Partial Differentiation

هو نوع من الاشتقاق نستخدمه عندما تكون لدينا دالة تحتوي على اكثر من متغير $f(x,y,z)$

Function of Two or more variables :-

* For Two variable $f(x,y)$

Ex : $f(x,y) = x^2 + y^2$

* for more variable $f(x,y,z)$

Ex : $f(x,y,z) = x^2 + y^2 + z^2$

By Domain and Sketching :-

Domin :- مجال الدالة يعني مجموعة القيم المسموح استخدامها كمدخلات للدالة بدون ما يصير خطأ رياضي

مثلاً $f(x) = \frac{1}{x}$ ما نكدر نقرضن $x \neq 0$ :: Domian كل الاعداد الحقيقية ماعدا الصفر

$f(x) = \sqrt{x} \rightarrow x \geq 0$

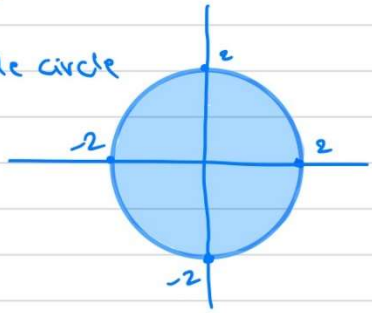
$f(x,y) = \sqrt{x+y} \rightarrow y + x \geq 0$

Ex1 // find Domain and sketch $f(x,y) = \sqrt{4 - x^2 - y^2}$

$4 - x^2 - y^2 \geq 0 \Rightarrow 4 \geq x^2 + y^2$

eq. of circle $x^2 + y^2 = r^2$

$\therefore x^2 + y^2 \leq 2^2$ inside circle



Ex2 // $f(x,y) = \ln(9 - x^2 - 9y^2)$

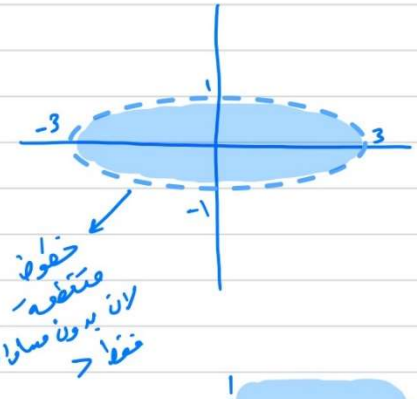
$9 - x^2 - 9y^2 > 0$

$9 > x^2 + 9y^2 \quad \div 9$

$1 > \frac{x^2}{9} + y^2$

eq. of Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

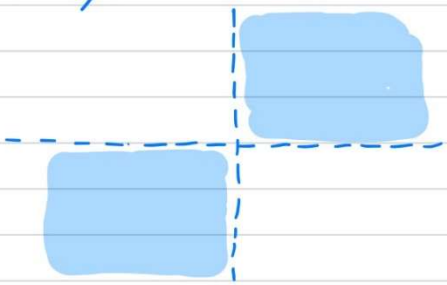
$\therefore 1 > \frac{x^2}{3^2} + \frac{y^2}{1^2}$



خطوط متقطعة لان بدون مساواة فقط >

Ex3 // $f(x,y) = \ln(xy)$

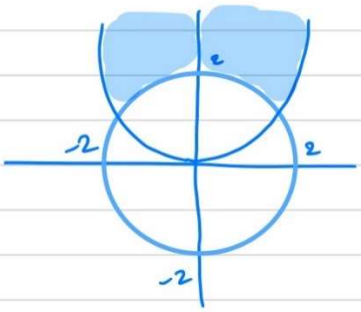
$xy > 0$



Ex4// $f(x,y) = \sqrt{x^2+y^2-4} + \sqrt{y-x^2}$

$y - x^2 \geq 0 \Rightarrow y \geq x^2$ داخل القوس perbola

$x^2 + y^2 - 4 \geq 0 \Rightarrow x^2 + y^2 \geq 4$ خارج الدائرة

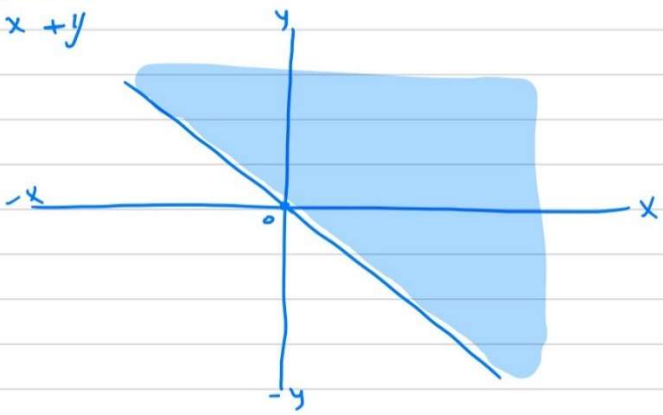


Ex5// $f(x,y) = \sqrt{x+y}$

$x + y \geq 0$

$x \geq -y$

$y \geq -x$

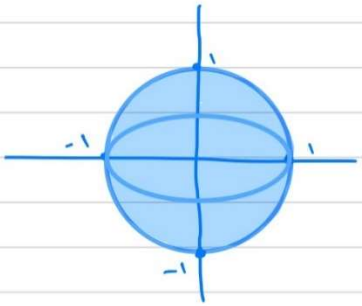


Ex6// $f(x,y,z) = \sqrt{1-x^2-y^2-z^2}$

$1 - x^2 - y^2 - z^2 \geq 0$

$1 \geq x^2 + y^2 + z^2$ eq. of sphere

inside sphere $r = 1$

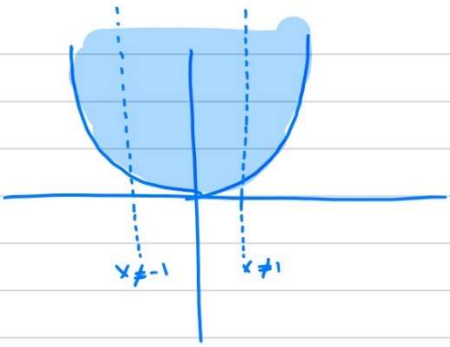


Ex7: $f(x,y) = \dfrac{\sqrt{y-x^2}}{1-x^2}$

$y \geq x^2$

$x \neq 1$

$x \neq -1$



H.W //

1- $f(x,y) = \sqrt{x} + \sqrt{y}$

2- $f(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2-16}}$

3- $f(x,y) = \dfrac{\sqrt{4-x^2-y^2}}{y^2-x^2}$

by level curves or contour curves.

$z = f(x,y)$

$z = k$ [k = any number]

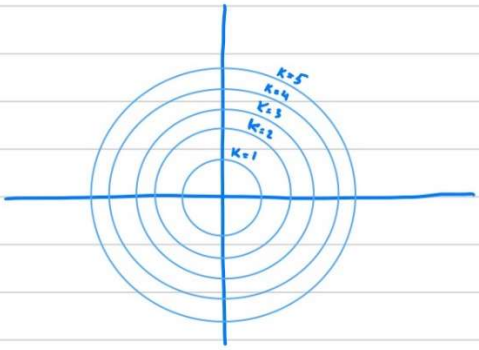
$\therefore f(x,y) = k$

$f(x,y,z) = 0 \Rightarrow f(x,y,k) = 0$

Ex 1// identify level curves of $f(x,y) = \sqrt{x^2+y^2}$

$z = \sqrt{x^2+y^2} = k$

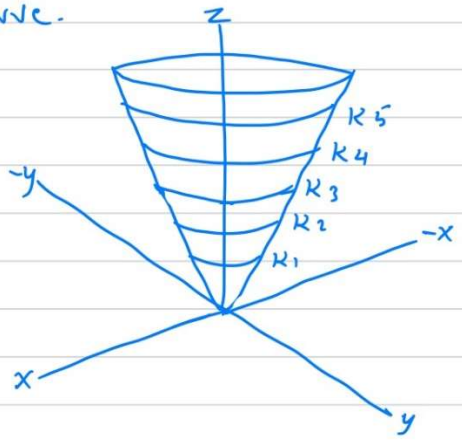
$\therefore k^2 = x^2 + y^2$



level curve : we would give z different values

$z = k = 1, 2, 3, 4, 5 \ldots$

when we sketch level curve we get a contour curve.



Ex 2 // $y^2 = 2x^2 + z$

$z = K$

$\therefore y^2 = 2x^2 + K$

at $K=0$ $\quad y = \mp\sqrt{2}\,x$

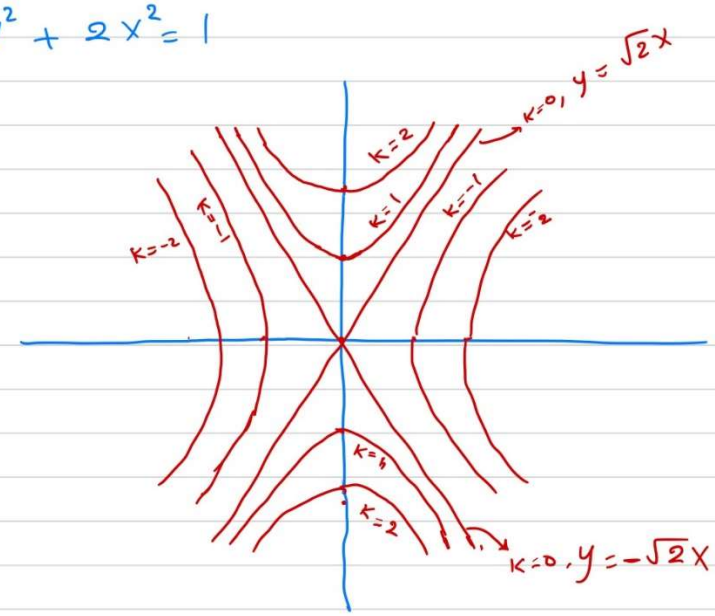
$\Rightarrow \frac{y^2}{K} - \frac{2x^2}{K} = 1$

at $K=2$ $\quad \frac{y^2}{2} - x^2 = 1$

at $K=-2$ $\quad -\frac{y^2}{2} + x^2 = 1$

at $K=1$ $\quad y^2 - 2x^2 = 1$

at $K=-1$ $\quad -y^2 + 2x^2 = 1$



Partial Derivatives

المشتقة الجزئية للدالة تعني اشتقاق f بالنسبة لـ x بثبات y ونشتق f بالنسبة إلى y بثبات x

Ex:- $f(x,y) = 2x^2y^3$

$f'_x = 4xy^3$

$f'_y = 6x^2y^2$

$$\therefore f_x(x,y) = \lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$f_y(x,y) = \lim_{h\to 0} \frac{f(x,y+h) - f(x,y)}{h}$$

$$f(x) \Rightarrow f'(x) = \frac{df}{dx}$$

$$\therefore f(x,y) \Rightarrow f'_x(x,y) = \frac{\partial f}{\partial x} \quad \& \quad f'_y(x,y) = \frac{\partial f}{\partial y}$$

Ex1 // find all of the first order partial derivatives for the following functions.

a/ $f(x,y) = x^4 + 6\sqrt{y} - 10$

$f_x(x,y) = 4x^3$

$f_y(x,y) = 0 + \frac{6}{2} y^{\frac{1}{2}-1} - 0 = \frac{3}{\sqrt{y}}$

b/ $w = x^2y - 10y^2z^3 + 43x - 7\tan(4y)$

$\frac{\partial w}{\partial x} = 2xy - 0 + 43 - 0 = 2xy + 43$

$\frac{\partial w}{\partial y} = x^2 - 20yz^3 + 0 - 7*4\sec^2(4y)$

$= x^2 - 20yz^3 - 28\sec^2(4y)$

$$\frac{\partial w}{\partial z} = 0 - 30y^2z^2 - 0 = -30y^2z^2$$

c/ $h(s,t) = t^7 \ln(s^2) + \frac{9}{t^3} - 7\sqrt{s^4}$ $\left[\frac{d[\ln g(x)]}{dx} = \frac{g'(x)}{g(x)}\right]$

$$h_s(s,t) = \frac{\partial h}{\partial s} = t^7\left(\frac{2s}{s^2}\right) + 0 - \frac{4}{7}s^{-\frac{3}{7}}$$

$$h_t(s,t) = \frac{\partial h}{\partial t} = 7t^6 \ln(s^2) - 27t^{-4} - 0$$

d/ $f(x,y) = \cos\left(\frac{4}{x}\right) e^{x^2y-5y^3}$

$$\frac{\partial f}{\partial x} = -\frac{4}{x^2}\left(-\sin\left(\frac{4}{x}\right)\right) \cdot e^{x^2y-5y^3} + \cos\left(\frac{4}{x}\right) e^{x^2y-5y^3}(2xy)$$

$$= \frac{4}{x^2}\sin\left(\frac{4}{x}\right) e^{x^2y-5y^3} + 2xy\cos\left(\frac{4}{x}\right) e^{x^2y-5y^3}$$

$$\frac{\partial f}{\partial y} = \cos\left(\frac{4}{x}\right) * \left(x^2 - 15y^2\right) e^{x^2y-5y^3}$$

$$= \left(x^2 - 15y^2\right)\cos\left(\frac{4}{x}\right) \cdot e^{x^2y-5y^3}$$

e/

$$z = \sqrt{x^2 + \ln(5x - 3y^2)}$$

نرفعه إلى اس ونشتق مشتقة القوس

$$\frac{\partial z}{\partial x} = \frac{1}{2}\left(x^2 + \ln(5x-3y^2)\right)^{-\frac{1}{2}} \cdot \left(2x + \frac{5}{5x-3y^2}\right)$$ ← مشتقة داخل القوس

$$= \left(x^2 + \ln(5x-3y^2)\right)^{-\frac{1}{2}} \cdot \left(\frac{5}{2(5x-3y^2)}\right)$$

$$\frac{\partial z}{\partial y} = \frac{1}{2}\left(x^2 + \ln(5x-3y^2)\right)^{-\frac{1}{2}} \cdot \left(\frac{-6y}{(5x-3y^2)}\right)$$

$$= \left(x^2 + \ln(5x-3y^2)\right)^{-\frac{1}{2}}\left(-\frac{3y}{5x-3y^2}\right)$$

Ex2 // Find $\frac{dy}{dx}$ for $3y^4 + x^7 = 5x$

$$12y^3\frac{dy}{dx} + 7x^6 = 5$$

$$\Rightarrow \frac{dy}{dx} = \frac{5 - 7x^6}{12y^3}$$

Ex3 // find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

$$x^2\sin(2y - 5z) = 1 + y\cos(6zx)$$

$\frac{\partial z}{\partial x}$

$$2x\sin(2y-5z) + \left(0\frac{\partial z}{\partial x} - 5\frac{\partial z}{\partial x}\right)x^2\cos(2y-5z)$$

$$= 0\frac{\partial z}{\partial x} - \left(6z + 6x\frac{\partial z}{\partial x}\right).y\sin(6zx)$$

$$\Rightarrow 2x\sin(2y-5z) - 5\frac{\partial z}{\partial x}x^2\cos(2y-5z)$$

$$= -6zy\sin(6zx) - 6yx\sin(6zx)\frac{\partial z}{\partial x}$$

$$\Rightarrow 2x\sin(2y-5z) + 6zy\sin(6zx) = \left(5x^2\cos(2y-5z)\right.$$

$$\left. - 6yx\sin(6zx)\right)\frac{\partial z}{\partial x}$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{2x\sin(2y-5z) + 6zy\sin(6zx)}{5x^2\cos(2y-5z) - 6yx\sin(6zx)}$$

$\frac{\partial z}{\partial y}$

$$x^2 \sin(2y-5z) = 1 + y\cos(6zx)$$

$$\left(2 - 5\frac{\partial z}{\partial y}\right) x^2 \cos(2y-5z) = 0\,\frac{\partial z}{\partial y} + \cos(6zx) - \left(6x\frac{\partial z}{\partial y}\right) y \sin(6zx)$$

$$2x^2\cos(2y-5z) - 5\cos(2y-5z)\frac{\partial z}{\partial y} = \cos(6zx) - 6xy\sin(6zx)\frac{\partial z}{\partial y}$$

$$\left[-5\cos(2y-5z) + 6xy\sin(6zx)\right]\frac{\partial z}{\partial y} = -2x^2\cos(2y-5z) + \cos(6zx)$$

$$\Rightarrow \frac{\partial z}{\partial y} = \frac{\cos(6zx) - 2x^2\cos(2y-5z)}{6xy\sin(6zx) - 5\cos(2y-5z)}$$

H.W//

■ find partial derivatives :-

a) $z = \frac{9u}{u^2+5v}$

b) $g(x,y,z) = \frac{x\sin y}{z^2}$

■ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$:-

$$x^3z^2 - 5xy^5z = x^2 + y^3$$