



Directional derivative.

$$D_{\vec{u}} f(x, y) = \lim_{h \rightarrow 0} \frac{f(x+ah, y+bh) - f(x, y)}{h} \quad \left. \begin{array}{l} \text{من نسب تغير و انتقام} \\ \text{التغيرات كلهما في} \\ \text{تن واحد.} \end{array} \right\}$$

$\vec{u} = (a, b)$ unit vector
 $D_{\vec{u}} f(x, y)$ directional derivative.

- The function of a single variable :-

$$g(z) = f(x_0 + az, y_0 + bz)$$

$$g'(z) = \lim_{h \rightarrow 0} \frac{g(z+h) - g(z)}{h}$$

at $z = 0$

$$\therefore g'(0) = \lim_{h \rightarrow 0} \frac{g(h) - g(0)}{h}$$

sub. in for $g(z)$:-

$$g'(0) = \lim_{h \rightarrow 0} \frac{f(x_0 + ah, y_0 + bh) - f(x_0, y_0)}{h} = D_{\vec{u}} f(x_0, y_0)$$

$$\therefore g'(0) = D_{\vec{u}} f(x_0, y_0)$$

$$\therefore D_{\vec{u}} f(x, y) = f_x(x, y) a + f_y(x, y) b$$

$$D_{\vec{u}} f(x, y, z) = f_x(x, y, z) a + f_y(x, y, z) b + f_z(x, y, z) c$$

x - اسیب - اسیب
 y - اسیب - اسیب
 z - اسیب - اسیب

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$$\vec{u} = \frac{\vec{v}}{|\vec{v}|}, \text{ or } \vec{u} = \frac{\vec{PQ}}{|\vec{PQ}|}$$

$$\vec{PQ} = (x_1 - x_0) \mathbf{i} + (y_1 - y_0) \mathbf{j} + (z_1 - z_0) \mathbf{k}$$

$$|\vec{PQ}| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

EX 1 // Find Each directional derivatives :-

a/ $D_{\vec{u}} f(2,0)$ where $f(x,y) = x e^{xy} + y$, \vec{u} is the unit vector of $\theta = \frac{2\pi}{3}$

$$\vec{u} = (a, b)$$

$$\vec{u} = (\cos \theta, \sin \theta) = \left(\cos \frac{2\pi}{3}, \sin \frac{2\pi}{3} \right) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2} \right)$$

$$D_{\vec{u}} f(x,y) = f_x(x,y)a + f_y(x,y)b$$

$$D_{\vec{u}} f(x,y) = \left(-\frac{1}{2} \right) \underset{x \text{ cosine}}{\left(e^{xy} - xye^{xy} \right)} + \left(\frac{\sqrt{3}}{2} \right) \underset{y \text{ cosine}}{\left(x^2 e^{xy} + 1 \right)}$$

$$D_{\vec{u}} f(2,0) = \left(-\frac{1}{2} \right) (e^0 - 0e^0) + \left(\frac{\sqrt{3}}{2} \right) (2^2 e^0 + 1)$$

$$D_{\vec{u}} f(2,0) = \left(-\frac{1}{2} \right) (1) + \left(\frac{\sqrt{3}}{2} \right) (5)$$

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Gradient formula

$$D_{\vec{u}} f(\vec{x}) = \nabla f \cdot \vec{u}$$

$$\vec{x} = (x, y, z) \text{ or } \vec{x} = (x, y)$$

$$\nabla f = (f_x, f_y, f_z) \text{ or } \nabla f = (f_x, f_y)$$

$$\begin{matrix} \downarrow & \downarrow & \downarrow \\ f_x & f_y & f_z \end{matrix}$$

$$\text{or } \nabla f = f_x \vec{i} + f_y \vec{j} + f_z \vec{k}$$

$$\nabla f = \frac{\partial f}{\partial x} \vec{i} + \frac{\partial f}{\partial y} \vec{j} + \frac{\partial f}{\partial z} \vec{k}$$

Ex1 //

a/ $D_{\vec{u}} f(\vec{x})$ for, $f(x, y) = x \cos(y)$ in direction of $\vec{v} = (2, 1)$

$$\nabla f(\cos y, -x \sin y)$$

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}, \|\vec{v}\| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\therefore \vec{u} = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right)$$

$$D_{\vec{u}} f(\vec{x}) = (\cos y, -x \sin y) \cdot \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right)$$

$$D_{\vec{u}} f(\vec{x}) = \frac{2}{\sqrt{5}} \cdot \cos y + \left(-\frac{1}{\sqrt{5}} \cdot x \sin y \right)$$

$$= \frac{1}{\sqrt{5}} (2 \cos y - x \sin y)$$

dot product
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b/ $D_{\vec{u}} f(\vec{x})$ for $f(x, y, z) = \sin(yz) + \ln(x^2)$ at $(1, 1, \pi)$
 $\vec{v} = (1, 1, -1)$

$$\nabla f(x, y, z) = \left(\frac{2}{x}, z \cos(yz), y \cos(yz) \right)$$

$$\nabla f(1, 1, \pi) = \left(\frac{2}{1}, \pi \cos(\pi), \cos(\pi) \right)$$

$$= (2, -\pi, -1)$$

$$|\vec{v}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$$

$$\vec{u} = \left[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \right]$$

$$D_{\vec{u}} f(1, 1, \pi) = (2, -\pi, -1) \cdot \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right)$$

$$D_{\vec{u}} f(1, 1, \pi) = \frac{2}{\sqrt{3}} - \frac{\pi}{\sqrt{3}} + \frac{1}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} [2 - \pi + 1] = \frac{3 - \pi}{\sqrt{3}}$$

c/ $f(x, y) = \cos\left(\frac{x}{y}\right)$, $\vec{v} = (3, -4)$.

$$\nabla f = \left(-\frac{1}{y} \sin\frac{x}{y}, \frac{x}{y^2} \sin\frac{x}{y} \right)$$

$$|\vec{v}| = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$$

$$\therefore u = \frac{3}{5}, -\frac{4}{5}$$

$$\therefore D_{\vec{u}} f = \left[-\frac{1}{y} \sin\frac{x}{y}, \frac{x}{y^2} \sin\frac{x}{y} \right] \cdot \left[\frac{3}{5}, -\frac{4}{5} \right]$$

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$$D_{\vec{u}} f = -\frac{3}{5y} \sin \frac{x}{y} - \frac{4x}{5y^2} \sin \frac{x}{y}$$

$$= -\frac{1}{5} \left(\frac{3}{y} + \frac{4x}{y^2} \right) \sin \frac{x}{y}$$

d/ $f(x, y, z) = x^2 y^3 - 4xz$, $\vec{v} = (-1, 2, 0)$

$$\nabla f = (2xy^3 - 4z, 3x^2y^2, -4x)$$

$$|\vec{v}| = \sqrt{(-1)^2 + 2^2 + 0} = \sqrt{5}$$

$$\vec{u} = \left(\frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right)$$

$$D_{\vec{u}} f = (2xy^3 - 4z, 3x^2y^2, -4x) \cdot \left(\frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right)$$

$$D_{\vec{u}} f = \left(-\frac{(2xy^3 - 4z)}{\sqrt{5}} + \frac{6x^2y^2}{\sqrt{5}} \right)$$

$$D_{\vec{u}} f = \frac{1}{\sqrt{5}} (4z - 2xy^3 + 6x^2y^2)$$

Ex 2 //

$$D_{\vec{u}} f (3, -1, 0) \text{ for } f(x, y, z) = 4x - y^2 e^{3x^2}$$

$$\vec{v} = (-1, 4, 2)$$

H.W