* Inverse transforms *

The symbol L^{-1} indicating the inverse trasforms

Ex1:-

a/ $L^{-1}\left[\frac{a}{s^2+a^2}\right] = \sin at$

b/ $L^{-1}\left[\frac{12}{s^2-9}\right] = L^{-1}\left[\frac{4*3}{s^2-3^2}\right] = 4\sinh 3t$

c/ $L^{-1}\left[\frac{s}{s^2+25}\right] = \cos 5t$

d/ $L^{-1}\left[\frac{3s+1}{s^2-s-6}\right]$ لايمكن حلها بأستخدام جدول الثوابت مباشر

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partial fraction:-

شروطه

① يجب ان يكون البسط درجة لقل في المقام واذا لم يكن نقسم أولاً

② نحلل المقام إلى اقواس .

Linear factor $(s+a)$ gives $\frac{A}{s+a}$

$(s+a)^2$ gives $\frac{A}{s+a} + \frac{B}{(s+a)^2}$

$(s+a)^3$ gives $\frac{A}{s+a} + \frac{B}{(s+a)^2} + \frac{C}{(s+a)^3}$

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Ex 2:- Determine $L^{-1}\left[\dfrac{5s+1}{s^2-s-12}\right]$

(1) $\dfrac{5s+1}{s^2-s-12} = \dfrac{5s+1}{(s-4)(s+3)}$ نقسم

$\Rightarrow \dfrac{A}{s-4} + \dfrac{B}{s+3} = \dfrac{5s+1}{(s-4)(s+3)}$] both side $*(s-4)(s+3)$

(2) $A(s+3) + B(s-4) = 5s+1$

(3) Let $(s+3)=0 \Rightarrow s=-3$

$\therefore A(0) + B(-3-4) = 5(-3)+1$

$\Rightarrow -7B = -14 \Rightarrow \boxed{B=2}$

(4) Let $(s-4)=0 \Rightarrow s=4$

$\therefore A(4+3) + 0 = (5*4)+1$

$7A = 21 \Rightarrow \boxed{A=3}$

(5) Sub. A, B in (1)

$\therefore \dfrac{5s+1}{s^2-s-12} = \dfrac{3}{s-4} + \dfrac{2}{s+3}$

$\therefore L^{-1}\left[\dfrac{5s+1}{s^2-s-12}\right] = 3e^{4t} + 2e^{-3t}$

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Ex3:- $L^{-1}\left[\dfrac{9s-8}{s^2-2s}\right]$

(1) $\dfrac{9s-8}{s^2-2s} = \dfrac{9s-8}{s(s-2)} = \dfrac{A}{s} + \dfrac{B}{s-2}\Bigg] * s(s-2)$

(2) $9s-8 = A(s-2) + Bs$

(3) let $(s-2)=0 \Rightarrow s=2$

$\therefore 9(2)-8 = A(0) + 2B$

$\boxed{B=5}$

(4) let $s=0$

$\therefore 0-8 = -2A + 0$

$\boxed{A=4}$

sub A, B in (1) $\Rightarrow \dfrac{4}{s} + \dfrac{5}{s-2}$

$\therefore L^{-1}\left[\dfrac{9s-8}{s^2-2s}\right] = L^{-1}\left[\dfrac{4}{s} + \dfrac{5}{s-2}\right]$

$\therefore f(t) = 4 + 5e^{2t}$

Ex3 / Express $F(s) = \frac{s^2 - 15s + 41}{(s+2)(s-3)^2}$ in partial

① $= \frac{A}{(s+2)} + \frac{B}{(s-3)} + \frac{C}{(s-3)^2}$] mult. by $(s+2)(s-3)^2$

$A(s-3)^2 + B(s-3)(s+2) + C(s+2)$
$= s^2 - 15s + 41$

② Let $(s-3) = 0 \Rightarrow s = 3$

$\therefore A(0) + B(0) + C(5) = 9 - 45 + 41$

$\Rightarrow \boxed{C = 1}$

③ let $(s+2) = 0 \Rightarrow s = -2$

$\therefore A(-2-3)^2 + B(0) + C(0) = 4 + 30 + 41$

$\Rightarrow \boxed{A = 3}$

④ Let $s = 0$

$9A - 6B + 2C = 41$ sub. A & C

$27 - 6B + 2 = 41$

$\Rightarrow \boxed{B = -2}$

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sub A, B, C in ①

$$L^{-1} \frac{3}{s+2} - \frac{2}{s-3} + \frac{1}{(s-3)^2}$$

$$L^{-1} \frac{3}{s+2} = 3e^{-2t}$$

$$L^{-1} \frac{2}{s-3} = 2e^{3t}$$

$$L^{-1} \frac{1}{(s-3)^2} \qquad \left(\because L^{-1} \frac{1}{s^2} = t\right)$$

$a = -3$

$\therefore$ we can use Theorem 2

if $L[f(t)] = F(s)$

then $L[e^{-at} f(t)] = F(s+a)$

$$\therefore F(s+a) = \frac{1}{(s-3)^2} = L[e^{-at} f(t)]$$

$$\therefore L^{-1}\left(\frac{1}{(s-3)^2}\right) = t e^{3t}$$

$$\therefore L^{-1}\left[\frac{s^2 - 15s + 41}{(s+2)(s-3)^2}\right] = 3e^{-2t} + 2e^{3t} + te^{3t}$$

The "cover up" rule

قاعدة التغطية :-

يتم استخدام هذه القاعدة بأخفاء احد العاملات في المقام

شروطها ان يكون المقام بعوامل خطية غير مكررة

Ex 4 : $F(s) = \dfrac{9s-8}{s(s-2)}$

$$= \frac{A}{s} + \frac{B}{s-2}$$

$A =$ cover up (s) in $f(s)$ $= \lim_{s\to 0} \left[\dfrac{9s-8}{s-2}\right] = 4$

$$\boxed{\therefore A = 4}$$

$B =$ coverup $(s-2)$ in $f(s)$ $\rightarrow (s-2)=0$ $= \lim_{s=2} \left[\dfrac{9s-8}{s}\right] = 5$

$$\therefore \boxed{B = 5}$$

$$\therefore \frac{9s-8}{s(s-2)} = \frac{4}{s} + \frac{5}{s-2}$$

$$L^{-1}\left[\frac{9s-8}{s(s-2)}\right] = 4t + e^{2t} = f(t)$$

Ex 5/

$$F(s) = \frac{s+17}{(s-1)(s+2)(s-3)}$$

$$= \frac{A}{s-1} + \frac{B}{s+2} + \frac{C}{s-3}$$

at A = cover up $(s-1)=0$

$$\therefore \lim_{s\to 1}\left[\frac{s+17}{(s+2)(s+3)}\right] = -\frac{18}{6}$$

$$\boxed{\therefore A = -3}$$

at B = cover up $(s+2)=0$

$$\therefore \lim_{s\to -2}\left[\frac{s+17}{(s-1)(s-3)}\right] = \frac{15}{15}$$

$$\boxed{\therefore B = 1}$$

at C = cover up $(s-3)=0$

$$\therefore \lim_{s\to 3}\left[\frac{s+17}{(s-1)(s+2)}\right] = \frac{20}{10}$$

$$\therefore \boxed{C = 2}$$

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$$\therefore F(s) = \frac{1}{s+2} + \frac{2}{s-3} - \frac{3}{s-1}$$

$$\therefore L^{-1}\left[\frac{s+17}{(s-1)(s+2)(s-3)}\right]$$

$$= e^{-2t} + 2e^{3t} - 3e^{t} = f(t)$$

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* Exercise *

① Express in partial fractions :-

a/ $\dfrac{22s+16}{(s+1)(s-2)(s+3)}$

b/ $\dfrac{s^2-11s+6}{(s+1)(s-2)^2}$

② Determine

a/ $L^{-1}\left[\dfrac{4s^2-17s-24}{s(s+3)(s-4)}\right]$

b/ $L^{-1}\left[\dfrac{5s^2-4s-7}{(s-3)(s^2+4)}\right]$

③ Find the inverse transforms :-

a/ $\dfrac{1}{2s-3}$ b/ $\dfrac{5}{(s-4)^3}$ c/ $\dfrac{3s+4}{s^2+9}$