



Equilibrium

- When a body is in equilibrium, the resultant of *all* forces acting on it is zero. Thus, the resultant force **R** and the resultant couple **M** are both zero, and we have the equilibrium equations

$$\mathbf{R} = \Sigma \mathbf{F} = \mathbf{0} \quad \mathbf{M} = \Sigma \mathbf{M} = \mathbf{0}$$

Free-body diagram and the mechanical effects

The name of the body	The effect of the body	Free-body diagram
Earth		
Flexible cables And ropes		
Cantilever beam		
Smooth surface		
Rollers, balls cylinders		
Smooth pins		



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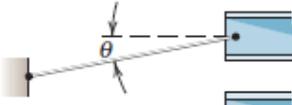
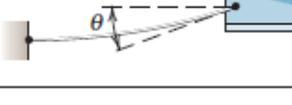
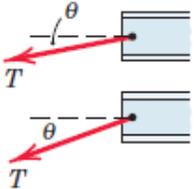
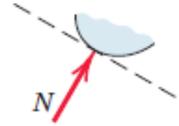
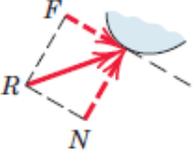
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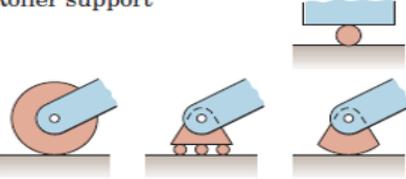
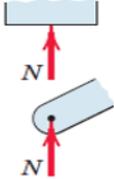
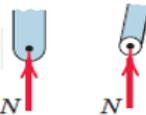
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Modeling the action of force

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
<p>1. Flexible cable, belt, chain, or rope</p> <p>Weight of cable negligible </p> <p>Weight of cable not negligible </p>	 <p>Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.</p>
<p>2. Smooth surfaces</p> 	 <p>Contact force is compressive and is normal to the surface.</p>
<p>3. Rough surfaces</p> 	 <p>Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant</p>

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
<p>4. Roller support</p> 	 <p>Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.</p>
<p>5. Freely sliding guide</p> 	 <p>Collar or slider free to move along smooth guides; can support force normal to guide only.</p>



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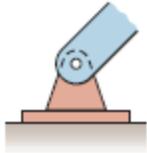
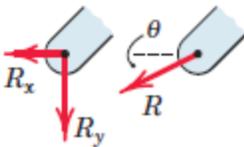
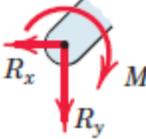
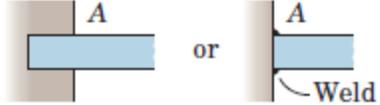
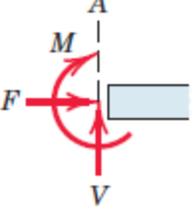
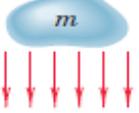
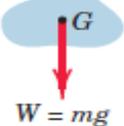
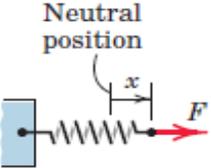
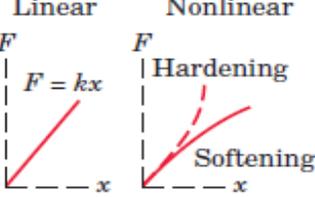
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MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (cont.)	
Type of Contact and Force Origin	Action on Body to Be Isolated
<p>6. Pin connection</p> 	<p>Pin free to turn A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ.</p>  <p>Pin not free to turn components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.</p> 
<p>7. Built-in or fixed support</p> 	<p>A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.</p> 
<p>8. Gravitational attraction</p> 	<p>The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.</p> 
<p>9. Spring action</p> <p>Neutral position</p>  <p>Linear $F = kx$</p> <p>Nonlinear</p> <p>Hardening</p> <p>Softening</p> 	<p>Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.</p> 



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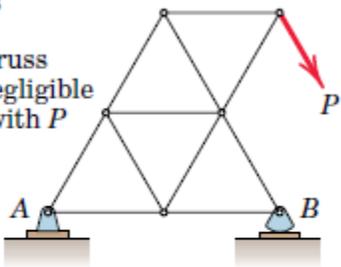
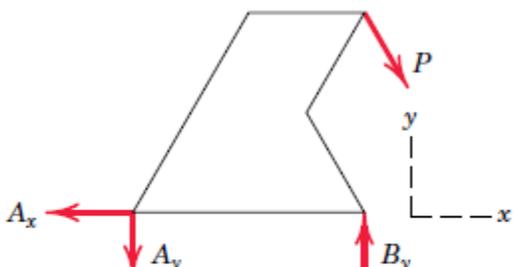
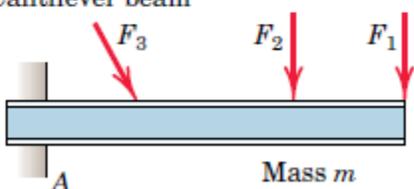
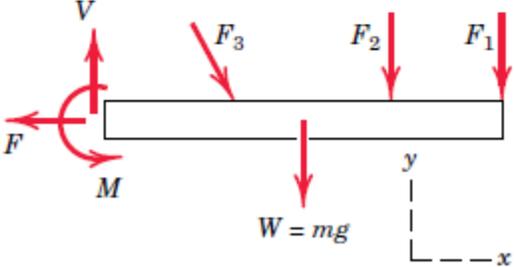


Construction of free- body diagrams

The free-body diagram shows all forces applied to the system by mechanical contact with other bodies.

- **Step 1.** Decide which system to isolate.
- **Step 2.** Next isolate the chosen system by drawing a diagram which represents its *complete external boundary*.
- **Step 3.** Identify all forces which act *on* the isolated system.
- **Step 4.** Show the choice of coordinate axes directly on the diagram.

Example of free-body diagrams

SAMPLE FREE-BODY DIAGRAMS	
Mechanical System	Free-Body Diagram of Isolated Body
<p>1. Plane truss</p> <p>Weight of truss assumed negligible compared with P</p> 	
<p>2. Cantilever beam</p>  <p>Mass m</p>	



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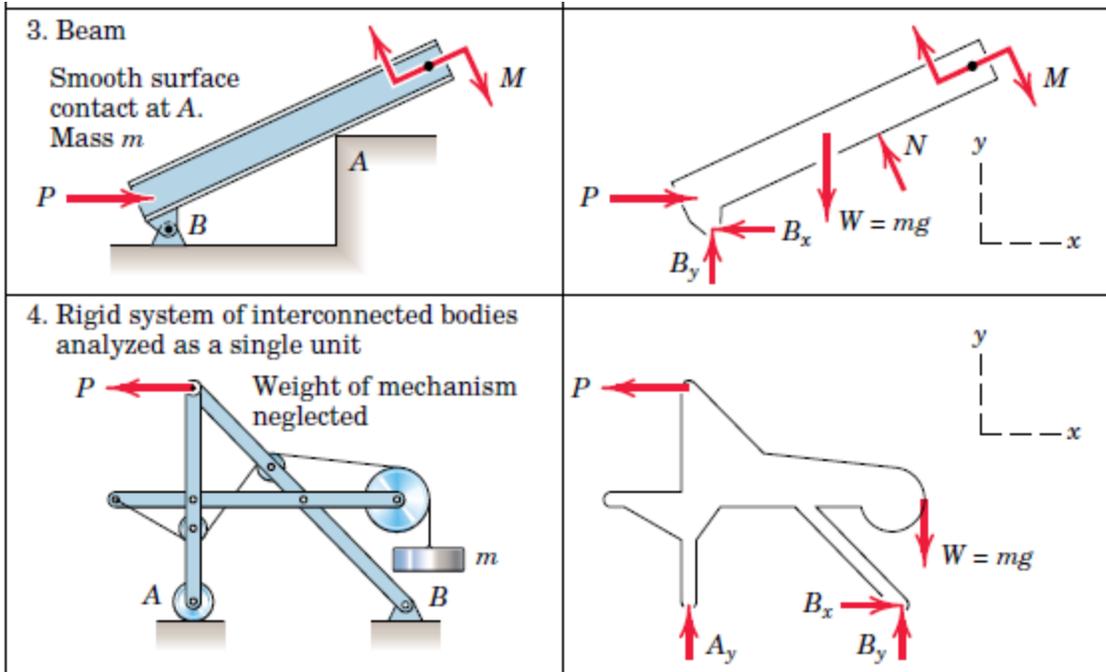
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Equilibrium conditions

- A body is in equilibrium if all forces and moments applied to it are in balance.

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M_O = 0$$



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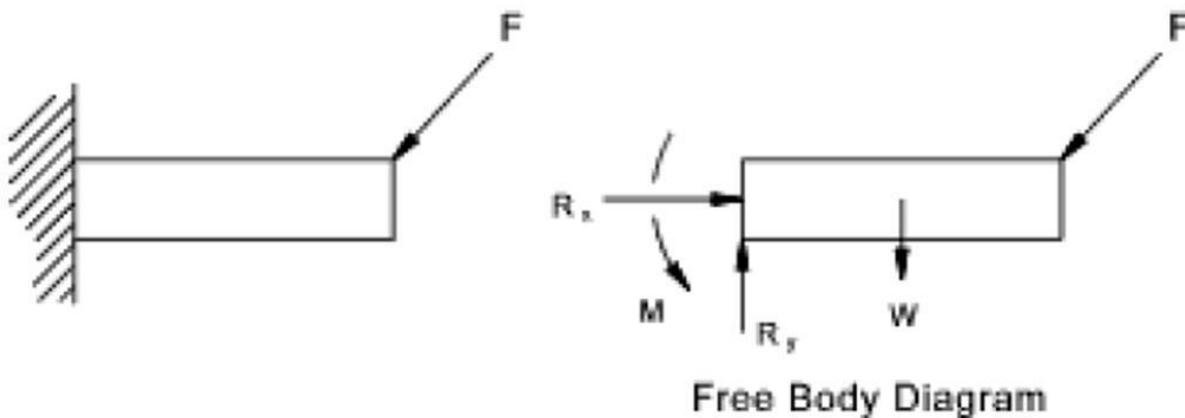
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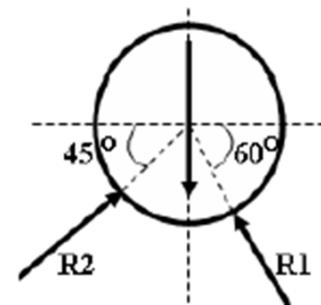
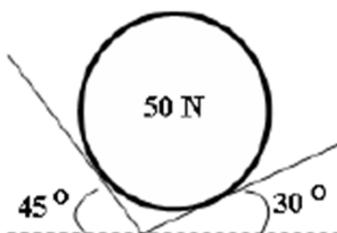
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Free body diagram : is a sketch to show all the forces and reactions acting on the body For example: The free body diagram includes external forces applied to the body and external reaction forces resulting from the method of supporting the body



Ex1: Draw Free – body diagram for the 50 N sphere shown in fig.





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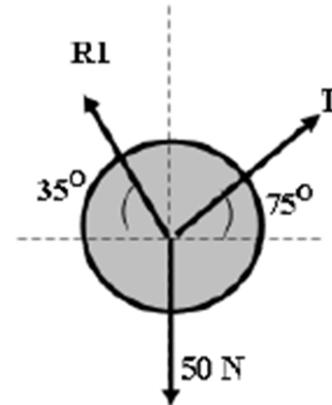
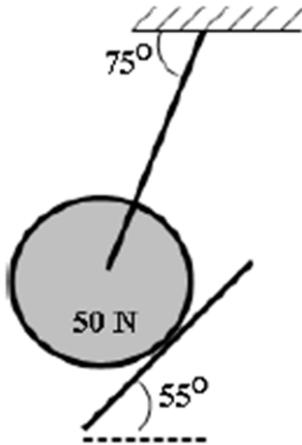
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Ex2: Draw Free – body diagram for the 50 N sphere shown in fig.



$$\rightarrow \sum F_x = 0$$

$$T \cos(75) - R_1 \cos(35) = 0$$

$$T \cos(75) = R_1 \cos(35)$$

$$T = \frac{\cos(35)}{\cos(75)} R_1 = 3.165 R_1 \quad \dots\dots(1)$$

$$+ \uparrow \sum F_y = 0$$

$$T \sin(75) + R_1 \sin(35) - 50 = 0$$

$$T \sin(75) + R_1 \sin(35) = 50 \quad \dots\dots(2)$$

Sub. Eq.(1) into Eq.(2)

$$(3.165 R_1) \sin(75) + R_1 \sin(35) = 50$$

$$3.63 R_1 = 50$$

$$R_1 = 13.774 N$$

Sub. R_1 into Eq.(1)

$$T = 3.165(13.774) = 43.6 N$$



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Examples

Mass at mid-point on beam (length L)



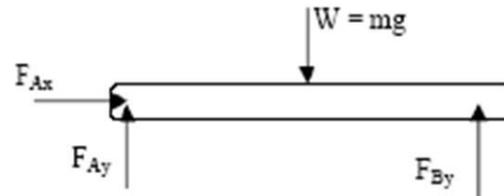
x-component forces

$$F_{Ax} = 0$$

y-component forces

$$F_{Ay} + F_{By} - W = 0$$

Free body diagram



moments about mid-point (or use A or B)

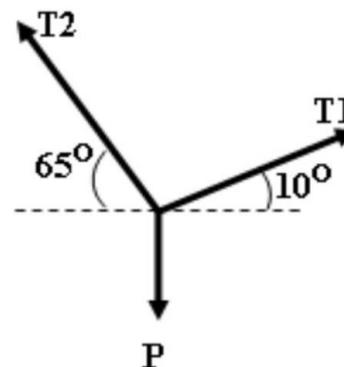
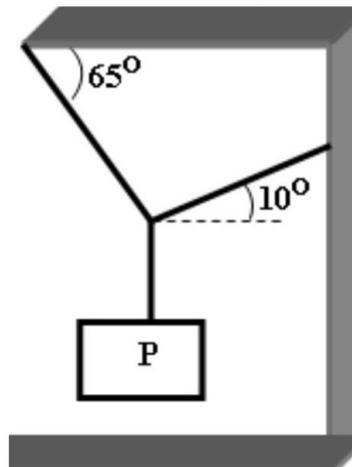
$$-\frac{1}{2}L F_{Ay} + \frac{1}{2}L F_{By} = 0$$

Final result

$$F_{Ax} = 0,$$

$$F_{Ay} = F_{By} = \frac{1}{2} W = \frac{1}{2} mg$$

Example: Draw Free body diagram for the ropes system shown in fig





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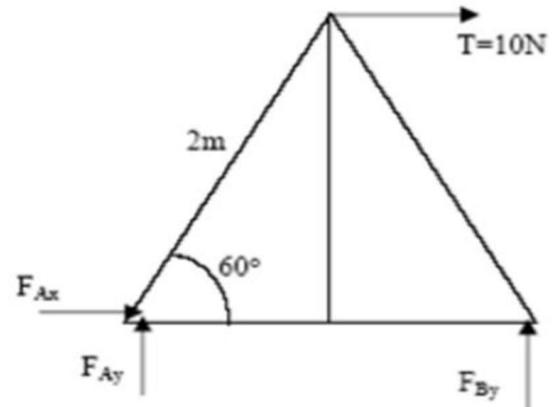
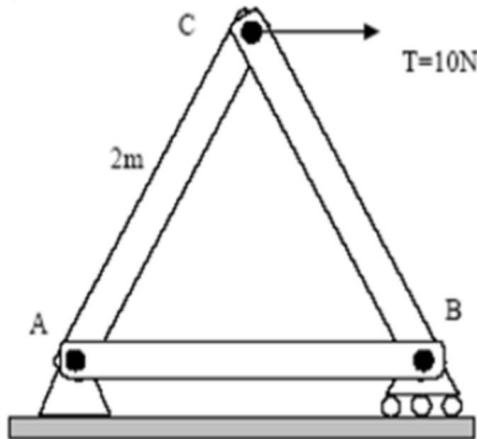
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Example : Draw Free body diagram for simple structure with a cable shown in fig



Two-and three-force members

- for a *two-force member* to be in equilibrium, the forces must be equal, opposite, and collinear.



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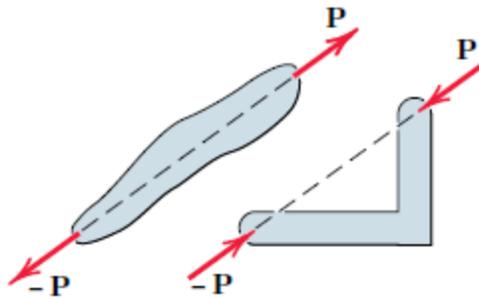
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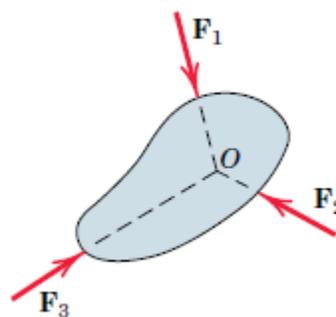
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Two-force members

- In a *three-force member*, which is a body under the action of three forces, equilibrium requires the lines of action of the three forces to be *concurrent*.
- The only exception occurs when the three forces are parallel. In this case we may consider the point of concurrency to be at infinity.



(a) Three-force member

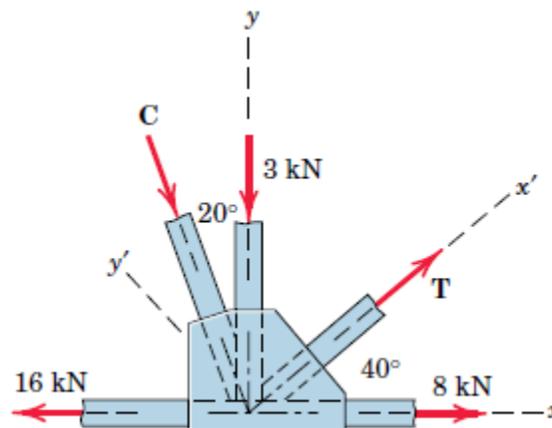


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Sample Problem 3/1

Determine the magnitudes of the forces C and T , which, along with the other three forces shown, act on the bridge-truss joint.



Solution. The given sketch constitutes the free-body diagram of the isolated section of the joint in question and shows the five forces which are in equilibrium.

Solution I (scalar algebra). For the x - y axes as shown we have

$$[\Sigma F_x = 0] \quad 8 + T \cos 40^\circ + C \sin 20^\circ - 16 = 0$$
$$0.766T + 0.342C = 8 \quad (a)$$

$$[\Sigma F_y = 0] \quad T \sin 40^\circ - C \cos 20^\circ - 3 = 0$$
$$0.643T - 0.940C = 3 \quad (b)$$

Simultaneous solution of Eqs. (a) and (b) produces

$$T = 9.09 \text{ kN} \quad C = 3.03 \text{ kN} \quad \text{Ans.}$$



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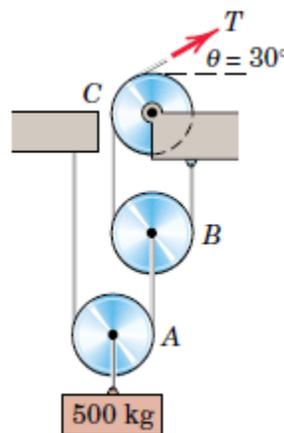
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Sample Problem 3/2

Calculate the tension T in the cable which supports the 500-kg mass with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C .



Solution. The free-body diagram of each pulley is drawn in its relative position to the others. We begin with pulley A , which includes the only known force. With the unspecified pulley radius designated by r , the equilibrium of moments about its center O and the equilibrium of forces in the vertical direction require

$$\textcircled{1} [\Sigma M_O = 0] \quad T_1 r - T_2 r = 0 \quad T_1 = T_2$$

$$[\Sigma F_y = 0] \quad T_1 + T_2 - 500(9.81) = 0 \quad 2T_1 = 500(9.81) \quad T_1 = T_2 = 2450 \text{ N}$$

From the example of pulley A we may write the equilibrium of forces on pulley B by inspection as

$$T_3 = T_4 = T_2/2 = 1226 \text{ N}$$



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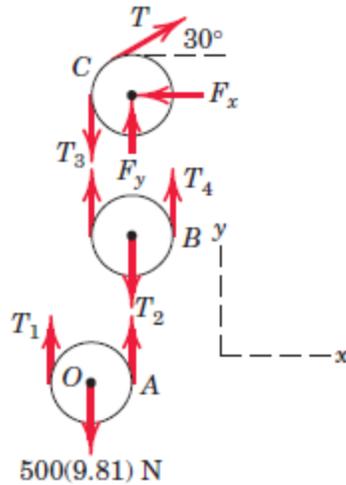
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For pulley C the angle $\theta = 30^\circ$ in no way affects the moment of T about the center of the pulley, so that moment equilibrium requires

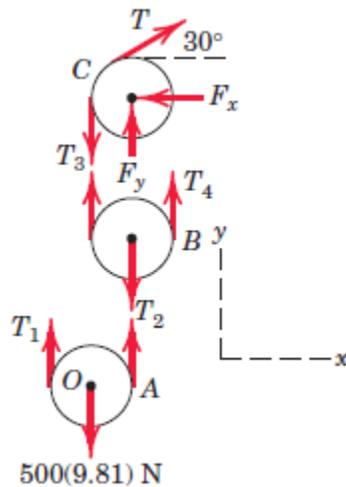
$$T = T_3 \quad \text{or} \quad T = 1226 \text{ N} \quad \text{Ans.}$$

Equilibrium of the pulley in the x - and y -directions requires

$$[\Sigma F_x = 0] \quad 1226 \cos 30^\circ - F_x = 0 \quad F_x = 1062 \text{ N}$$

$$[\Sigma F_y = 0] \quad F_y + 1226 \sin 30^\circ - 1226 = 0 \quad F_y = 613 \text{ N}$$

$$[F = \sqrt{F_x^2 + F_y^2}] \quad F = \sqrt{(1062)^2 + (613)^2} = 1226 \text{ N} \quad \text{Ans.}$$





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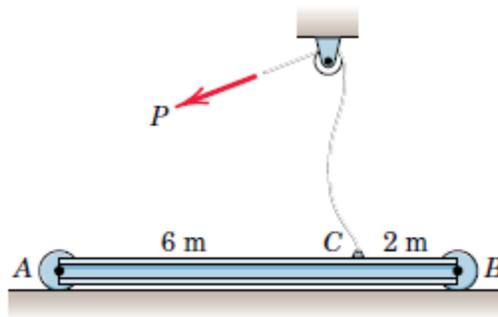
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Sample Problem 3/3

The uniform 100-kg I-beam is supported initially by its end rollers on the horizontal surface at A and B . By means of the cable at C it is desired to elevate end B to a position 3 m above end A . Determine the required tension P , the reaction at A , and the angle θ made by the beam with the horizontal in the elevated position.



Solution. In constructing the free-body diagram, we note that the reaction on the roller at A and the weight are vertical forces. Consequently, in the absence of other horizontal forces, P must also be vertical. From Sample Problem 3/2 we see immediately that the tension P in the cable equals the tension P applied to the beam at C .

Moment equilibrium about A eliminates force R and gives

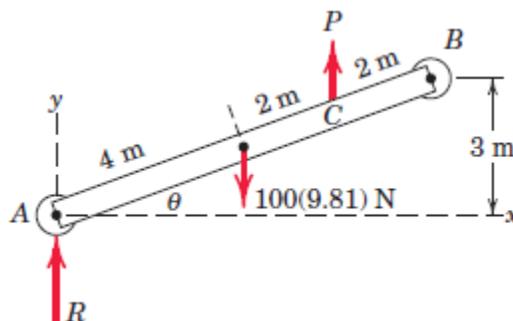
$$\textcircled{1} \quad [\Sigma M_A = 0] \quad P(6 \cos \theta) - 981(4 \cos \theta) = 0 \quad P = 654 \text{ N} \quad \text{Ans.}$$

Equilibrium of vertical forces requires

$$[\Sigma F_y = 0] \quad 654 + R - 981 = 0 \quad R = 327 \text{ N} \quad \text{Ans.}$$

The angle θ depends only on the specified geometry and is

$$\sin \theta = 3/8 \quad \theta = 22.0^\circ \quad \text{Ans.}$$





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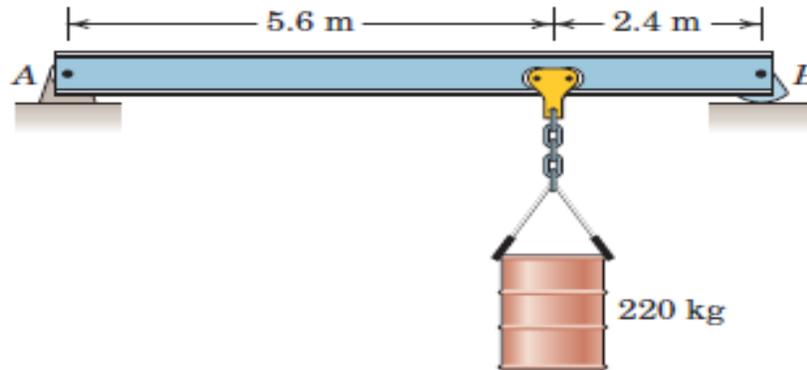
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3/6 The 450-kg uniform I-beam supports the load shown. Determine the reactions at the supports.



Problem 3/6

3/6

$\sum F_x = 0 \Rightarrow A_x = 0$

$\sum M_A = 0 \Rightarrow -450(9.81)4 - 220(9.81)(5.6) + B_y(8) = 0$

$\therefore B_y = 3720 \text{ N}$

$\sum F_y = 0 \Rightarrow A_y - 450(9.81) - 220(9.81) + 3720 = 0$

$\therefore A_y = 2850 \text{ N}$



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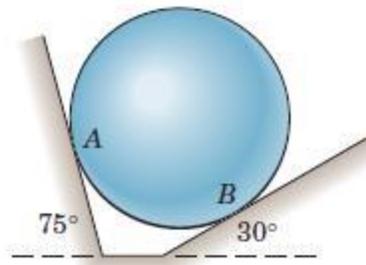
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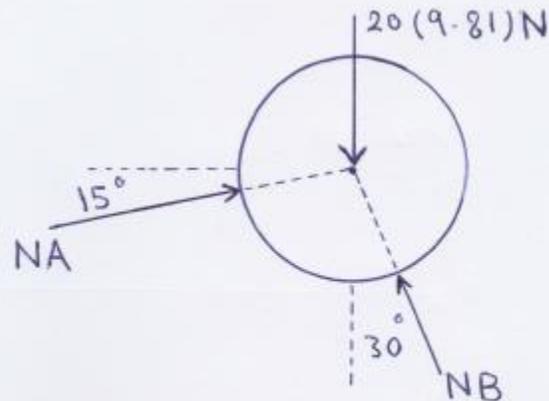


3/8 The 20-kg homogeneous smooth sphere rests on the two inclines as shown. Determine the contact forces at A and B.



Problem 3/8

3/8



$$\sum F_x = 0$$

$$N_A \cos 15 - N_B \sin 30 = 0 \quad \dots \textcircled{1}$$

$$\sum F_y = 0$$

$$N_A \sin 15 + N_B \cos 30 - 20(9.81) = 0 \quad \dots \textcircled{2}$$

Solving eq ① and ② get :

$$N_A = 101.6 \text{ N}$$



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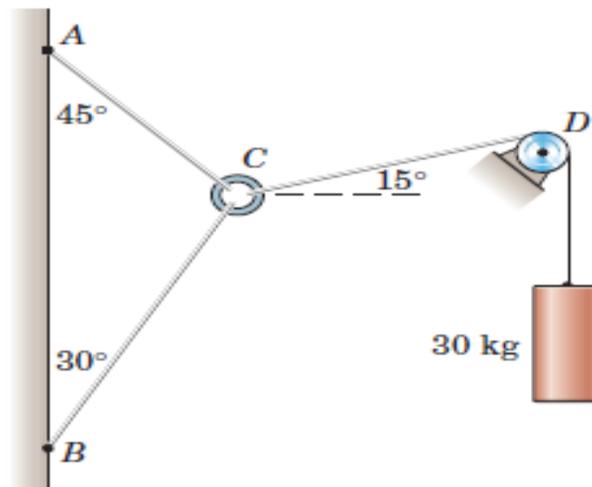
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3/15 Three cables are joined at the junction ring C . Determine the tensions in cables AC and BC caused by the weight of the 30-kg cylinder.

Ans. $T_{AC} = 215 \text{ N}$, $T_{BC} = 264 \text{ N}$



Problem 3/15

3/15

$\sum F_x = 0$

$$-T_{AC} \cos 45 - T_{BC} \cos 60 + 30(9.81) \cos 15 = 0$$



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$$\sum F_y = 0$$

$$T_{AC} \sin 45 - T_{BC} \sin 60 + 30(9.81) \sin 15 = 0$$

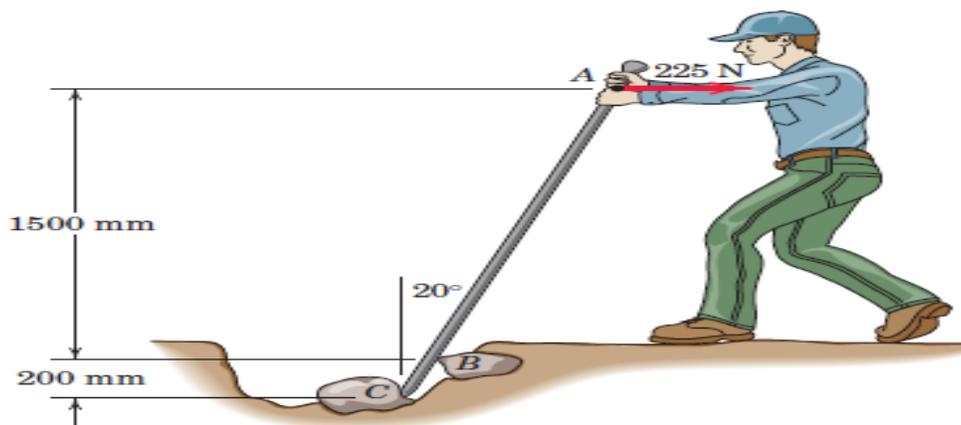
Solving the two equation get :

$$T_{AC} = 215 \text{ N}$$

$$T_{BC} = 264 \text{ N}$$

3/21 While digging a small hole prior to planting a tree, a homeowner encounters rocks. If he exerts a horizontal 225-N force on the prybar as shown, what is the horizontal force exerted on rock C? Note that a small ledge on rock C supports a vertical force reaction there. Neglect friction at B. Complete solutions (a) including and (b) excluding the weight of the 18-kg prybar.

Ans. (a) $F_C = 1705 \text{ N}$, (b) $F_C = 1464 \text{ N}$



Problem 3/21



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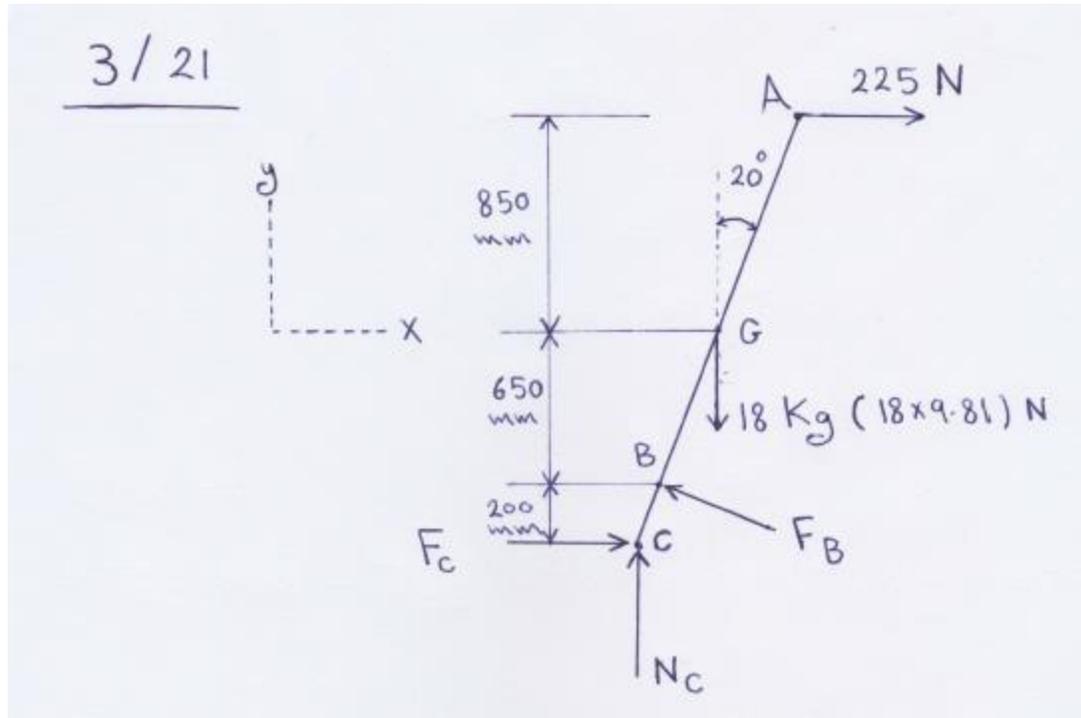
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a) including the weight of the 18 Kg prybar

$$\uparrow \sum M_c = 0$$

$$225(1.7) + 18(9.81)(0.85 \tan 20) - F_B \left(\frac{0.2}{\cos 20} \right) = 0$$

$$\therefore F_B = 2053.836 \text{ N}$$

$$\sum F_x = 0 \Rightarrow F_c - F_B \cos 20 + 225 = 0$$

$$\therefore F_c = 1705 \text{ N}$$



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b) Exclude the weight of the 18 Kg prybar

$$\uparrow \sum M_C = 0$$

$$225 (1.7) - F_B \left(\frac{0.2}{\cos 20} \right) = 0$$

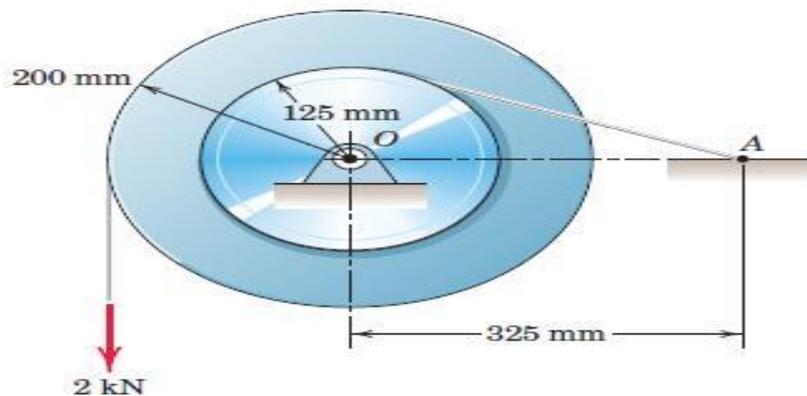
$$\therefore F_B = 1797.162 \text{ N}$$

$$\sum F_x = 0 \Rightarrow F_C - F_B \cos 20 + 225 = 0$$

$$\therefore F_C = 1464 \text{ N}$$

3/31 The two light pulleys are fastened together and form an integral unit. They are prevented from turning about their bearing at O by a cable wound securely around the smaller pulley and fastened to point A . Calculate the magnitude R of the force supported by the bearing O for the applied 2-kN load.

Ans. $R = 4.38 \text{ kN}$



Problem 3/31



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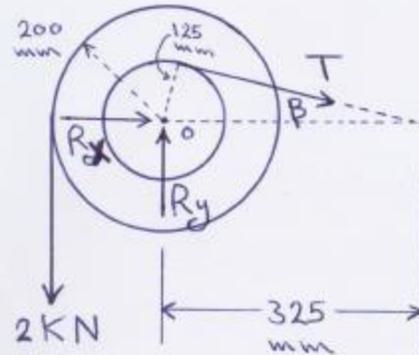


3/31

$$\sin \beta = \frac{125}{325}$$
$$\therefore \beta = 22.619^\circ$$

$$\sum M_o = 0$$

$$2(200) - T(125) = 0 \Rightarrow T = 3.2 \text{ KN}$$



$$\sum F_x = 0 \Rightarrow 3.2 \cos 22.619 + R_x = 0$$

$$\therefore R_x = -2.95 \text{ KN}$$

$$\sum F_y = 0 \Rightarrow R_y - 2 - 3.2 \sin 22.619 = 0$$

$$\therefore R_y = 3.23 \text{ KN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{2.95^2 + 3.23^2} = 4.38 \text{ KN}$$