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1. Linear Kinematics

Linear kinematics describes the **motion of segment centres of gravity** in terms of displacement, velocity, and acceleration.

Velocity

$$Velocity = \frac{Change\ in\ displacement}{Change\ in\ Time} \qquad (3.20)$$

Acceleration

$$Acceleration = \frac{Change\ in\ velocity}{Change\ in\ Time} \qquad (3.21)$$

These equations represent **finite difference numerical differentiation methods** used to derive motion variables from displacement data.

In gait analysis, displacement data are usually smoothed using **digital filtering techniques** to reduce measurement noise.

Example of Centre of Gravity Data

Table 3.6 is an example of the center of gravity data for one time frame for the Man. DST file in GaitLab.

Choosing the centre of gravity (CG) of one segment (the right foot) and one direction (Z) yields the curves seen in Figure 3.12. This figure shows how Equations 3.20 and 3.21 function. Study the position curve at the top of Figure 3.12 first.

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You will notice that at t = 0.0 s (heel strike), the foots CG is slightly above zero. It drops down almost to zero and then does not change much until about 0.6 s. This period is obviously when the foot is flat on the ground.

At heel strike, the velocity is slightly **negative** because the CG is still moving downward. During foot-flat (0.1-0.6 s), **the velocity is almost zero** because the **displacement is not changing** (see Equation 3.20). The acceleration is **quite large and positive** just after heel strike when the foot is experiencing an upward force from the ground. During the foot-flat phase, when the velocity is not changing much, the acceleration is almost zero (see Equation 3.21).

Important observations from the curves:

- Velocity becomes zero at **maximum or minimum points of displacement**.
- Acceleration becomes zero at **turning points of the velocity curve**.

Table 3.6 Three-Dimensional Displacement, Velocities and Acceleration Data of the Segment Centres of Gravity at Time = 0.00 s (Right Heel Strike) for a Normal Male

Segment	X	Y	Z	v_X	v_Y	v_Z	a_X	a_Y	a_Z
R. Thigh	-0.121	0.173	0.761	0.68	-0.12	0.08	8.2	-1.3	2.6
L. Thigh	-0.238	0.324	0.748	0.77	-0.09	-0.02	8.7	0.5	0.3
R. Calf	0.012	0.199	0.347	0.52	-0.05	0.00	4.0	0.6	1.4
L. Calf	-0.436	0.327	0.355	0.68	0.03	0.09	9.2	-1.4	1.5
R. Foot	0.115	0.217	0.074	0.19	0.03	-0.23	-2.1	1.0	-0.8
L. Foot	-0.579	0.316	0.097	0.32	0.06	0.26	6.0	0.7	3.3

Note. The XYZ positions refer to the global coordinate system defined in Figure 3.4. The velocities (in m/s) are v_X, v_Y and v_Z, while a_X, a_Y and a_Z are the accelerations (in m/s^2).

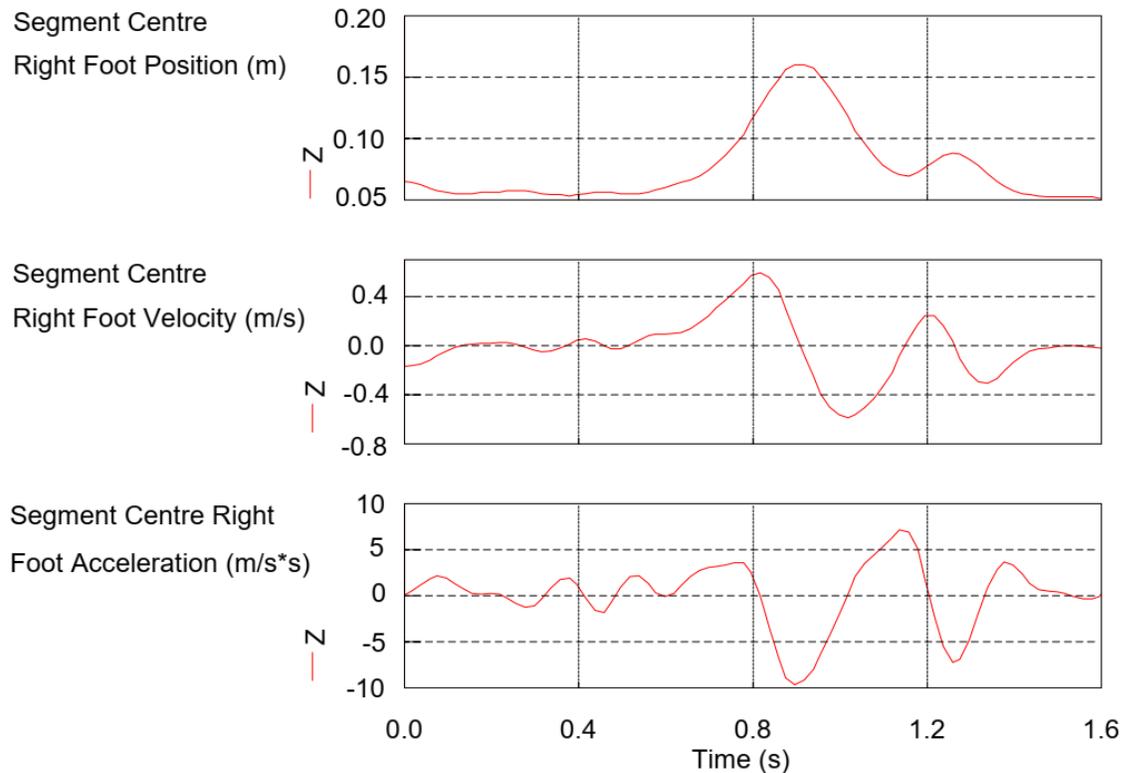|	Al-Mustaqbal University / College of Engineering	
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Normal adult male

Figure 3.12 The displacement, velocity and acceleration of the right foots center of gravity in the Z direction as a function of time

2. Angular Kinematics

Angular kinematics describes **segment orientation and rotational motion** in three-dimensional space.

Two main descriptions are used:

1. **Anatomical Joint Angles**
2. **Segment Euler Angles**

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Anatomical Joint Angles

Joint angles describe the **orientation of the distal segment relative to the proximal segment**.

Three principal rotations occur at joints:

1. Flexion / Extension

Rotation about the **mediolateral axis of the proximal segment** (i.e., the z axes in

Figure 3.10).

2. Internal / External Rotation

Rotation about the **longitudinal axis of the distal segment** (i.e., the x axes in Figure 3.10).

3. Abduction / Adduction

Rotation about a **floating axis perpendicular to the other two axes**

These angle definitions can be a little more easily understood by referring to Figure 3.13, which illustrates the left knee.

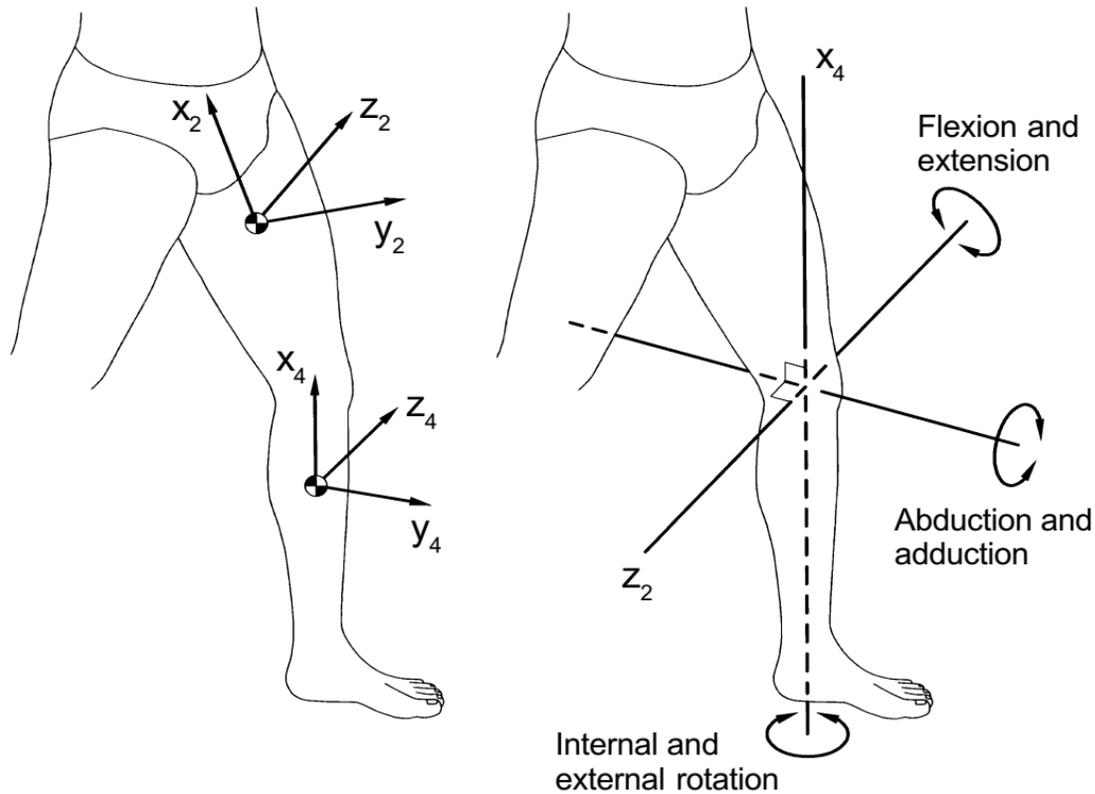|	Al-Mustaqbal University / College of Engineering	
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Figure 3.13 The axis of rotation for the left knee. The proximal and distal reference frames are shown on the left, while the axes are highlighted on the right There are three separate ranges of motion: Flexion and extension take place about the mediolateral axis of the left thigh (z2); internal and external ritation take place about the longitudinal axi of the left calf (x$_4$); and abduction and adduction take place about an axis that is perpendicular to both z$_2$ and x$_4$.Note that these three axes do not form a right-handed triad, because z$_2$ and x$_4$ are not necessarily at right angles to one another.

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The first three columns of Table 3.7 show the anatomical joint angles for t=0.00 s for the Man.DST file in GaitLab, whereas Figure 3.14 concentrates on the right hip joint. The mathematical details describing how the rotation axes of Figure 3.13 can be used to generate the data in Figure 3.14 and Table 3.7 are quite complex and are included in Appendix B.

Table 3.7 Angular Kinematics (Anatomical Joint Angles, Segment Angular Velocities and Accelerations) at Time = 0.00 s for a Normal Male

Joint/Segment	Flx/Ext	Abd/Add	Int/Ext	Omega_x	Omega_y	Omega_z	Omdot_x	Omdot_y	Omdot_z
R. Hip/Thigh	18.9	-5.0	-3.7	0.15	0.11	-0.08	9.3	6.5	-0.3
L. Hip/Thigh	-21.8	2.1	-13.3	0.61	-0.02	0.48	3.2	15.5	5.1
R. Knee/Calf	10.4	4.1	-1.7	-0.15	0.95	0.88	5.2	82.8	12.7
L. Knee/Calf	18.9	5.1	9.0	-0.19	0.60	-1.25	21.0	-9.4	-20.3
R. Ankle/Foot	-7.6	-6.8	-16.8	0.03	0.40	2.57	-0.7	3.1	-1.9
L. Ankle/Foot	-9.7	-8.3	-19.0	-1.80	-1.18	2.16	7.3	-16.6	38.4

Note. The first three columns are the anatomical joint angles. Refer to the text and Figure 3.13 for their definitions. The middle three columns, the segment angular velocities, and the last three columns, the segment angular accelerations, are kinematic quantities derived from the segment Euler angles (refer to text and Figure 3.16) and are relative to the segment reference frames (Figure 3.10).

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Normal adult male

Figure 3.14 The orientation of the right thigh relative to the pelvis, that is, the right hip joint angles plotted, as a function of time. Top is flexion (positive) and extension (negative), middle is internal (+ve) and external rotation (-ve), and bottom is abduction (+ve) and adduction (-ve).
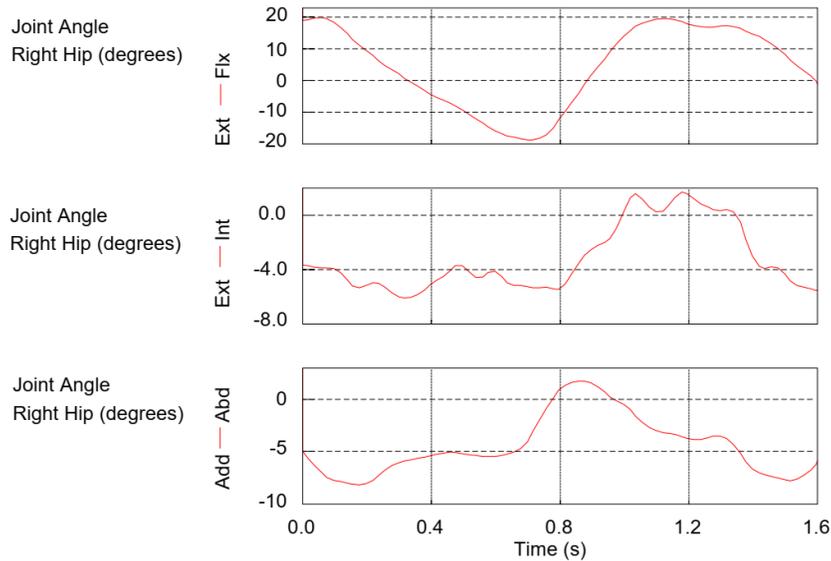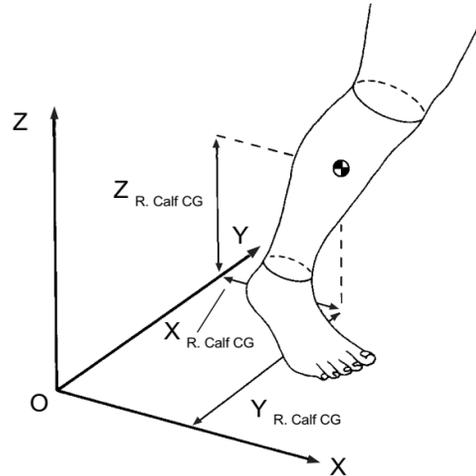
Joint Angle
Right Hip (degrees)

Joint Angle
Right Hip (degrees)

Joint Angle
Right Hip (degrees)

Figure 3.15 The three linear degrees of freedom ($X_{R.\ Calf}$, $Y_{R.\ Calf}$ and $Z_{R.\ Calf}$) defining the position of the right calf's centre of gravity in terms of the global reference system XYZ

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3. Segment Euler Angles

Segment Euler angles describe the **orientation of a body segment relative to the global reference frame**.

A rigid body in three-dimensional space has **six degrees of freedom**:

- Three translational coordinates (X, Y, Z)
- Three rotational coordinates (Euler angles)

Figure reference

- **Figure 3.15**: Linear degrees of freedom defining the CG position of a segment.
- **Figure 3.16**: Euler angle rotations defining segment orientation.

The Euler rotation sequence is:

1. Rotation ϕ about the **global Z axis**
2. Rotation θ about the **line of nodes**
3. Rotation ψ about the **segment z-axis**

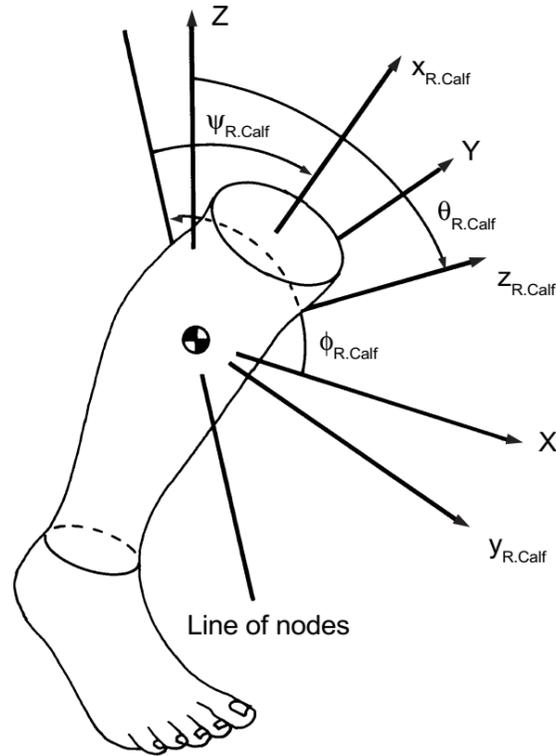|	Al-Mustaqbal University / College of Engineering	
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Figure 3.16 The three angular degrees of freedom (or Euler angles $\phi_{R.Calf}$, $\theta_{R.Calf}$, and $\psi_{R.Calf}$) defining the orientation of the right calf's reference axes ($x_{R.Calf}$, $y_{R.Calf}$, and $z_{R.Calf}$) relative to the global reference system XYZ. Note that the calf's CG has been moved to coincide with the origin of XYZ. The three Euler angle rotations take place in the following order: (a) $\phi_{R.Calf}$ about the Z axis; (b) $\theta_{R.Calf}$ about the line of nodes; and (c) $\psi_{R.Calf}$ about the $z_{R.Calf}$ axis. The line of nodes is perpendicular to both Z and $z_{R.Calf}$.

4. Angular Velocity and Angular Acceleration

Once Euler angles are determined, angular motion variables can be calculated.

Angular velocity

$$\text{Angular velocity} = \text{Function [Euler angles and their first derivatives]} \quad (3.22)$$

$$\text{Angular acceleration} = \text{Function [Euler angles and their first and second derivatives]} \quad (3.23)$$

Key characteristics:

- Angular velocity symbol: **ω**
- Angular acceleration symbol: **ώ**

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- Units:
  - rad/s
  - rad/s²

Angular velocity is calculated as the change in angular position, or the angular displacement, that occurs during a given period of time:

$$\text{angular velocity} = \frac{\text{change in angular position}}{\text{change in time}}$$

$$\omega = \frac{\Delta \text{ angular position}}{\Delta \text{ time}}$$

$$\text{angular velocity} = \frac{\text{angular displacement}}{\text{change in time}}$$

$$\omega = \frac{\theta}{\Delta t}$$

Another way to express change in angular position is angular position2 angular position1, in which angular position1 represents the body's position at one point in time and angular position2 represents the body's position at a later point:

$$\omega = \frac{\text{angular position}_2 - \text{angular position}_1}{\text{time}_2 - \text{time}_1}$$

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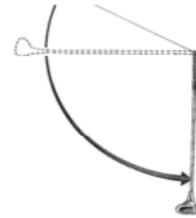## SAMPLE PROBLEM II.I

A golf club is swung with an average angular acceleration of 1.5 rad/s^2.
What is the angular velocity of the club when it strikes the ball at the end
of a 0.8 s swing? (Provide an answer in both radian and degree-based units.)

Known

$$\alpha = 1.5 \text{ rad/s}^2$$
$$t = 0.8 \text{ s}$$

Solution

The formula to be used is the equation relating angular acceleration, angular velocity, and time:

$$\alpha = \frac{\omega_2 - \omega_1}{t}$$

Substituting in the known quantities yields the following:

$$1.5 \text{ rad/s}^2 = \frac{\omega_2 - \omega_1}{0.8 \text{ s}}$$

It may also be deduced that the angular velocity of the club at the beginning of the swing was zero:

$$1.5 \text{ rad/s}^2 = \frac{\omega_2 - 0}{0.8 \text{ s}}$$

$$(1.5 \text{ rad/s}^2)(0.8 \text{ s}) = \omega_2 - 0$$

$$\omega_2 = 1.2 \text{ rad/s}$$

In degree-based units:

$$\omega_2 = (1.2 \text{ rad/s})(57.3 \text{ deg/rad})$$

$$\omega_2 = 68.8 \text{ deg/s}$$

5. Ground Reaction Forces (GRF)

Three-dimensional gait analysis requires a **force plate** to measure the interaction between the foot and the ground.

A force plate records six parameters:

1. Force in X direction, F_X
2. Force in Y direction, F_Y
3. Force in Z direction, F_Z
4. Position of resultant force in X direction, D_X

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5. Position of resultant force in Y direction, D_Y

6. Torque about Z axis, T_Z

Figure 3.18a depicts a foot on the plate, whereas Figure 3.18b shows the previously listed six pieces of information associated with the contact between the foot and plate.

Figure 3.18 The force plate used to measure the reaction forces of the ground acting on a subject's foot: (a) view of foot and plate showing XYZ global reference frame; (b) the resultant force $\mathbf{F}_R$ of the plate on the foot has three orthogonal components– F_X, F_Y, and F_Z. The position of this resultant force is specified by the coordinates DX and DY, and $\mathbf{T}_Z$ is the torque applied to the foot about the vertical Z axis.
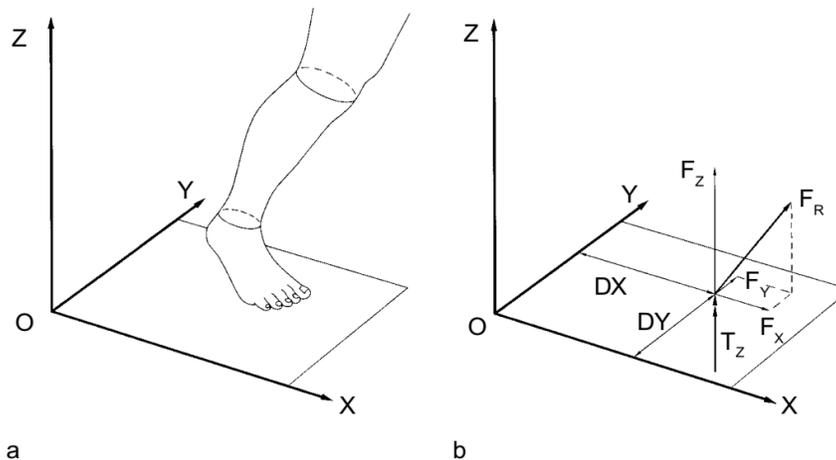
Table 3.8 The Three-Dimensionl Force Plate Information, as Defined in Figure 3.18b, Showing the Data for Plates 1 and 2 Between 0.68 and 0.72 s

Time = 0.68 s Number	F_X(N)	F_Y(N)	F_Z(N)	DX(m)	DY(m)	T_Z(Nm)
Plate 1	108	-1	645	0.196	0.204	2.774
Plate 2	0	0	0	0.000	0.000	0.000

Time = 0.70 s Number	F_X(N)	F_Y(N)	F_Z(N)	DX(m)	DY(m)	T_Z(Nm)
Plate 1	117	-4	617	0.198	0.205	2.546
Plate 2	-15	30	271	0.672	0.272	0.609

Time = 0.72 s Number	F_X(N)	F_Y(N)	F_Z(N)	DX(m)	DY(m)	T_Z(Nm)
Plate 1	120	-2	538	0.199	0.206	1.417
Plate 2	-79	30	352	0.677	0.274	0.159

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Table 3.8 shows the ground reaction force information for both plates between 0.68 and 0.72 s for the Man.DST file in GaitLab.

Note that at 0.68 s, all the values for Plate 2 are zero. This simply means that the subjects left foot has not yet made contact with this plate. Note too that the DX value for Plate 2 is about 0.47 m greater than that for Plate 1. This simply reflects the fact that Plate 2 is that much further from the origin of the global reference frame (refer to Figure 3.4). Note also that FZ in Plate 1 is decreasing (with increasing time) as the subjects right foot approaches toe-off.

The ground reaction force data may be plotted as a function of time as seen in Figure 3.19. There are a few observations that need to be made when studying the FX, F Y, and FZ curves in this figure. The range (in newtons) for the vertical force F Z is almost three times that of the fore-aft force FX. The mediolateral force F Y has a range of less than one tenth of FZ. The subjects weight is a little over 600N, so FZ, which has the characteristic double hump, exceeds body weight two different times during the stance phase.

As can be seen from all three curves, the stance phase lasts a little over 0.75 s. It should be pointed out that the male subject in this example was walking slightly slower than his normal pace (stance time is normally a little over 0.6 s). The fore-aft force FX is negative (i.e., acting in a backward or aft direction) for the first half of the cycle. Thereafter it becomes positive as the subject pushes off, driving backward on the plate, and, in accordance with Newtons third law of motion, experiencing a forward force. The mediolateral force F Y is negative for the first 0.5 s, indicating that just after heel strike, the foot was acting on the plate in the positive Y direction (refer to Figure 3.18, and remember that all these forces F X, FY, FZ reflect the force of the plate on the subjects foot and not vice versa). During most of the stance phase FY is positive, which means that the plate is pushing inward (or in a medial direction) on the subjects foot.

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Calculation of Joint Forces and Moments

Now that we have the ground reaction force data, we can go on to calculate the resultant forces and moments acting at the joints of the subjects lower extremities ankles, knees, and hips. First, you need to understand the concept of a Free Body Diagram (FBD). An FBD is a diagram in which the segment or body or interest (say, the right foot) is removed (or set free) from its environment and the external forces and moments acting on it are drawn in. Figure 3.20 shows an FBD for the right foot.
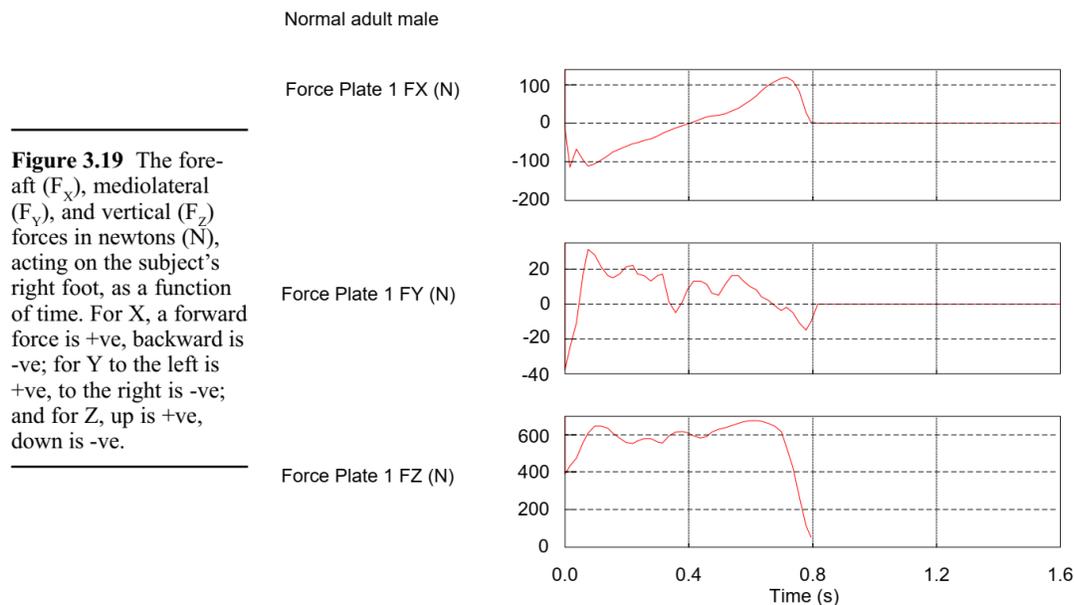
Figure 3.19 The fore-aft (F_X), mediolateral (F_Y), and vertical (F_Z) forces in newtons (N), acting on the subject's right foot, as a function of time. For X, a forward force is +ve, backward is -ve; for Y to the left is +ve, to the right is -ve; and for Z, up is +ve, down is -ve.

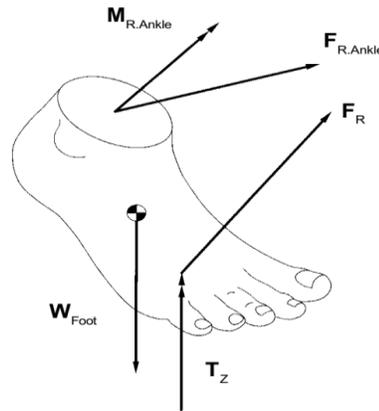|	Al-Mustaqbal University / College of Engineering	
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Figure 3.20 Free Body Diagram for the right foot during pushoff. The *external* forces acting on the foot are its weight $\mathbf{W}_F$, the resultant ground reaction $\mathbf{F}_R$, and the force of the calf on the foot at the ankle joint $\mathbf{F}_{R.Ankle}$. The external moments acting on the foot are the ground reaction torque about the Z axis $\mathbf{T}_Z$, and the moment of the calf on the foot at the ankle joint $\mathbf{M}_{R.Ankle}$. Note that the torque and moment are indicated as vectors with double arrow points. Compare this FBD for the right foot with Figure 3.18 and make sure you understand the similarities and differences

Joint moments are calculated using the **rotational form of Newton's second law**.

$$\Sigma M = \frac{d}{dt} \text{ (Angular Momentum)}$$

The moments acting on a segment include:

- The **joint moment**
- The **moment produced by the ground reaction force**
- Moments generated by other external forces

Moments created by forces are calculated as:

$$Moment = Force \times Lever\ Arm$$

Where:

- **Force** is the applied force
- **Lever arm** is the perpendicular distance from the line of action of the force to the center of rotation.

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Inverse Dynamics Procedure

The calculation of joint forces and moments is performed using **Inverse Dynamics**.

In this method, calculations proceed **from the distal segment to the proximal segment**.

The procedure is:

1. Analyze the **foot** to determine the ankle joint force and moment.
2. Use these results to analyze the **shank (calf)** and determine the knee joint force and moment.
3. Continue with the **thigh** to determine the hip joint force and moment.

Thus, joint kinetics are obtained sequentially along the lower limb.

Representation of Joint Forces

Joint forces are **three-dimensional vectors**, therefore they are resolved into anatomical components.

Force Components

- **Mediolateral force**
- **Anterior–posterior force**
- **Proximal–distal force**

These components describe the direction and magnitude of the forces acting across the joint.

Representation of Joint Moments

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Joint moments are also expressed relative to anatomical axes.

Moment Components

- **Flexion–Extension moment**
- **Abduction–Adduction moment**
- **Internal–External rotation moment**

Expressing moments in anatomical terms allows direct interpretation of joint function during movement.

Interpretation of Joint Moments in Gait

Analysis of joint moments during walking shows that:

- The **flexion–extension moment** is typically the largest component.
- **Abduction–adduction moments** may also be significant in certain phases of gait.
- Human walking is fundamentally a **three-dimensional movement**, not purely planar.

Limitations of Joint Force Calculations

The calculated joint forces represent the **resultant force acting across the joint**.

This resultant includes contributions from:

- Muscle forces
- Ligament forces
- Bone contact forces

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However, it is **not possible to determine individual muscle forces directly** from these equations because the number of unknown forces exceeds the number of available equations.

To estimate muscle forces, additional methods may be used, such as:

- **Optimization techniques**
- **Electromyography (EMG)**