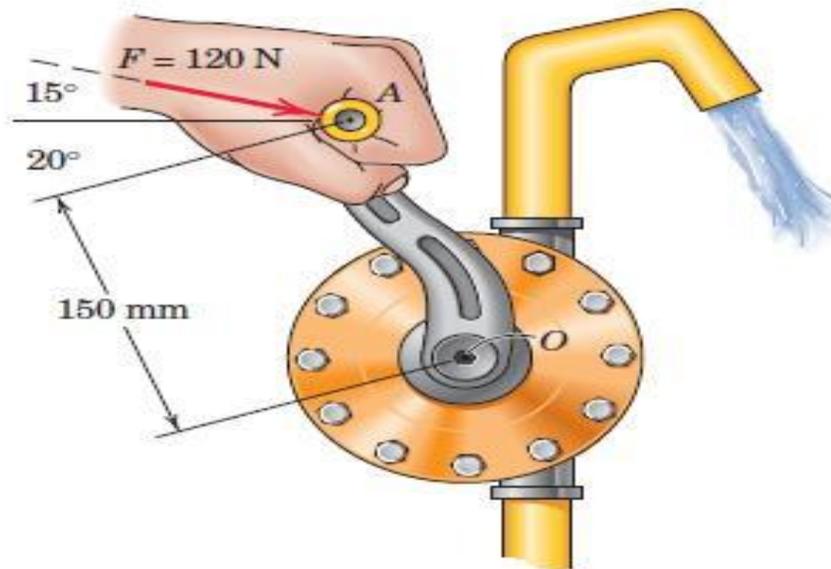|	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

2/33 In steadily turning the water pump, a person exerts the 120-N force on the handle as shown. Determine the moment of this force about point O.
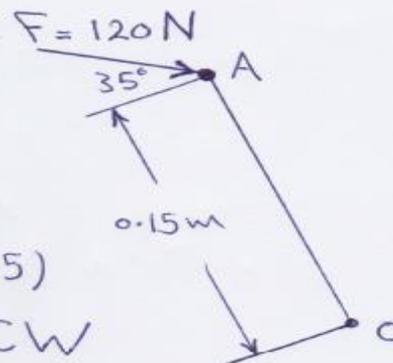
Ans. $M_O = 14.74$ N·m CW

Problem 2/33

2 / 33

$F = 120$ N

$35°$

A

0.15 m

O

$\curvearrowright_+ \; M_o = F_x \, (d)$

$= 120 \cos 35 \, (0.15)$

$= 14.74$ N.m CW

	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1ˢᵗ term – Lecture 3	

2/40 Elements of the lower arm are shown in the figure. The mass of the forearm is 2.3 kg with mass center at G. Determine the combined moment about the elbow pivot O of the weights of the forearm and the 3.6-kg homogeneous sphere. What must the biceps tension force be so that the overall moment about O is zero?

T

O — 50 mm

55°

G

150 mm

2.3(9.81) N

A

3.6(9.81) N

├── 325 mm ──┤
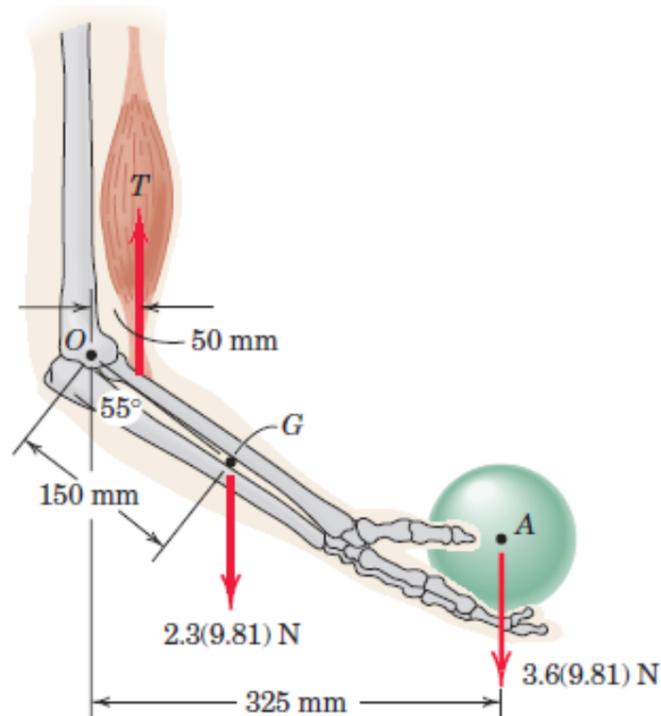
Problem 2/40

	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

$\underline{2/40}$

The combined moment about the elbow pivot O of the weights of the forearm and the 3.6 Kg sphere is:

$+\curvearrowright M_O = 2.3(9.81)(0.150 \sin 55) + 3.6(9.81)(0.325)$

$\qquad = 14.25$ N.m   CW

$\curvearrowright_+ \quad \Sigma M_O = 0 \quad \Rightarrow \quad -T(0.5) + 14.25 = 0$

$\therefore T = 28.5$ N

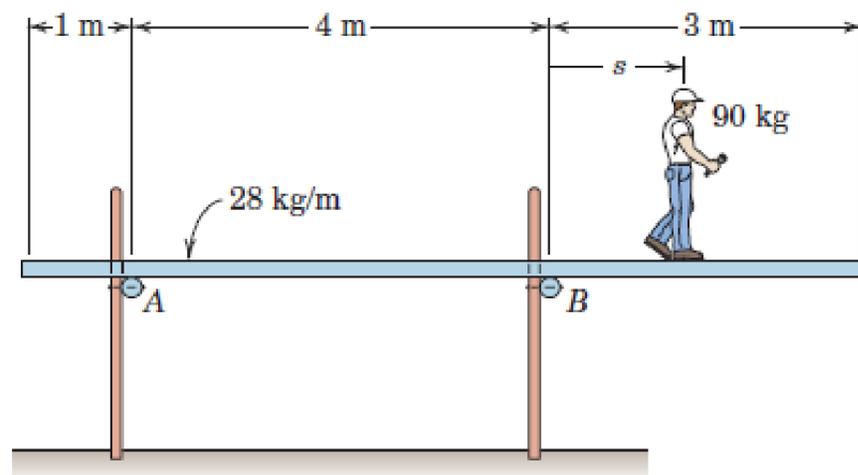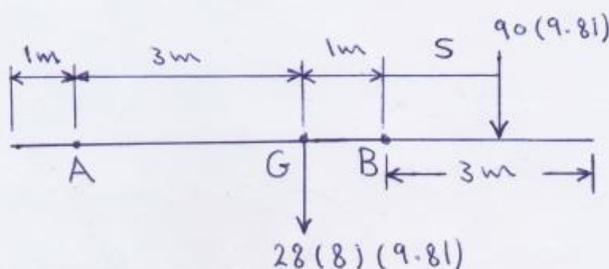|	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1ˢᵗ term – Lecture 3	

2/44 The uniform work platform, which has a mass per unit length of 28 kg/m, is simply supported by cross rods A and B. The 90-kg construction worker starts from point B and walks to the right. At what location s will the combined moment of the weights of the man and platform about point B be zero?

Problem 2/44

2/44

$$\circlearrowleft M_B = 28(8)(9.81)(1) - 90(9.81)S = 0$$

$$S = 2.49 \text{ m}$$

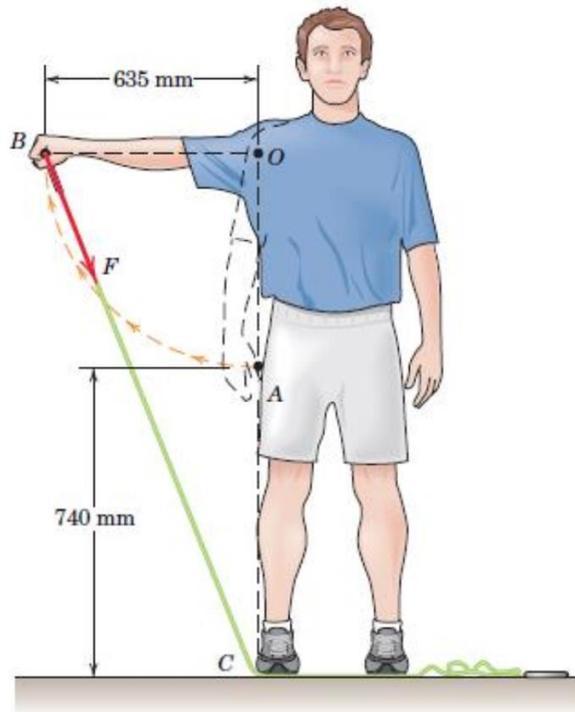|	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

2/49 An exerciser begins with his arm in the relaxed vertical position OA, at which the elastic band is unstretched. He then rotates his arm to the horizontal position OB. The elastic modulus of the band is $k =$ 60 N/m—that is, 60 N of force is required to stretch the band each additional meter of elongation. Determine the moment about O of the force which the band exerts on the hand B.

Ans. $M_O = 26.8$ N·m CCW

	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

$\underline{2/49}$

$\overline{OC} = 635 + 740 = 1375 \text{ mm}$

$F = KX$

$= 60\left(\sqrt{0.635^2 + 1.375^2} - 0.740\right)$

$= 46.5 \text{ N}$

$\theta = \tan^{-1}\dfrac{1.375}{0.635} = 65.2°$

$\circlearrowleft M_o = 46.5 \sin 65.2 \ (0.635)$

$\qquad = 26.8 \text{ N.m} \quad CCW$
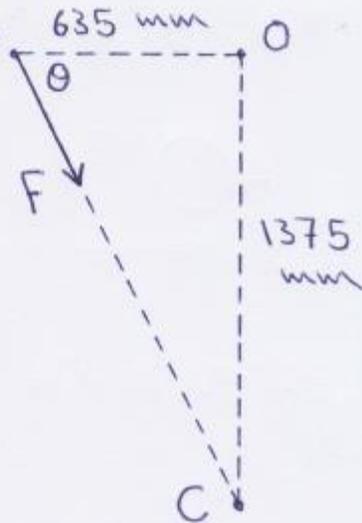
	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

2/50 (*a*) Calculate the moment of the 90-N force about point *O* for the condition $\theta = 15°$. Also, determine the value of θ for which the moment about *O* is (*b*) zero and (*c*) a maximum.

O

$F = 90$ N

800 mm

θ

A

600 mm

Problem 2/50

2/50

O

800 mm

90 N

θ F_y

F_x

a)

A

600 mm
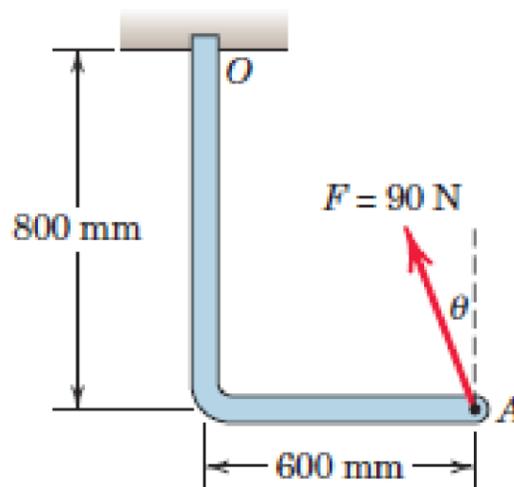
$\overset{+}{\curvearrowright} M_o = F_y \,(0.6) - F_x \,(0.8)$

$M_o = 90 \cos 15 \,(0.6) - 90 \sin 15 \,(0.8)$

$= 33.5 \text{ N.m}$

	Al-Mustaqbal University / College of Engineering	
	Prosthetics & Orthotics Eng. Department	
	First Class	
	Subject (**Physics**)	
	Code (**UOMU013015**)	
	Asst. Lec. Mariam Ghassan Al-maroof	
	1st term – Lecture 3	

O

800 mm

90N

θ

600 mm

b)

$$\theta = \tan^{-1}\left(\frac{600}{800}\right) = 36.9°$$

c)

$$\alpha = \tan^{-1}\frac{800}{600}$$

$$\alpha = 36.9°$$

$$\theta = 90 + \alpha$$

$$= 90 + 36.9$$

$$= 126.9$$

O

α

800 mm

θ

90 N

600 mm

A

Email (mariam.ghassan.ghaffar@uomus.edu.iq)