Subject Name: Strength of Materials II

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Lecturer: Assist.Lect. Fatima Rahim Abass

Email: Fatima.Rahim.Abass@uomus.edu.iq

Lecture No. 1
Lecture Title: Stress and strain

1.1. INTRODUCTION

When an external force acts on a body, the body tends to undergo some deformation. Due to cohesion between the molecules, the body resists deformation. This resistance by which material of the body opposes the deformation is known as **strength of material.** Within a certain limit (*i.e.*, in the elastic stage) the resistance offered by the material is proportional to the deformation brought out on the material by the external force. Also within this limit the resistance is equal to the external force (or applied load). But beyond the elastic stage, the resistance offered by the material is less than the applied load. In such a case, the deformation continues, until failure takes place.

Within elastic stage, the resisting force equals applied load. This resisting force per unit area is called stress or intensity of stress.

تعرف مقاومة المادة (strength of material) على انها قدرة مادة الجسم على مقاومة التشوه (deformation) اذا تعرضت لتاثير قوة خارجية.

STRESS

1.2. STRESS

The force of resistance per unit area, offered by a body against deformation is known as stress. The external force acting on the body is called the *load or force*. The load is applied on the body while the stress is induced in the material of the body. A loaded member remains in equilibrium when the resistance offered by the member against the deformation and the applied load are equal.

Mathematically stress is written as, $\sigma = \frac{P}{A}$

where σ = Stress (also called intensity of stress),

P = External force or load, and

A = Cross-sectional area.

- يعرف الاجهاد (stress) على انه قوة المقاومة التي تبديها المادة ضد التشوه (deformation) على وحدة المساحة.
- القوة الخارجية المؤثرة على الجسم تسمى الحمل (Load) او القوة (Force).

Units of Stress

1.2.1. Units of Stress. The unit of stress depends upon the unit of load (or force) and unit of area. In M.K.S. units, the force is expressed in kgf and area in metre square (*i.e.*, m^2). Hence unit of stress becomes as kgf/m^2. If area is expressed in centimetre square (*i.e.*, cm^2), the stress is expressed as kgf/cm^2.

In the S.I. units, the force is expressed in newtons (written as N) and area is expressed as m^2. Hence unit of stress becomes as N/m^2. The area is also expressed in millimetre square then unit of force becomes as N/mm^2.

$$1\ N/m^2 = 1\ N/(100\ cm)^2 = 1\ N/10^4\ cm^2$$

$$= 10^{-4}\ N/cm^2 \text{ or } 10^{-6}\ N/mm^2 \qquad \left(\because \frac{1}{cm^2} = \frac{1}{10^2\ mm^2}\right)$$

في نظام الوحدات العالمي (S.I. Units) وحدات الاجهاد هي (N/m^2)

Units of Stress

$\therefore \quad 1 \text{ N/mm}^2 = 10^6 \text{ N/m}^2.$

Also $\quad 1 \text{ N/m}^2 = 1 \text{ Pascal} = 1 \text{ Pa}.$

The large quantities are represented by kilo, mega, giga and terra. They stand for :

Kilo = 10^3 and represented by ...... k

Mega = 10^6 and represented by ...... M

Giga = 10^9 and represented by ...... G

Terra = 10^{12} and represented by ...... T.

Thus mega newton means 10^6 newtons and is represented by MN. The symbol 1 MPa stands for 1 mega pascal which is equal to 10^6 pascal (or 10^6 N/m^2).

The small quantities are represented by milli, micro, nana and pica. They are equal to

Milli = 10^{-3} and represented by ...... m

Micro = 10^{-6} and represented by ...... μ

Nana = 10^{-9} and represented by ...... η

Pica = 10^{-12} and represented by ...... p.

Notes. 1. Newton is a force acting on a mass of one kg and produces an acceleration of 1 m/s^2 *i.e.*,

$1 \text{ N} = 1 \text{ (kg)} \times 1 \text{ m/s}^2.$

2. The stress in S.I. units is expressed in N/m^2 or N/mm^2.

3. The stress 1 N/mm^2 = 10^6 N/m^2 = MN/m^2. Thus one N/mm^2 is equal to one MN/m^2.

4. One pascal is written as 1 Pa and is equal to 1 N/m^2.

Strain

1.3. STRAIN

When a body is subjected to some external force, there is some change of dimension of the body. The ratio of change of dimension of the body to the original dimension is known as strain. Strain is dimensionless.

Strain may be :

1. Tensile strain,
2. Compressive strain,
3. Volumetric strain, and
4. Shear strain.

If there is some increase in length of a body due to external force, then the ratio of increase of length to the original length of the body is known as *tensile strain*. But if there is some decrease in length of the body, then the ratio of decrease of the length of the body to the original length is known as *compressive strain*. The ratio of change of volume of the body to the original volume is known as *volumetric strain*. The strain produced by shear stress is known as shear strain.

- يعرف الانفعال (strain) على انه نسبة تغير ابعاد الجسم الى ابعاده الاصلية نتيجة تعرضه الى قوة خارجية ويرمز له بالرمز (e) أو (ϵ).
- الانفعال بدون وحدات

Types of stresses

1.4. TYPES OF STRESSES

The stress may be normal stress or a shear stress.

Normal stress is the stress which acts in a direction perpendicular to the area. It is represented by σ (sigma). The normal stress is further divided into tensile stress and compressive stress.

1.4.1. Tensile Stress. The stress induced in a body, when subjected to two equal and opposite pulls as shown in Fig. 1.1 (*a*) as a result of which there is an increase in length, is known as tensile stress. The ratio of increase in length to the original length is known as *tensile strain*. The tensile stress acts normal to the area and it pulls on the area.

- الاجهاد يمكن ان يكون اجهاد طبيعي (Normal Stress) او اجهاد قص (Shear Stress)
- الاجهاد الطبيعي وهو الذي يؤثر عموديا على المساحة ويرمز له (σ) يقسم الى اجهاد شد (tensile stress) واجهاد ضغط (compressive stress)
- اجهاد الشد (tensile stress) ينجم عن تعرض الجسم الى قوتين متساويتين بالمقدار ومتعاكستين بالاتجاه وتؤثران الى الخارج كما في الشكل (1.1a) وينتج عن ذلك زيادة في الطول .
- النسبة بين الزيادة في الطول الى الطول الاصلي يسمى انفعال الشد (tensile strain) .

Types of Stresses

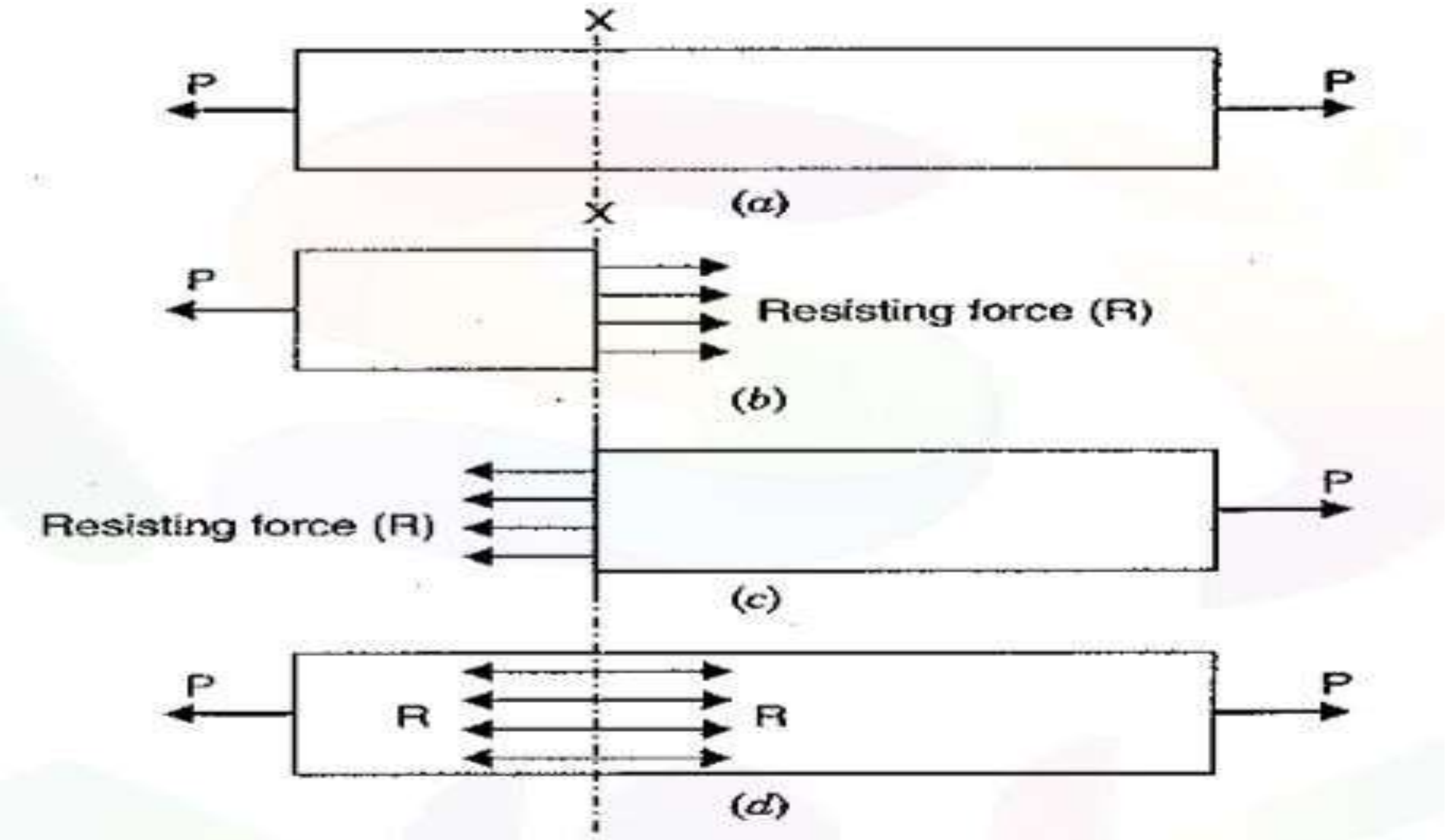


Fig. 1.1

Types of stresses

Let P = Pull (or force) acting on the body,

A = Cross-sectional area of the body,

L = Original length of the body,

dL = Increase in length due to pull P acting on the body,

σ = Stress induced in the body, and

e = Strain (*i.e.*, tensile strain).

Fig. 1.1 (*a*) shows a bar subjected to a tensile force P at its ends. Consider a section x-x, which divides the bar into two parts. The part left to the section x-x, will be in equilibrium if P = Resisting force (R). This is shown in Fig. 1.1 (*b*). Similarly the part right to the section x-x, will be in equilibrium if P = Resisting force as shown in Fig. 1.1 (*c*). This resisting force per unit area is known as stress or intensity of stress.

Types of Stresses

$$\therefore \quad \text{Tensile stress} = \sigma = \frac{\text{Resisting force } (R)}{\text{Cross-sectional area}} = \frac{\text{Tensile load } (P)}{A} \qquad (\because \; P = R)$$

or

$$\sigma = \frac{P}{A} \qquad \ldots(1.1)$$

And tensile strain is given by,

$$e = \frac{\text{Increase in length}}{\text{Original length}} = \frac{dL}{L}. \qquad \ldots(1.2)$$

Types of Stresses

1.4.2. Compressive Stress. The stress induced in a body, when subjected to two equal and opposite pushes as shown in Fig. 1.2 (*a*) as a result of which there is a decrease in length of the body, is known as compressive stress. And the ratio of decrease in length to the original length is known as *compressive strain*. The compressive stress acts normal to the area and it pushes on the area.

Let an axial push P is acting on a body is cross-sectional area A. Due to external push P, let the original length L of the body decreases by dL.

- اجهاد الضغط (Compressive stress) ينجم عن تعرض الجسم الى قوتين متساويتين بالمقدار ومتعاكستين بالاتجاه يضغطان الى الداخل كما في شكل (1.2a) وينتج عن ذلك نقص في طول الجسم .
- النسبة بين النقص في الطول الى الطول الاصلي تسمى انفعال الضغط (Compressive strain).

Types of Stresses

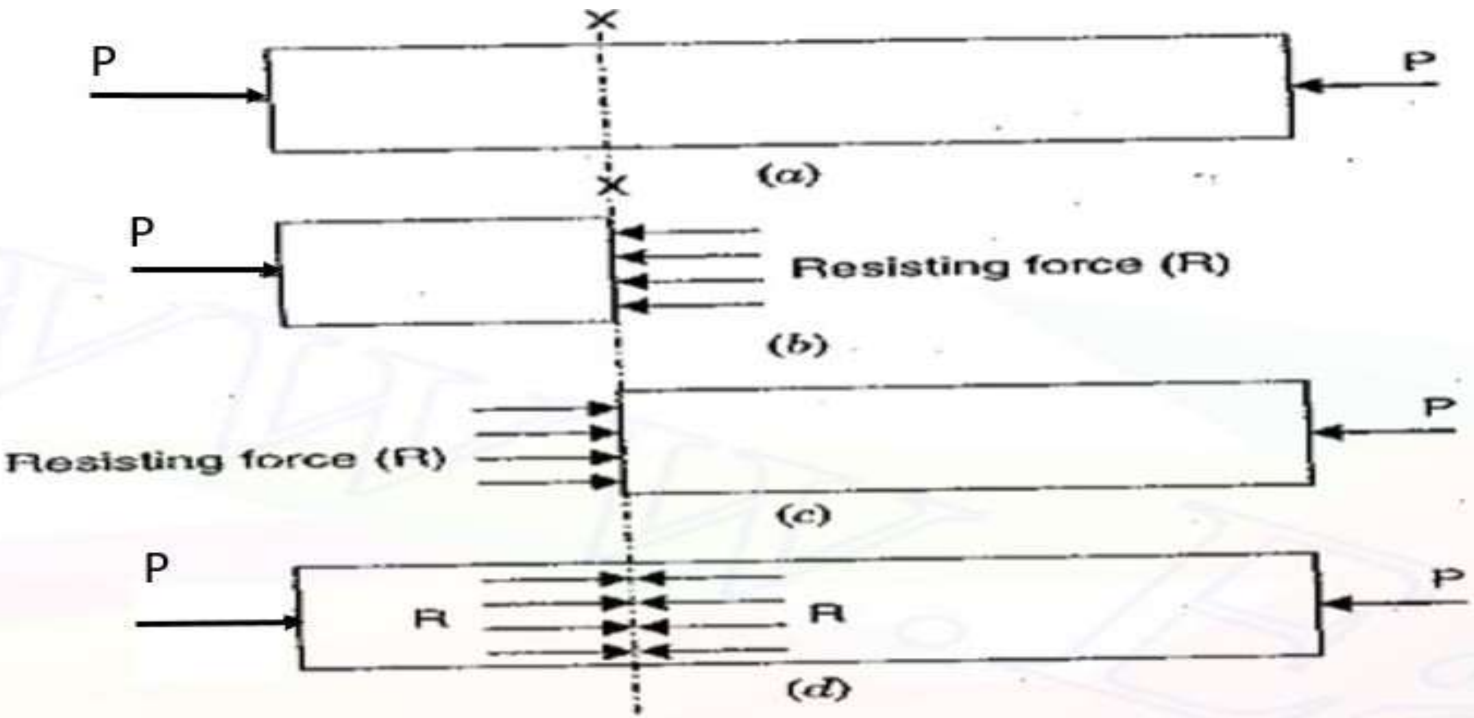


Fig. 1.2

Then compressive stress is given by,

$$\sigma = \frac{\text{Resisting Force } (R)}{\text{Area } (A)} = \frac{\text{Push } (P)}{\text{Area } (A)} = \frac{P}{A}.$$

And compressive strain is given by,

$$e = \frac{\text{Decrease in length}}{\text{Original length}} = \frac{dL}{L}.$$

Types of stresses

1.4.3. Shear Stress. The stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section as shown in Fig. 1.3 as a result of which the body tends to shear off across the section, is known as shear stress. The corresponding strain is known as *shear strain*. The shear stress is the stress which acts tangential to the area. It is represented by τ.

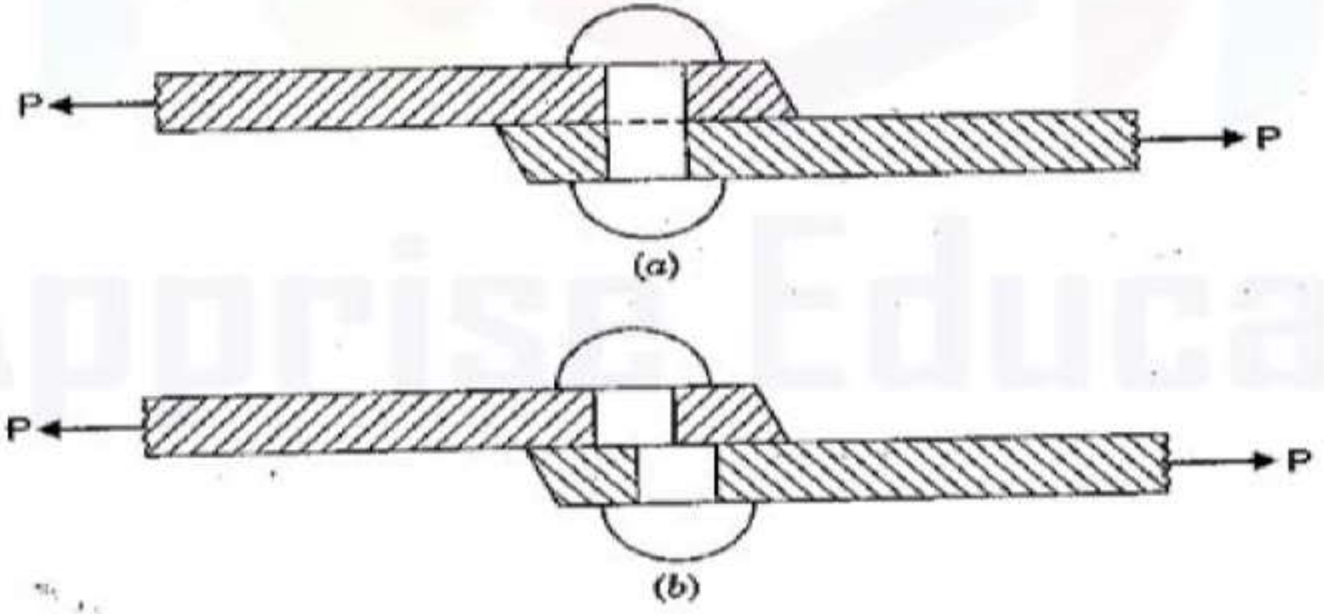
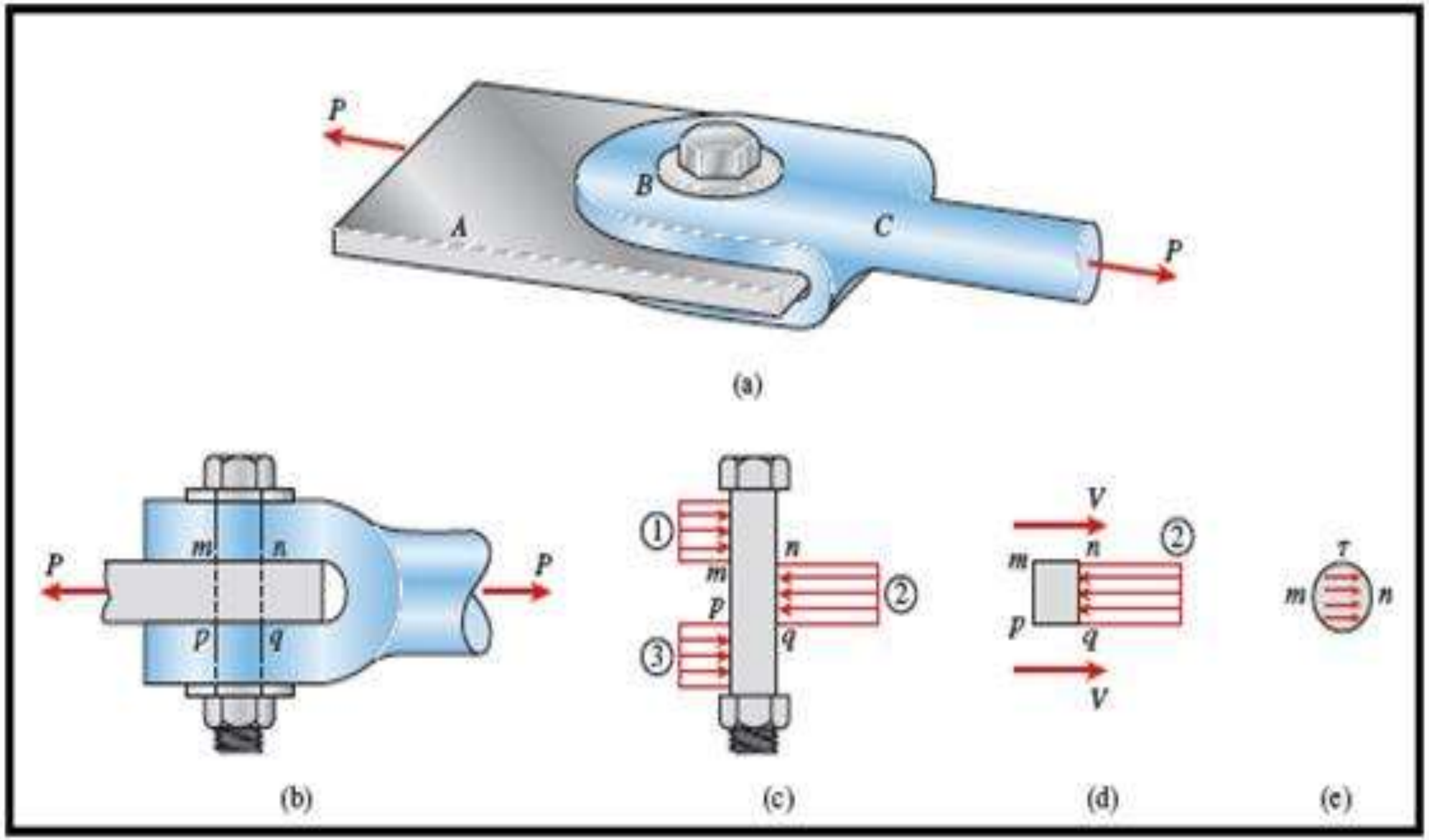


Fig. 1.3

- يتكون اجهاد القص (Shear stress) في الجسم عندما يتعرض الى تأثير قوتين متساويتين بالمقدار ومتعاكستين بالاتجاه وتؤثران بشكل عرضي وليستا على نفس المحور كما في شكل (1.3) ، ويعبر عنه بالرمز (τ) ويلفظ (تاو).
- ينتج عن ذلك قص وسيلة ربط القطعتين اذا تجاوز الاجهاد مقاومة مادة وسيلة الربط .



$$\text{shear stress}\,(\tau) = \frac{\text{shear load}}{\text{area resisting shear}} = \frac{P}{A}$$

Elasticity

1.5. ELASTICITY AND ELASTIC LIMIT

المرونة وحد المرونة

When an external force acts on a body, the body tends to undergo some deformation. If the external force is removed and the body comes back to its origin shape and size (which means the deformation disappears completely), the body is known as *elastic body*. This property, stress to the corresponding strain is a constant within the elastic limit. This constant is represented by E and is known as modulus of elasticity or Young's modulus of elasticity.

$$\therefore \quad \frac{\text{Normal stress}}{\text{Corresponding strain}} = \text{Constant} \quad \text{or} \quad \frac{\sigma}{e} = E$$

where σ = Normal stress, e = Strain and E = Young's modulus

or

$$e = \frac{\sigma}{E} \qquad \text{...[1.7 (A)]}$$

The above equation gives the stress and strain relation for the normal stress in one direction.

- اذا اثرت قوة خارجية على جسم فسيحصل فيه بعض التشوه ، فاذا ازيلت القوة وعاد الجسم الى شكله وابعاده الاصلية وهذا يعني زوال التشوه بشكل كامل فان هذا الجسم يصنف على انه جسم مرن (Elastic Body).
- العلاقة بين الاجهاد (stress) والانفعال (strain) ضمن حدود المرونة يعبر عنها بثابت (constant) يسمى معامل المرونة (modulus of elasticity) .
- يعرف معامل المرونة بمعامل يونج (Young modulus) ويعبر عنه بالرمز (E).

$$E = \frac{\sigma}{e}$$

Hooke's Law
قانون هوك

Hooke's Law states that when a material is loaded within elastic limit, the stress is proportional to the strain produced by the stress. This means the ratio of the stress to the corresponding strain is a constant within the elastic limit. This constant is known as Modulus of Elasticitv or Modulus of Rigiditv or Elastic Modulii.

ينص قانون هوك على ان المادة اذا ما تم تحميلها ضمن حدود المرونة فان الاجهاد الناتج يتناسب مع الانفعال الناتج بتاثير ذلك الاجهاد ، ذلك يعني ان النسبة بين الاجهاد الى الانفعال تكون ثابتة ضمن حدود المرونة ، هذا الثابت يسمى معامل المرونة او معامل الصلابة.

$$\text{stress } (\sigma) \propto \text{ strain } (\varepsilon)$$

$$\frac{\text{stress}}{\text{strain}} = \text{constant}^{*}$$

Modulus of Elasticity

معامل المرونة (معامل يونك)

1.7. MODULUS OF ELASTICITY (OR YOUNG'S MODULUS)

The ratio of tensile stress or compressive stress to the corresponding strain is a constant. This ratio is known as Young's Modulus or Modulus of Elasticity and is denoted by E.

$$\therefore \quad E = \frac{\text{Tensile stress}}{\text{Tensile strain}} \quad \text{or} \quad \frac{\text{Compressive stress}}{\text{Compressive strain}}$$

or

$$E = \frac{\sigma}{e} \qquad \ldots(1.5)$$

النسبة بين اجهاد الشد او الضغط والانفعال تعرف بمعامل يونج او معامل المرونة (E) ووحداته هي نفس وحدات الاجهاد مثلا (N/m^2)

1.8. FACTOR OF SAFETY

It is defined as the ratio of ultimate tensile stress to the working (or permissible) stress. Mathematically it is written as

$$\text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Permissible stress}} \qquad \ldots(1.7)$$

معامل الامان يعرف على انه النسبة بين اجهاد الشد الاقصى الى الاجهاد المسموح به (الاجهاد العامل)

Examples

Problem 1.1. *A rod 150 cm long and of diameter 2.0 cm is subjected to an axial pull of 20 kN. If the modulus of elasticity of the material of the rod is 2×10^5 N/mm^2 ; determine :*

(i) the stress,

(ii) the strain, and

(iii) the elongation of the rod.

- عمود طوله 150 cm وقطره 2 cm تعرض الى قوة شد محورية مقدارها 20 KN . اذا كان معامل مرونة مادة العمود هو 2×10^5 N/mm^2 . احسب :

i. الاجهاد. الانفعال.

ii.

iii. الاستطالة في العمود .

Sol. Given : Length of the rod, $L = 150$ cm

Diameter of the rod, $D = 2.0$ cm $= 20$ mm

$\therefore$ Area, $A = \frac{\pi}{4}(20)^2 = 100\pi$ mm^2

Axial pull, $P = 20$ kN $= 20{,}000$ N

Modulus of elasticity, $E = 2.0 \times 10^5$ N/mm^2

(*i*) The stress (σ) is given by equation (1.1) as

$$\sigma = \frac{P}{A} = \frac{20000}{100\pi} = 63.662 \text{ N/mm}^2. \quad \textbf{Ans.}$$

(*ii*) Using equation (1.5), the strain is obtained as

$$E = \frac{\sigma}{e}.$$

$\therefore$ Strain,

$$e = \frac{\sigma}{E} = \frac{63.662}{2 \times 10^5} = 0.000318. \quad \textbf{Ans.}$$

(*iii*) Elongation is obtained by using equation (1.2) as

$$e = \frac{dL}{L}.$$

$\therefore$ Elongation, $dL = e \times L$

$$= 0.000318 \times 150 = 0.0477 \text{ cm}. \quad \textbf{Ans.}$$

Examples

Problem 1.2. *Find the minimum diameter of a steel wire, which is used to raise a load of 4000 N if the stress in the rod is not to exceed 95 MN/m².*

جد اقل قطر لسلك معدني يستخدم لرفع حمل مقداره 4000 N اذا كان الاجهاد في السلك يجب ان لا يتجاوز $95\ MN/m^2$

Sol. Given : Load, $P = 4000$ N

Stress, $\sigma = 95\ MN/m^2 = 95 \times 10^6\ N/m^2$ $\quad (\because\ M = Mega = 10^6)$

$= 95\ N/mm^2$ $\quad (\because\ 10^6\ N/m^2 = 1\ N/mm^2)$

Let D = Diameter of wire in mm

$\therefore$ Area, $A = \frac{\pi}{4} D^2$

Now stress $= \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$

$$95 = \frac{4000}{\frac{\pi}{4} D^2} = \frac{4000 \times 4}{\pi D^2} \quad \text{or} \quad D^2 = \frac{4000 \times 4}{\pi \times 95} = 53.61$$

$\therefore$ $D = 7.32$ mm. **Ans.**

Examples

Problem 1.3. *Find the Young's Modulus of a brass rod of diameter 25 mm and of length 250 mm which is subjected to a tensile load of 50 kN when the extension of the rod is equal to 0.3 mm.*

جد معامل المرونة لعمود براص قطره 25 mm وطوله 250 mm معرض لقوة شد مقدارها 50 KN عندما كانت الاستطالة في العمود تساوي 0.3 mm .

Sol. Given : Dia. of rod, $D = 25$ mm

$\therefore$ Area of rod, $A = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2$

Tensile load, $P = 50 \text{ kN} = 50 \times 1000 = 50{,}000$ N

Extension of rod, $dL = 0.3$ mm

Length of rod, $L = 250$ mm

Stress (σ) is given by equation (1.1), as

$$\sigma = \frac{P}{A} = \frac{50{,}000}{490.87} = 101.86 \text{ N/mm}^2.$$

Strain (e) is given by equation (1.2), as

$$e = \frac{dL}{L} = \frac{0.3}{250} = 0.0012.$$

Using equation (1.5), the Young's Modulus (E) is obtained, as

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{101.86 \text{ N/mm}^2}{0.0012} = 84883.33 \text{ N/mm}^2$$

$= 84883.33 \times 10^6 \text{ N/m}^2$. **Ans.** $(\because 1 \text{ N/mm}^2 = 10^6 \text{ N/m}^2)$

$= 84.883 \times 10^9 \text{ N/m}^2 = 84.883 \text{ GN/m}^2$. **Ans.** $(\because 10^9 = \text{G})$

Examples

Problem 1.4. *A tensile test was conducted on a mild steel bar. The following data was obtained from the test :*

اجري اختبار شد على عمود من الصلب
وتم الحصول على النتائج التالية :

(i) Diameter of the steel bar	= 3 cm
(ii) Gauge length of the bar	= 20 cm
(iii) Load at elastic limit	= 250 kN
(iv) Extension at a load of 150 kN	= 0.21 mm
(v) Maximum load	= 380 kN
(vi) Total extension	= 60 mm
(vii) Diameter of the rod at the failure	= 2.25 cm.

Determine :(a) the Young's modulus, (b) the stress at elastic limit, (c) the percentage elongation, and (d) the percentage decrease in area.

المطلوب حساب (a) معامل المرونة (b) الاجهاد عند حد المرونة (c)نسبة الاستطالة (d) نسبة النقص في المساحة

Examples

Sol. Area of the rod, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (3)^2 \text{ cm}^2$

$$= 7.0685 \text{ cm}^2 = 7.0685 \times 10^{-4} \text{ m}^2. \qquad \left[\because \text{ cm}^2 = \left(\frac{1}{100} \text{ m}\right)^2\right]$$

(*a*) To find Young's modulus, first calculate the value of stress and strain within elastic limit. The load at elastic limit is given but the extension corresponding to the load at elastic limit is not given. But a load of 150 kN (which is within elastic limit) and corresponding extension of 0.21 mm are given. Hence these values are used for stress and strain within elastic limit

$$\therefore \quad \text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{150 \times 1000}{7.0685 \times 10^{-4}} \text{ N/m}^2 \qquad (\because 1 \text{ kN} = 1000 \text{ N})$$

$$= 21220.9 \times 10^4 \text{ N/m}^2$$

and
$$\text{Strain} = \frac{\text{Increase in length (or Extension)}}{\text{Original length (or Gauge length)}}$$

$$= \frac{0.21 \text{ mm}}{20 \times 10 \text{ mm}} = 0.00105$$

$\therefore$ Young's Modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{21220.9 \times 10^4}{0.00105} = 20209523 \times 10^4 \text{ N/m}^2$$

$$= 202.095 \times 10^9 \text{ N/m}^2 \qquad (\because 10^9 = \text{Giga} = \text{G})$$

$$= 202.095 \text{ GN/m}^2. \quad \textbf{Ans.}$$

Examples

(b) The stress at the elastic limit is given by,

$$\text{Stress} = \frac{\text{Load at elastic limit}}{\text{Area}} = \frac{250 \times 1000}{7.0685 \times 10^{-4}}$$

$$= 35368 \times 10^4 \text{ N/m}^2$$

$$= 353.68 \times 10^6 \text{ N/m}^2 \qquad (\because \; 10^6 = \text{Mega} = \text{M})$$

$$= \mathbf{353.68 \text{ MN/m}^2}. \quad \textbf{Ans.}$$

(c) The percentage elongation is obtained as,

Percentage elongation

$$= \frac{\text{Total increase in length}}{\text{Original length (or Gauge length)}} \times 100$$

$$= \frac{60 \text{ mm}}{20 \times 10 \text{ mm}} \times 100 = \mathbf{30\%}. \quad \textbf{Ans.}$$

Examples

(*d*) The percentage decrease in area is obtained as,

Percentage decrease in area

$$= \frac{(\text{Original area} - \text{Area at the failure})}{\text{Original area}} \times 100$$

$$= \frac{\left(\frac{\pi}{4} \times 3^2 - \frac{\pi}{4} \times 2.25^2\right)}{\frac{\pi}{4} \times 3^2} \times 100$$

$$= \left(\frac{3^2 - 2.25^2}{3^2}\right) \times 100 = \frac{(9 - 5.0625)}{9} \times 100 = 43.75\%. \quad \textbf{Ans.}$$

Home Work

1. A rod 200 cm long and diameter 3 cm is subjected to an axial pull of 30 kN. If the Young's modulus of the material of the rod is 2×10^5 N/mm^2 , determine:

i. Stress.
ii. Strain.
iii. Elongation of the rod.

2. Find the Young's modulus of a rod of diameter 30 mm and of length 300 mm which is subjected to a tensile load of 60 kN and the extension of the rod is equal to 0.4 mm.

3. An axial pull of 40000 N is acting on a bar consisting of three sections of length 30 cm,25 cm and 20 cm and of diameters 2 cm,4 cm and 5 cm respectively. If Young's modulus = 2×10^5 n/mm^2 , determine:

i. Stress in each section.
ii. Total extension of the bar.