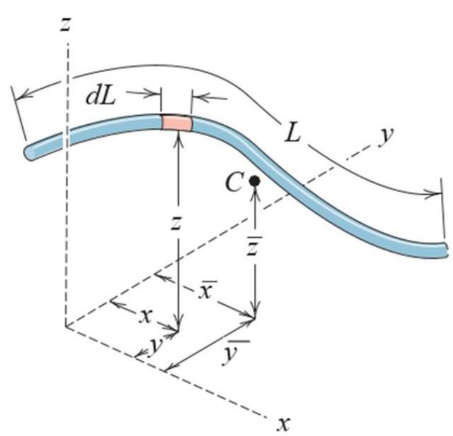
Center of Gravity and Centroid application

Centroid

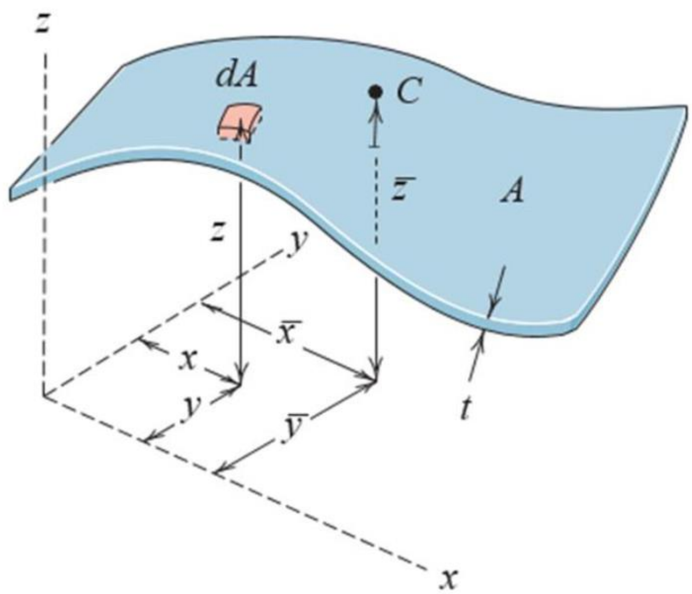
Centroid: The centroid is a point which defines the geometrical center of an object.

1-Lines. For a slender rod or wire of length L , cross-sectional area A, and density *p*, **Fig.1** , the body approximates a line segment , and **dm = *p*A dL.** If *P* and A are constant over the length of the rod, the coordinates of the center of mass also become the coordinates of centroid C of the line segment, which, from Eqs, may be written:

$$\bar{x} = \frac{\int x\, dL}{L} \qquad \bar{y} = \frac{\int y\, dL}{L} \qquad \bar{z} = \frac{\int z\, dL}{L}$$



Email (mariam.ghassan.ghaffar@uomus.edu.iq)



2-Areas. When a body of density p has a small but constant thickness t, we can model it as a surface area A, **Fig 2.** The mass of an element becomes **dm = p*t* dA** . Again, if p and t are constant over the entire area, the coordinates of the center of mass of the body also become the coordinates of the centroid C of the surface area, and d from Eqs. The coordinates may be written:

$$\bar{x} = \frac{\int x\, dA}{A} \qquad \bar{y} = \frac{\int y\, dA}{A} \qquad \bar{z} = \frac{\int z\, dA}{A}$$

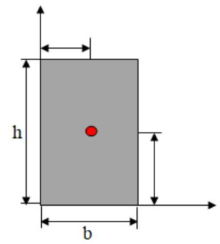
Email (mariam.ghassan.ghaffar@uomus.edu.iq)

Areas uniform:

1- Rectangle:

Area=b*h

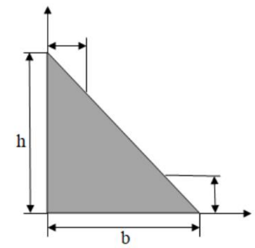
The centroid (b/2,h/2)



2- triangle:

Area=(1/2)b*h

The centroid (b/3,h/3)



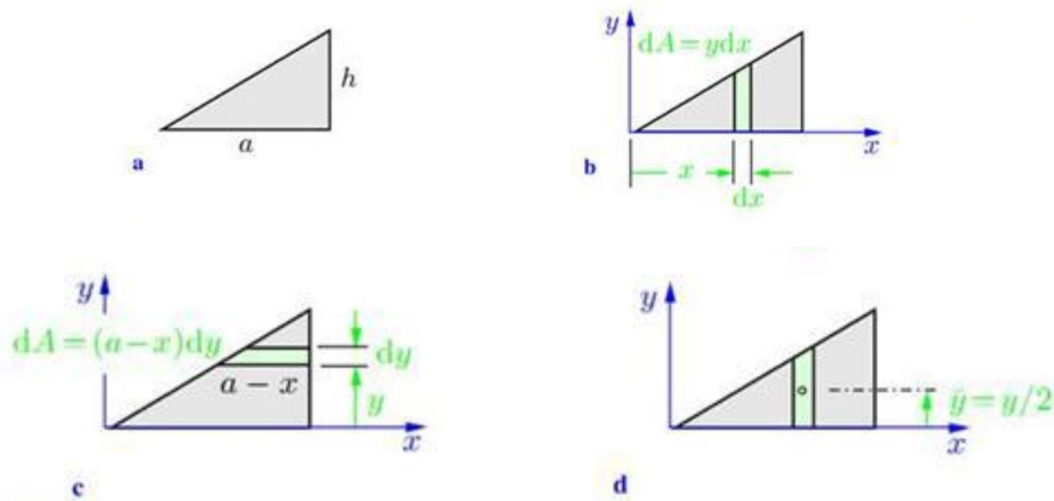
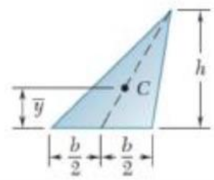
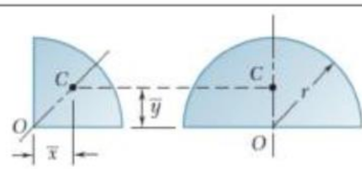
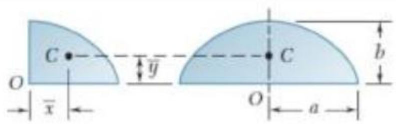
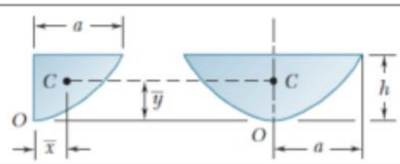
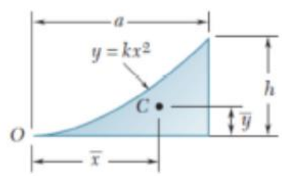
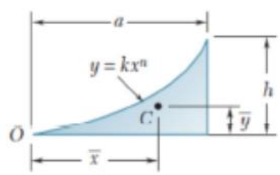
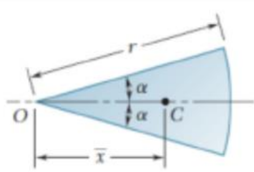


Fig. 4.9

$$x_c = \frac{1}{A}\int x\,\mathrm{d}A, \quad y_c = \frac{1}{A}\int y\,\mathrm{d}A.$$

Email (mariam.ghassan.ghaffar@uomus.edu.iq)

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

Email (mariam.ghassan.ghaffar@uomus.edu.iq)

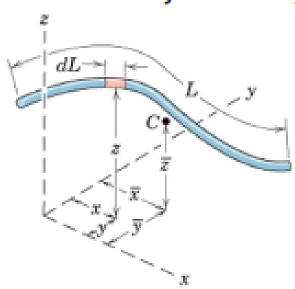
Al-Mustaqbal University / College of Engineering
Prosthetics & Orthotics Eng. Department
First Class
Subject **(MECHANICS-DYNAMIC)**
Code **(UOMU0103022)**
Asst. Lec. Mariam Ghassan Al-maroof
1st term – Lecture 1

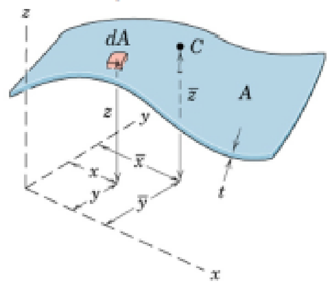
Center of Mass and Centroids

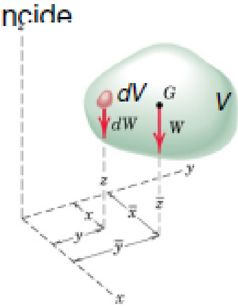
Centroids of Lines, Areas, and Volumes

Centroid is a geometrical property of a body

→ When density of a body is uniform throughout, centroid and CM coincide







Lines: Slender rod, Wire

Cross-sectional area = A

ρ and A are constant over L

$dm = \rho A dL$; Centroid = CM

$$\bar{x} = \frac{\int x dL}{L} \quad \bar{y} = \frac{\int y dL}{L} \quad \bar{z} = \frac{\int z dL}{L}$$

Areas: Body with small but constant thickness t

Cross-sectional area = A

ρ and A are constant over A

$dm = \rho t dA$; Centroid = CM

$$\bar{x} = \frac{\int x dA}{A} \quad \bar{y} = \frac{\int y dA}{A} \quad \bar{z} = \frac{\int z dA}{A}$$

Numerator = First moments of Area

Volumes: Body with volume V

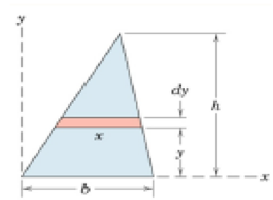
ρ constant over V

$dm = \rho dV$ Centroid = CM

$$\bar{x} = \frac{\int x dV}{V} \quad \bar{y} = \frac{\int y dV}{V} \quad \bar{z} = \frac{\int z dV}{V}$$

Examples: Centroids

Locate the centroid of the triangle along h from the base



Solution:

$dA = x dy \qquad \frac{x}{(h-y)} = \frac{b}{h}$

Total Area $A = \frac{1}{2} bh \qquad y = y_c$

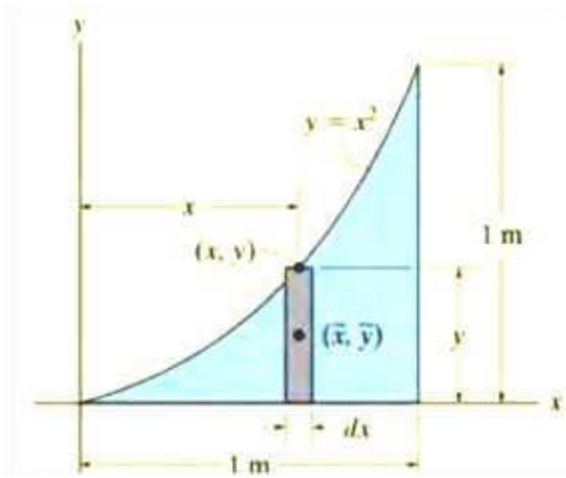
$$\bar{x} = \frac{\int x_c dA}{A} \quad \bar{y} = \frac{\int y_c dA}{A} \quad \bar{z} = \frac{\int z_c dA}{A}$$

$$A\bar{y} = \int y_c dA \quad \Rightarrow \frac{bh}{2}\bar{y} = \int_0^h y \frac{b(h-y)}{y} dy = \frac{bh^2}{6}$$

$$\boxed{\bar{y} = \frac{h}{3}}$$

Email (mariam.ghassan.ghaffar@uomus.edu.iq)

Example : Locate the centroid of the area shown.



Area and Moment Arms. The area of the element is $dA = y\,dx$, and its centroid is located at $\tilde{x} = x$, $\tilde{y} = y/2$.

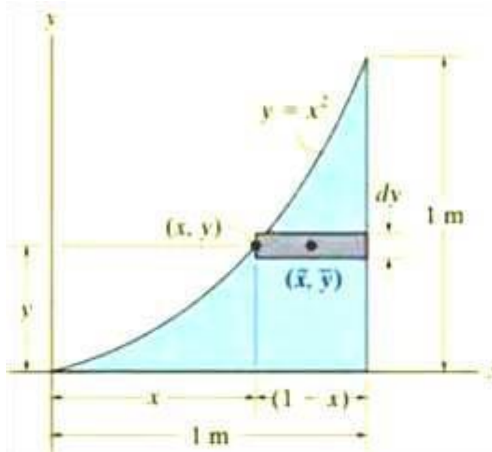
Integrations. Applying Eqs. 9–4 and integrating with respect to x yields

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{1\text{m}} xy\,dx}{\int_0^{1\text{m}} y\,dx} = \frac{\int_0^{1\text{m}} x^3\,dx}{\int_0^{1\text{m}} x^2\,dx} = \frac{0.250}{0.333} = 0.75\text{ m}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1\text{m}} (y/2)y\,dx}{\int_0^{1\text{m}} y\,dx} = \frac{\int_0^{1\text{m}} (x^2/2)x^2\,dx}{\int_0^{1\text{m}} x^2\,dx} = \frac{0.100}{0.333} = 0.3\text{ m}$$

Area and Moment Arms. The area of the element is $dA = (1 - x)\,dy$, and its centroid is located at

$$\tilde{x} = x + \left(\frac{1-x}{2}\right) = \frac{1+x}{2},\ \tilde{y} = y$$



$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{1\text{m}} [(1+x)/2](1-x)\,dy}{\int_0^{1\text{m}} (1-x)\,dy} = \frac{\frac{1}{2}\int_0^{1\text{m}} (1-y)\,dy}{\int_0^{1\text{m}} (1-\sqrt{y})\,dy} = \frac{0.250}{0.333} = 0.75\text{ m}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1\text{m}} y(1-x)\,dy}{\int_0^{1\text{m}} (1-x)\,dy} = \frac{\int_0^{1\text{m}} (y - y^{3/2})\,dy}{\int_0^{1\text{m}} (1-\sqrt{y})\,dy} = \frac{0.100}{0.333} = 0.3\text{ m}$$

Email (mariam.ghassan.ghaffar@uomus.edu.iq)

Example . *Find the centre of gravity of a 100 mm × 150 mm × 30 mm T-section.*

Solution. As the section is symmetrical about *Y-Y* axis, bisecting the web, therefore its centre of gravity will lie on this axis. Split up the section into two rectangles *ABCH* and *DEFG* as shown in Fig 6.10.

Let bottom of the web *FE* be the axis of reference.

(*i*) *Rectangle ABCH*

$$a_1 = 100 \times 30 = 3000 \text{ mm}^2$$

and $$y_1 = \left(150 - \frac{30}{2}\right) = 135 \text{ mm}$$

(*ii*) *Rectangle DEFG*

$$a_2 = 120 \times 30 = 3600 \text{ mm}^2$$

and $$y_2 = \frac{120}{2} = 60 \text{ mm}$$

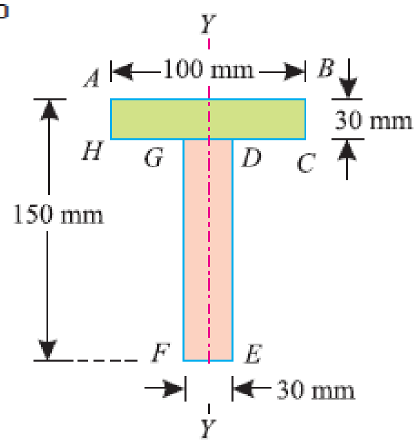


Fig. 6.10.

We know that distance between centre of gravity of the section and bottom of the flange *FE*,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(3000 \times 135) + (3600 \times 60)}{3000 + 3600} \text{ mm}$$

$$= 94.1 \text{ mm} \qquad \textbf{Ans.}$$

Email (mariam.ghassan.ghaffar@uomus.edu.iq)

Example *Find the centre of gravity of a channel section 100 mm × 50 mm × 15 mm.*

Solution. As the section is symmetrical about *X-X* axis, therefore its centre of gravity will lie on this axis. Now split up the whole section into three rectangles *ABFJ*, *EGKJ* and *CDHK* as shown in Fig. 6.11.

Let the face *AC* be the axis of reference.

(*i*) *Rectangle ABFJ*

$$a_1 = 50 \times 15 = 750 \text{ mm}^2$$

and $$x_1 = \frac{50}{2} = 25 \text{ mm}$$

(*ii*) *Rectangle EGKJ*

$$a_2 = (100 - 30) \times 15 = 1050 \text{ mm}^2$$

and $$x_2 = \frac{15}{2} = 7.5 \text{ mm}$$

(*iii*) *Rectangle CDHK*

$$a_3 = 50 \times 15 = 750 \text{ mm}^2$$

and $$x_3 = \frac{50}{2} = 25 \text{ mm}$$

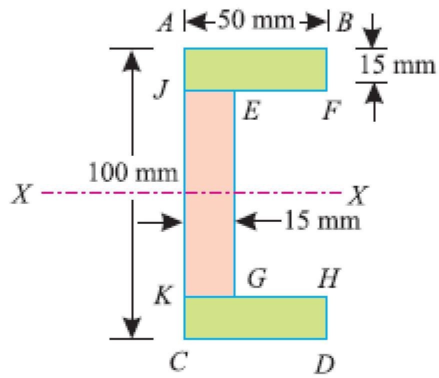


Fig. 6.11.

We know that distance between the centre of gravity of the section and left face of the section *AC*,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(750 \times 25) + (1050 \times 7.5) + (750 \times 25)}{750 + 1050 + 750} = 17.8 \text{ mm} \quad \textbf{Ans.}$$

Email (mariam.ghassan.ghaffar@uomus.edu.iq)

Example *Find the centroid of an unequal angle section 100 mm × 80 mm × 20 mm.*

Solution. As the section is not symmetrical about any axis, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the angle section. Split up the section into two rectangles as shown in Fig. 6.13.

Let left face of the vertical section and bottom face of the horizontal section be axes of reference.

(*i*) *Rectangle* 1

$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$

$$x_1 = \frac{20}{2} = 10 \text{ mm}$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

(*ii*) *Rectangle* 2

$$a_2 = (80 - 20) \times 20 = 1200 \text{ mm}^2$$

$$x_2 = 20 + \frac{60}{2} = 50 \text{ mm}$$

and $$y_2 = \frac{20}{2} = 10 \text{ mm}$$

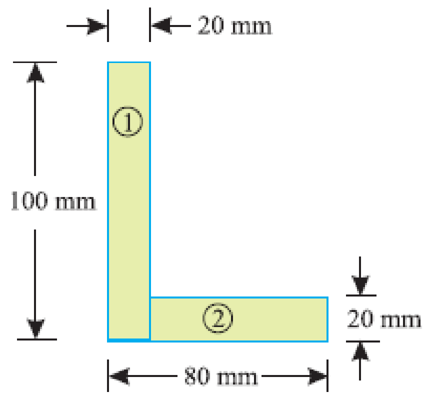


Fig. 6.13.

We know that distance between centre of gravity of the section and left face,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} = 25 \text{ mm} \qquad \textbf{Ans.}$$

Similarly, distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} = 35 \text{ mm} \qquad \textbf{Ans.}$$

Email (mariam.ghassan.ghaffar@uomus.edu.iq)

Centroids and Centers of Gravity of Composite Areas

A composite body consists of a series of connected (simpler) shaped bodies, which may be rectangular, triangular, semicircular, etc.

Such a body can often be divided into its composite parts and provided the area and location of the center of gravity of each of these parts are known.

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} \qquad \bar{y} = \frac{\sum \tilde{y}A}{\sum A}$$

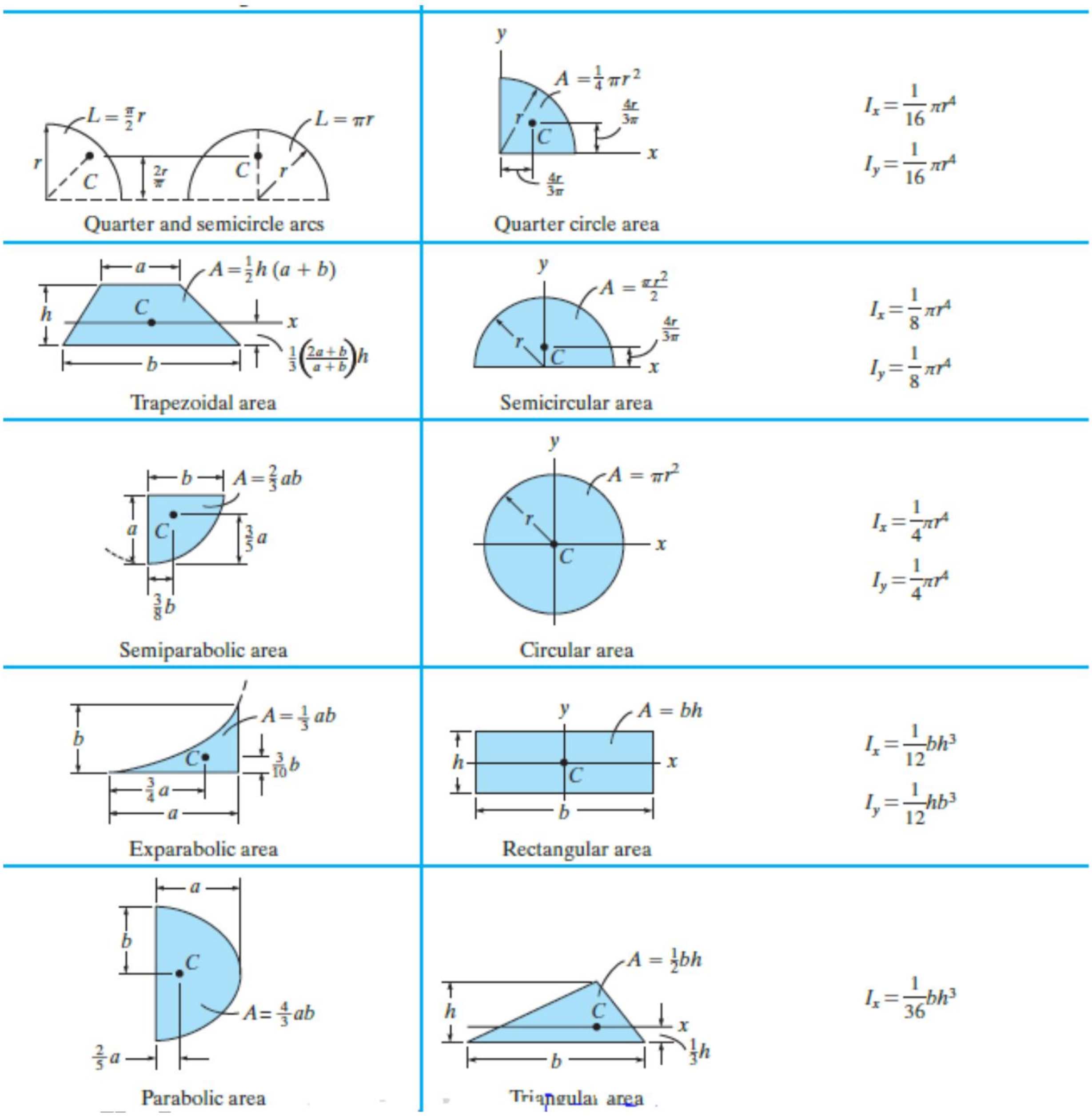
Note:-

1-If a composite body has a hole, then considers the composite body without the hole and consider the hole as an additional composite part having negative size.

2-If an object is symmetrical about an axis, the centroid of the object lies on this axis.

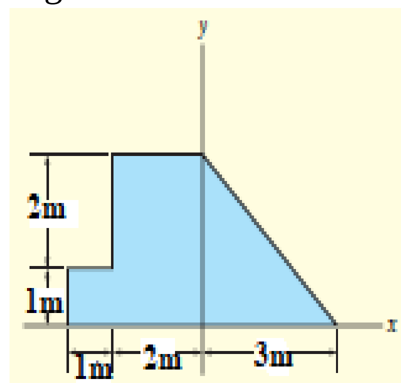
Email (mariam.ghassan.ghaffar@uomus.edu.iq)

Geometric Properties

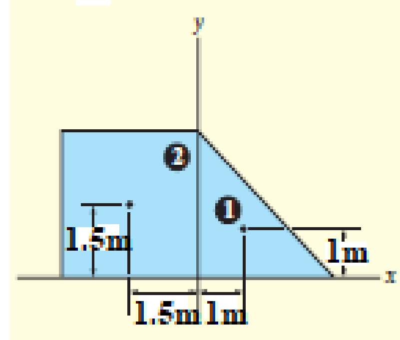


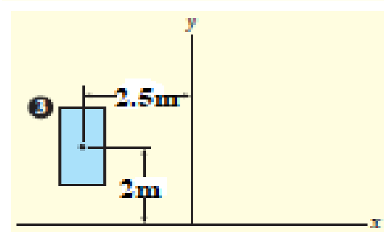
Email (mariam.ghassan.ghaffar@uomus.edu.iq)

Example (1): Locate the centroid of the plate area shown in figure



Solution:





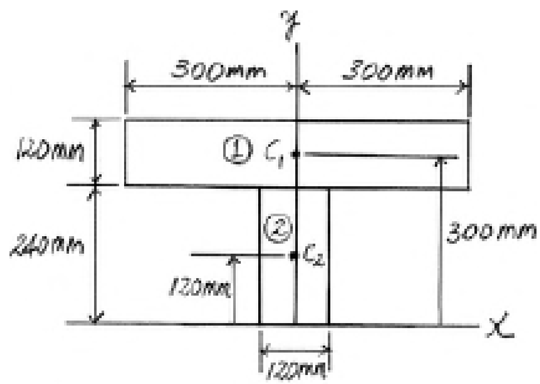
Segment	$A(m^2)$	$\tilde{x}(m)$	$\tilde{y}(m)$	$\tilde{x}A(m^3)$	$\tilde{y}A(m^3)$
1	$\frac{1}{2}(2)(3)=4.5$	1	1	4.5	4.5
2	3(3)=9	-1.5	1.5	-13.5	13.5
3	-2(1) = -2	-2.5	2	5	-4
	ΣA=11.5			$\Sigma\,\tilde{x}A$=-4	$\Sigma\,\tilde{y}A$=14

Email (mariam.ghassan.ghaffar@uomus.edu.iq)

$$\bar{x} = \Sigma = \frac{\tilde{x}A}{\Sigma A} = \frac{-4}{11.5} = -0.348\text{m}$$

$$\bar{y} = \Sigma = \frac{\tilde{y}A}{\Sigma A} = \frac{14}{11.5} = 1.22\text{m}$$

Example(2): Locate the centroid for the beams cross sectional area



<u>Solution:-</u>

$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A}$$

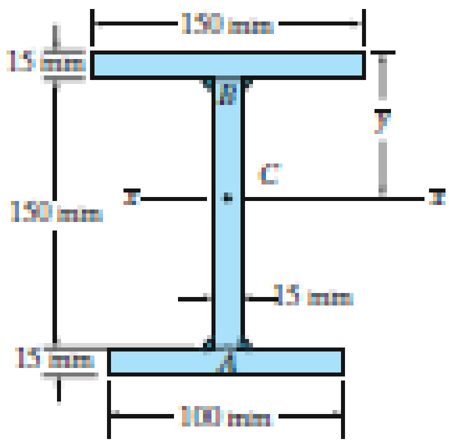
$$= \frac{300(120)(600) + 120(240)(120)}{120(600) + 240(120)}$$

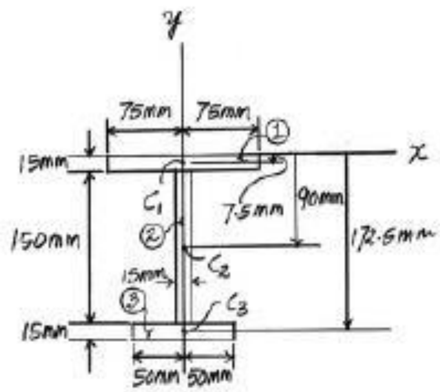
$$= 247.57mm$$

$$\bar{x} = 0$$

Email (mariam.ghassan.ghaffar@uomus.edu.iq)

Example(3): Determine the location of the centroid of the beam having cross sectional area shown below





Solution:

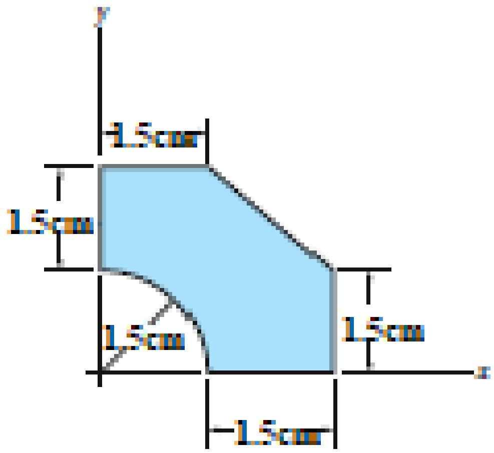
$$\bar{x} = 0$$

$$\bar{y} = \Sigma = \frac{\tilde{y}A}{\Sigma A}$$

$$= \frac{7.5(15)(150) + 90(150)(15) + 172.5(15)(100)}{(15)(150) + (150)(15) + (15)(100)}$$

$$= 79.7\text{mm}$$

H.W: Determine the location of the centroid of the area.



Email (mariam.ghassan.ghaffar@uomus.edu.iq)