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Lecture No. 4

Strength of Material

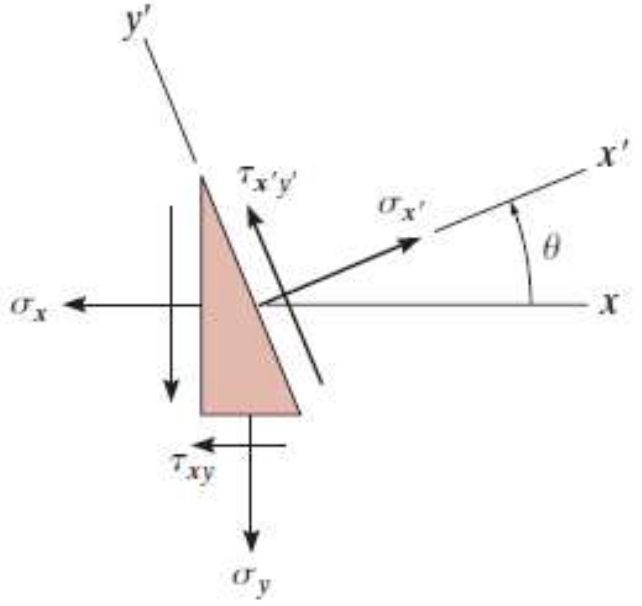
Mohr’s Circle—Plane Stress

Stress

In this section, we will show how to apply the equations for plane stress transformation using a graphical solution that is often convenient to use and easy to remember. Furthermore, this approach will allow us to “visualize” how the normal and shear stress components and vary as the plane on which they act is oriented in different directions

$$\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right) = \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$



then the parameter θ can be *eliminated* by squaring each equation and adding the equations together. The result is

$$\left[\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

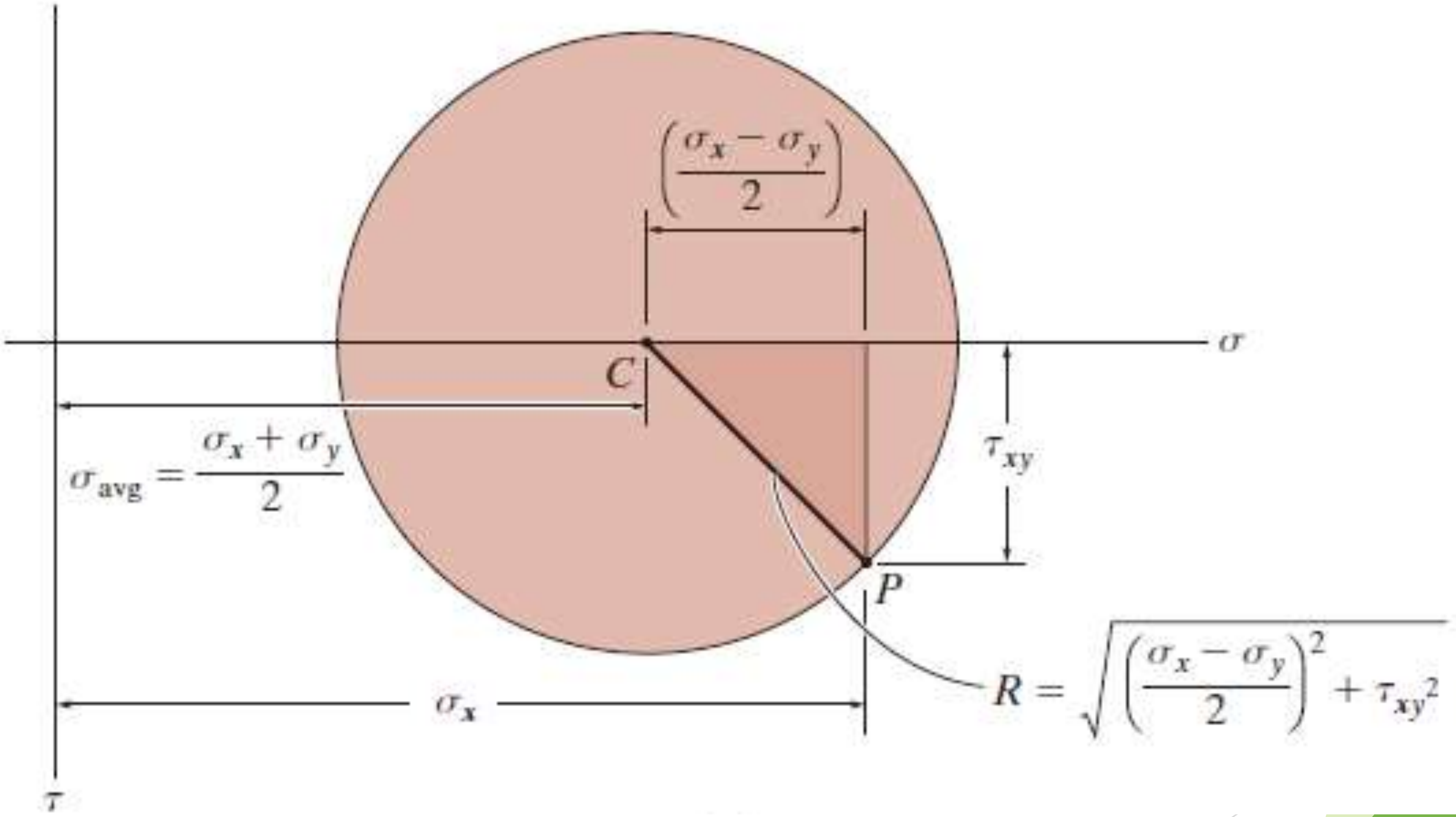
For a specific problem σ_x, σ_y, τ_{xy} are *known constants*. Thus the above equation can be written in a more compact form as

$$(\sigma_{x'} - \sigma_{avg})^2 + \tau_{x'y'}^2 = R^2$$

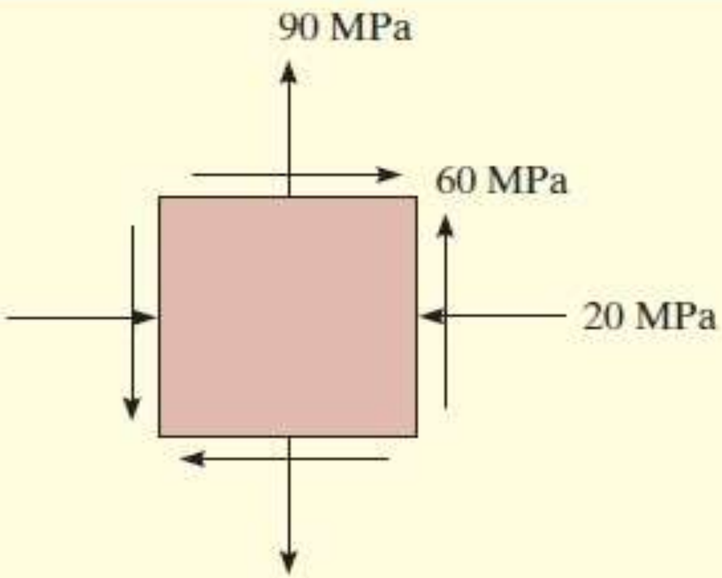
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

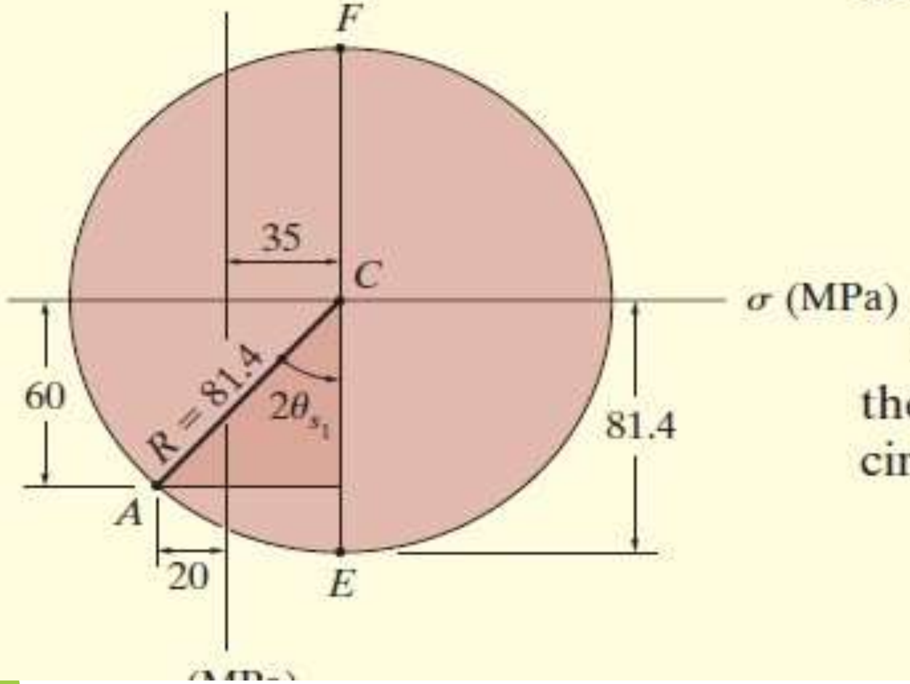
If we establish coordinate axes, the normal stress *positive to the right* and the shear stress *positive downward*, and then plot the above eq. it will be seen that this equation represents a *circle* having a radius *R* and center on the axis of normal stress at point C. This circle is called *Mohr's circle*, because it was developed by the German engineer Otto Mohr.



$\left(\frac{\sigma_x - \sigma_y}{2}\right)$
σ
C
τ_{xy}
$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$
P
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
σ_x
τ



(a)



The state of plane stress at a point is shown on the element in Fig. 9–19*a*. Determine the maximum in-plane shear stress at this point.

SOLUTION

Construction of the Circle. From the problem data,

$$\sigma_x = -20 \text{ MPa} \qquad \sigma_y = 90 \text{ MPa} \qquad \tau_{xy} = 60 \text{ MPa}$$

The σ, τ axes are established in Fig. 9–19*b*. The center of the circle C is located on the σ axis, at the point

$$\sigma_{avg} = \frac{-20 + 90}{2} = 35 \text{ MPa}$$

Point C and the reference point $A(-20, 60)$ are plotted. Applying the Pythagorean theorem to the shaded triangle to determine the circle's radius CA, we have

$$R = \sqrt{(60)^2 + (55)^2} = 81.4 \text{ MPa}$$

(b)

Maximum In-Plane Shear Stress. The maximum in-plane shear stress and the average normal stress are identified by point E (or F) on the circle. The coordinates of point $E(35, 81.4)$ give

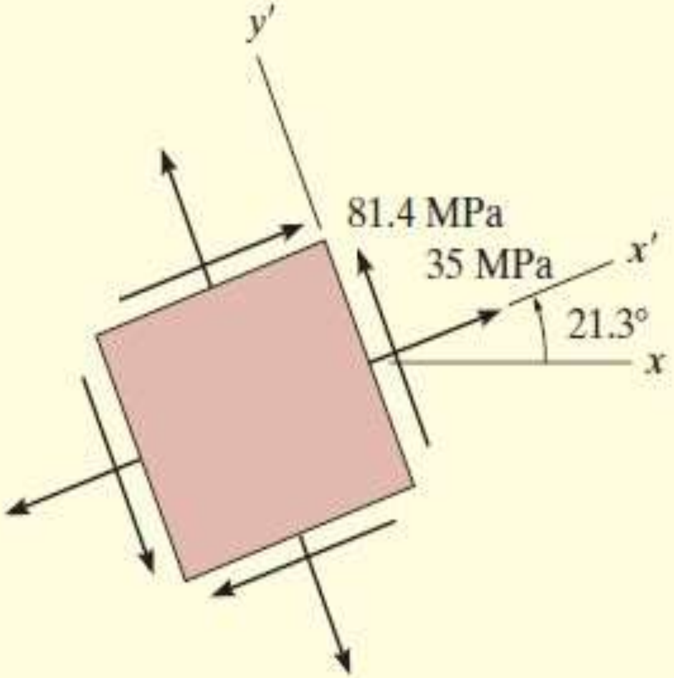
$$\tau_{\substack{\max \\ \text{in-plane}}} = 81.4 \text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_{\text{avg}} = 35 \text{ MPa} \qquad \textit{Ans.}$$

The angle θ_{s_1}, measured *counterclockwise* from CA to CE, can be found from the circle, identified as $2\theta_{s_1}$. We have

$$2\theta_{s_1} = \tan^{-1}\left(\frac{20 + 35}{60}\right) = 42.5^\circ$$

$$\theta_{s_1} = 21.3^\circ \qquad \textit{Ans.}$$

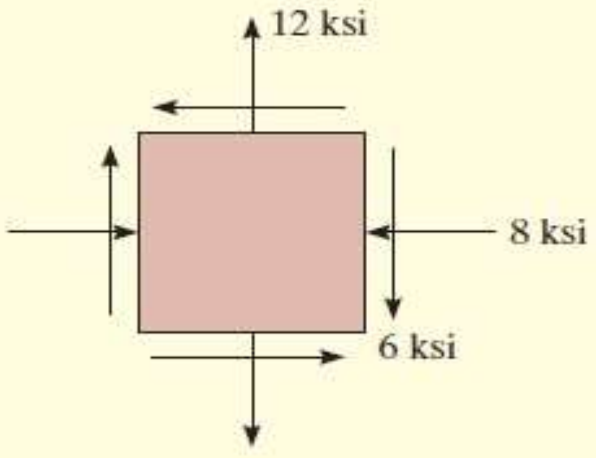


(c)

Fig. 9–19

This *counterclockwise* angle defines the direction of the x' axis, Fig. 9–19c. Since point E has *positive* coordinates, then the average normal stress and the maximum in-plane shear stress both act in the *positive* x' and y' directions as shown.

The state of plane stress at a point is shown on the element in Fig. 9–20*a*. Represent this state of stress on an element oriented 30° counterclockwise from the position shown.



(a)

SOLUTION

Construction of the Circle. From the problem data,

$$\sigma_x = -8 \text{ ksi} \qquad \sigma_y = 12 \text{ ksi} \qquad \tau_{xy} = -6 \text{ ksi}$$

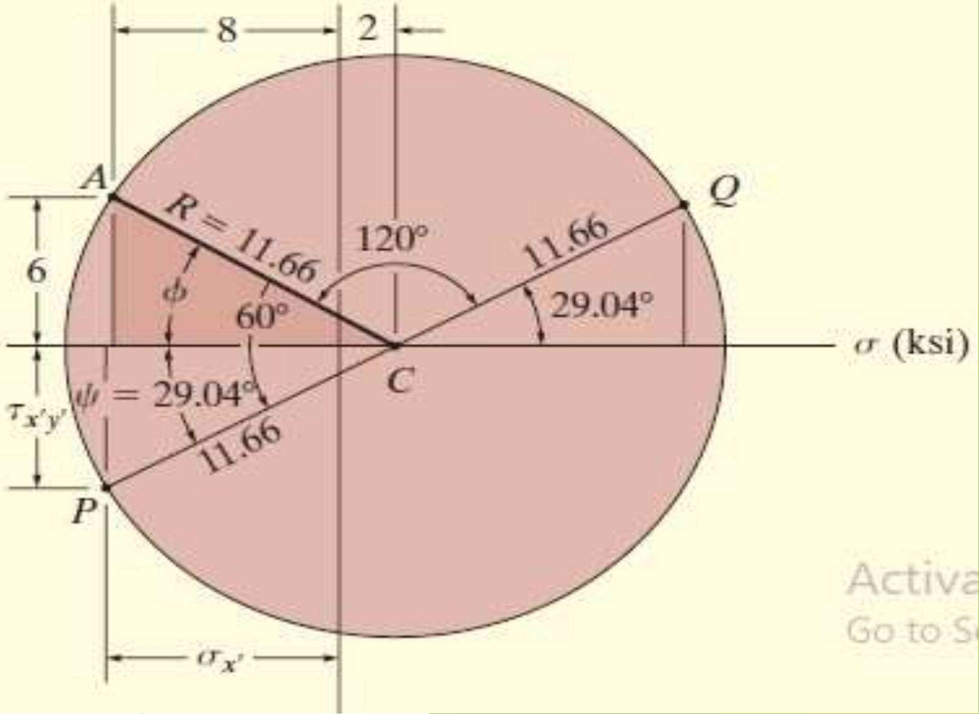
The σ and τ axes are established in Fig. 9–20*b*. The center of the circle C is on the σ axis at

$$\sigma_{avg} = \frac{-8 + 12}{2} = 2 \text{ ksi}$$

The reference point for $\theta = 0°$ has coordinates $A(-8, -6)$. Hence from the shaded triangle the radius CA is

$$R = \sqrt{(10)^2 + (6)^2} = 11.66$$

Stresses on 30° Element. Since the element is to be rotated 30° *counterclockwise*, we must construct a radial line CP, $2(30°) = 60°$ *counterclockwise*, measured from CA ($\theta = 0°$), Fig. 9–20*b*. The coordinates of point $P(\sigma_{x'}, \tau_{x'y'})$ must now be obtained. From the geometry of the circle,



$$\phi = \tan^{-1}\frac{6}{10} = 30.96^\circ \qquad \psi = 60^\circ - 30.96^\circ = 29.04^\circ$$

τ (ksi)

(b)

$$\sigma_{x'} = 2 - 11.66 \cos 29.04^\circ = -8.20 \text{ ksi} \qquad \textit{Ans.}$$

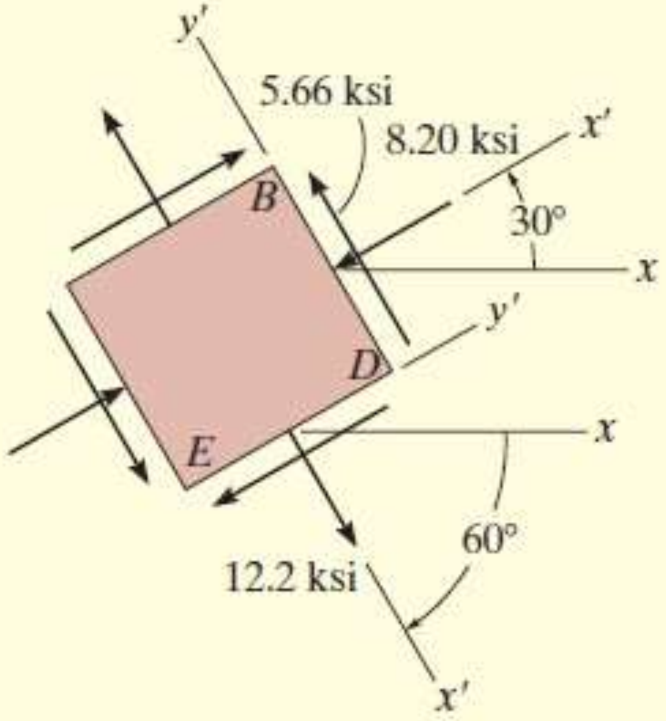
$$\tau_{x'y'} = 11.66 \sin 29.04^\circ = 5.66 \text{ ksi} \qquad \textit{Ans.}$$

These two stress components act on face *BD* of the element shown in Fig. 9–20*c* since the x' axis for this face is oriented 30° *counterclockwise* from the x axis.

The stress components acting on the adjacent face *DE* of the element, which is 60° *clockwise* from the positive x axis, Fig. 9–20*c*, are represented by the coordinates of point *Q* on the circle. This point lies on the radial line *CQ*, which is 180° from *CP*. The coordinates of point *Q* are

$$\sigma_{x'} = 2 + 11.66 \cos 29.04^\circ = 12.2 \text{ ksi} \qquad \textit{Ans.}$$

$$\tau_{x'y'} = -(11.66 \sin 29.04) = -5.66 \text{ ksi} \quad \text{(check)} \qquad \textit{Ans.}$$



(c)

NOTE: Here $\tau_{x'y'}$ acts in the $-y'$ direction.

Fig. 9–20