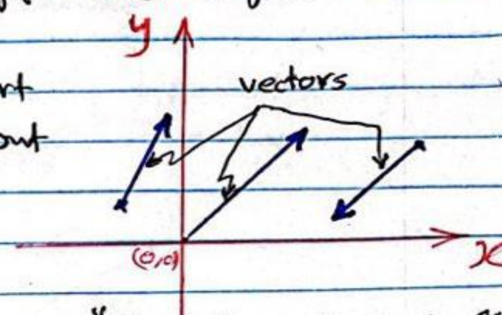
Vectors :- المتجهات

- The vector is only two pieces of information:
1- Direction
2- Length or Magnitude
المتجه عبارة عن مكونين وهما : اتجاه وطول أو كمية .

- We can graph a vector by an arrow that we can visualize on x-y plane and we can capture it by the arrow length and angle

- Vectors on graph could start From not just an origion, but From any where -



"Examples of Vectors"

Examples of Vectors :-

- To answer the question "What is the current temperature?" We use a single number (scalar); likewise the question about a mass ;
- While to answer the question "What is the current velocity of the wind?", we need more than just a single number. We need magnitude (speed) and direction. This where vectors come to handy.

Position, displacement, velocity, acceleration, Force, momentum & torque are all physical quantities that can be represented mathmatically by vectors.

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Vector Denoting :-

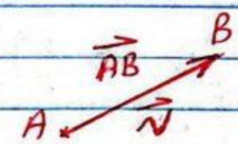
- Vectors are writing with arrow on top on equations.

Ex)

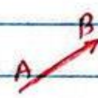
- Velocity vector $\longrightarrow \vec{v}$
- Force vector $\longrightarrow \vec{F}$

Note) Any varrable symbol with no arrow on top means scalar.

- A vector can be geometrically represented by a direction line segment with a head & a tail ;



so vector $\vec{AB}$ is a vector from point A to B.

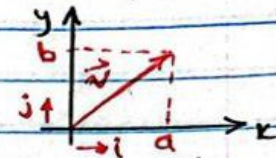
- Also, we can denote vector $\vec{AB}$ by a small case letter $\vec{v}$
- The length of the arrow  Corresponds to the magnitude of the vector.
- The arrow points in the direction of the vector.

How to represent the vector mathmatically :-

Vector in plane :- كيف تمثل المتجه رياضياً

We can write vectors as Columns. Let us take a very important special vector as example :-

$$i = \begin{pmatrix}1\\0\end{pmatrix} \quad , \quad j = \begin{pmatrix}0\\1\end{pmatrix}$$

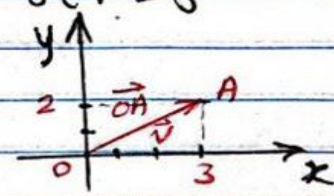


Any vector in the xy-plane can be written in terms of i & j using the triangle law & scalar multiplication.

$$\vec{v} = ai + bj = a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$$

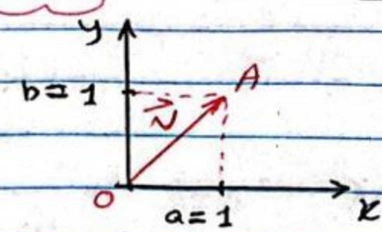
Ex:

$$\begin{pmatrix}3\\2\end{pmatrix} = 3\begin{pmatrix}1\\0\end{pmatrix} + 2\begin{pmatrix}0\\1\end{pmatrix} = 3i + 2j$$



Notes

* IF $a = b = 1$, then $\vec{v} = i + j$ is a "unit vector"



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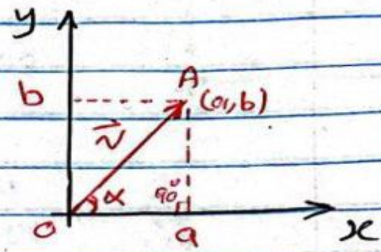
Finding the length/magnitude and the direction of vector :- إيجاد طول (مقدار) و اتجاه المتجه

IF $\vec{V} = a\,i + b\,j$ ——① , the the length/magnitude of a vector $\vec{V}$ is :-



$$|V| = \sqrt{a^2 + b^2} \text{ ——②}$$

(نظرية فيثاغورس)

- It's a Pythagorean theorem

$$\left.\begin{array}{l} a = |V| \cos\alpha \\ b = |V| \sin\alpha \end{array}\right\} \text{——③}$$

$$\tan\alpha = \frac{b}{a}$$

Substitute eq. ③ in ① yields ;

$$\vec{V} = |V| (\cos\alpha\, i + \sin\alpha\, j)$$

$\vec{V}$ — vector symbol
$|V|$ — vector length
i, j — unit vector components (basis/Fundamental vector components)
α — vector angle with x-axis

Ex/ Find a vector in plane of length (7 units) & makes angle (35°) with x-axis ?

Solution/

since $|\vec{V}| = 7$ & $\alpha = 35°$

$\therefore \vec{V} = 7 \times (\cos 35\; i + \sin 35\; j)$

$\boxed{\vec{V} = 5.7\,i + 4\,j}$ Ans

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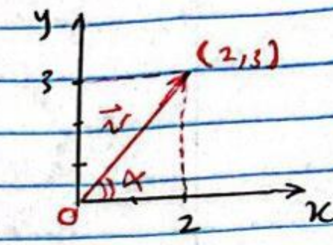
<u>EX)</u> Find the angle between the vector $\vec{v} = 2i + 3j$ and the x-axis?

<u>Solution</u>



$|\vec{v}| = \sqrt{a^2 + b^2}$; $a = 2$, $b = 3$

$|\vec{v}| = \sqrt{2^2 + 3^2} = \sqrt{13}$

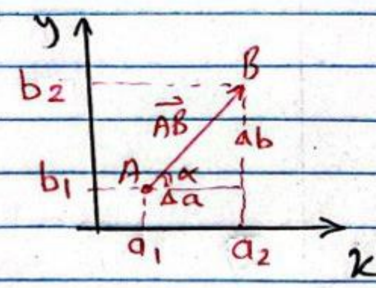
$a = |v| \cos\alpha \Longrightarrow \cos\alpha = \dfrac{a}{|v|} = \dfrac{2}{\sqrt{13}}$

$\therefore \alpha = \cos^{-1} \dfrac{2}{\sqrt{13}} = \boxed{56.3^\circ}$ <u>Ans</u>

<u>Vectors with tail not in origion :-</u>

<u>متجهات نقطة ابتداءه ليس ع (0,0)</u>

Vectors can be start not from the origion, but from any where like A to B



$\therefore \vec{AB} = \Delta a\, i + \Delta b\, j$

$\vec{AB} = (a_2 - a_1)\, i + (b_2 - b_1)\, j$

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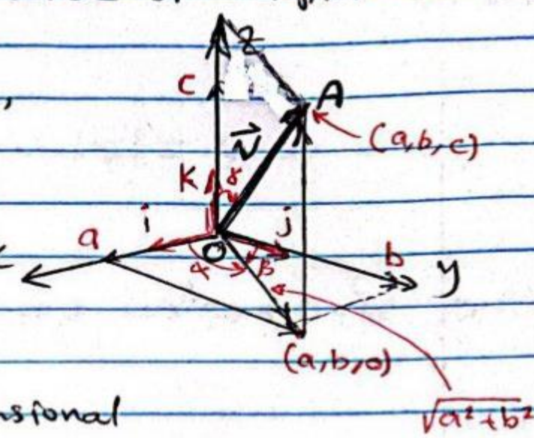
Vectors in a space :-

- This could be in three or higher dimensions

- Similar to the 2-D case, but we now have three basis vectors i, j & k.
- From these three components unit vectors we can describe any vector in three-dimensional space.



$$\boxed{\vec{V} = \vec{OA} = a\,i + b\,j + c\,K},$$

where :-

i, j, K = Basis or Fundamental unit vector.
a, b, c = Directional numbers (scalars).
α, β, γ = Directional angles.

$$\boxed{|V| = |OA| = \sqrt{a^2+b^2+c^2}},$$

$a = |V| * \cos\alpha$
$b = |V| * \cos\beta$
$c = |V| * \cos\gamma$

$$\boxed{\frac{\vec{V}}{|V|} = \cos\alpha\, i + \cos\beta\, j + \cos\gamma\, K} = \text{unit vector with direction of } \vec{V}$$

And,

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

Ex) Find a vector in space of length (5 units) that makes angles (70°) with x-axis, (85°) with y-axis?

Solution

$\alpha = 70^\circ$, $\beta = 85^\circ$, $|V| = 5$

$\gamma = ?$, $\vec{V} = ?$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \implies \cos^2 70 + \cos^2 85 + \cos^2\gamma = 1$

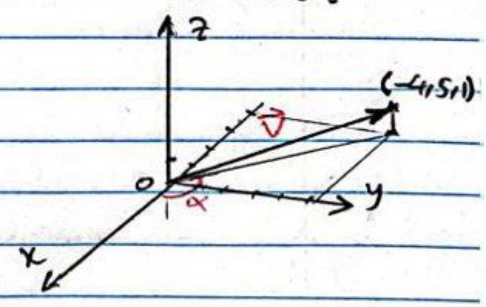
$\therefore \boxed{\cos\gamma = 0.935}$

$\therefore \vec{V} = |V|(\cos\alpha\, i + \cos\beta\, j + \cos\gamma\, k)$

$= 5(\cos 70\, i + \cos 85\, j + 0.935\, k)$

$\boxed{\vec{V} = 1.7\, i + 0.436\, j + 4.675\, k}$ Ans

Ex) Find the angle between the vector $\vec{V} = -4i + 5j + k$ and the x-axis?



Solution

$a = -4$, $b = 5$, $c = 1$

$|V| = \sqrt{a^2 + b^2 + c^2}$

$|V| = \sqrt{(-4)^2 + (5)^2 + (1)^2} = \sqrt{42}$

$\cos\alpha = \frac{a}{|V|} \implies \alpha = \cos^{-1}\frac{a}{|V|} = \cos^{-1}\frac{-4}{\sqrt{42}}$

$\boxed{\alpha = 128^\circ}$

Scalar product :- (Dot product)

ضرب القيم العددية

Let $\vec{A} = a_1 i + a_2 j + a_3 k$

And $\vec{B} = b_1 i + b_2 j + b_3 k$

Then, $\vec{A} \cdot \vec{B} = |A|\,|B| \cos\theta$

where θ — is the angle between $\vec{A}$ & $\vec{B}$

Properties :-

1- $\vec{A} \cdot \vec{A} = |A|^2$

2- $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} = a_1 b_1 + a_2 b_2 + a_3 b_3$

3- $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|A|\,|B|} = \dfrac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} + \sqrt{b_1^2 + b_2^2 + b_3^2}}$

4- $\vec{A} \perp \vec{B} \Rightarrow \vec{A} \cdot \vec{B} = 0$ [Orthogonal Vectors] متجهات متعامدة

5- $ai + bj \perp bi - aj$

EX) Find the angle "θ" between $\vec{A} = i - 2j - 2k$ & $\vec{B} = 6i + 3j + 2k$?

Solution

$\vec{A} \cdot \vec{B} = (1 * 6) + (-2 * 3) + (-2 * 2) = \boxed{-4}$

$|A| = \sqrt{1^2 + (-2)^2 + (-2)^2} = \sqrt{9} = \boxed{3}$

$|B| = \sqrt{6^2 + 3^2 + 2^2} = \sqrt{49} = \boxed{7}$

$\Rightarrow |A|\,|B| = \boxed{21}$

$\therefore \cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|A|\,|B|} \Rightarrow \cos^{-1} \dfrac{-4}{21} \approx \boxed{101^\circ}$ Ans

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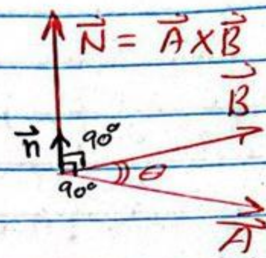
Vector Product :- (Cross-product)

ضرب المتجهات

Normal vector is what yields from vector product or cross product.



$$\vec{N} = \vec{A} \times \vec{B} = \vec{n}\,|A|\,|B| \sin\theta$$

where $\vec{n}$ -- is a normal unit vector

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$; where, $\vec{A} = a_1 \hat{i} + a_2 \hat{j} + a_3 k$, $\vec{B} = b_1 \hat{i} + b_2 \hat{j} + b_3 k$

Properties :-

1- $\vec{A} \times \vec{A} = 0 \longrightarrow$ " $\sin 0 = 0$ "

2- $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

3- $\vec{A} \parallel \vec{B} \Longrightarrow \vec{A} \times \vec{B} = 0 \longrightarrow \sin 0 = 0$

4- Area of $\Delta\, ABC = \frac{1}{2} |\vec{A} \times \vec{B}|$

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Ex) Find $\vec{A} \times \vec{B}$ & $\vec{B} \times \vec{A}$ if

$\vec{A} = 2i + j + k$

$\vec{B} = -4i + 3j + k$

Solution)

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} i - \begin{vmatrix} 2 & 1 \\ -4 & 1 \end{vmatrix} j + \begin{vmatrix} 2 & 1 \\ -4 & 3 \end{vmatrix} k$$

$$= (1 \times 1 - (3 \times 1))\, i - (2 \times 1 - (-4 \times 1))\, j + (2 \times 3 - (-4 \times 1))\, k$$

$$\boxed{\vec{A} \times \vec{B} = -2i - 6j + 10k}$$

but,

$$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B} = \boxed{2i + 6j - 10k}$$

Triple Product :- الضرب الثلاثي المكرر

A- Scalar triple product :-

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \cdot \vec{C}$$

Note)

1- Box volume is $\longrightarrow$ $V_{box} = |\vec{A} \cdot \vec{B} \times \vec{C}|$

2- Pyramid Volume is $\longrightarrow$ $V_{Py} = \frac{1}{6} |\vec{A} \cdot \vec{B} \times \vec{C}|$

B- Vector triple Product :-

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C}) \times B - (\vec{A} \cdot \vec{B})\, \vec{C}$$

Note)

$i \cdot i = j \cdot j = k \cdot k = 1$; $i \times i = j \times j = k \times k = 0$

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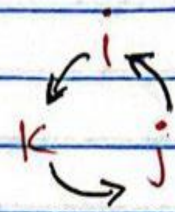
$i \cdot j = j \cdot k = k \cdot i = 0$



$i \times j = k$

$j \times k = i$

$k \times i = j$

HW#2

1- Find the Length & direction of these vectors & the angles make with the x-axis?

a- $5i + 12j$

b- $\sqrt{3}\, i + j$

2- Find a vector 6 units long in the direction of $\vec{A} = 2i + 2j - k$

3- Find the area of the triangle whose vertices are $A(1,-1,0)$, $B(2,1,-1)$, & $C(-1,1,2)$?

4- If $\vec{A} = 2i - j$ & $B = i + 3j - 2k$, find $A \times B$, then calculate $(A \times B) \cdot A$?

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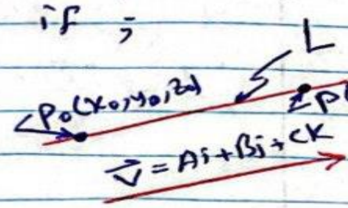
Equation of line in a space

معادلة الخط المستقيم في الفضاء

If L is a line in a space that passes through a point $P_o(x_o, y_o, z_o)$ & it parallel to a vector $\vec{V} = Ai + Bj + CK$, Then $P(x, y, z)$ is any point lies on L only if ;

$$\overrightarrow{P_oP} = t\vec{V} \quad \text{—— (1)}$$



where ;

t — time parameter

So eq. (1) can be written as ;

$$(x - x_o)i + (y - y_o)j + (z - z_o)K = t(Ai + Bj + CK)$$

$$x - x_o = At \longrightarrow x = At + x_o$$
$$y - y_o = Bt \longrightarrow y = Bt + y_o$$
$$z - z_o = Ct \longrightarrow z = Ct + z_o$$

← Eq. of line is a space

Ex 1

Find parametric equations for the line through the point $(-2, 0, 4)$ parallel to the vector $\vec{V} = 2i + 4j - 2K$

Solution

$$P_o(x_o, y_o, z_o) = (-2, 0, 4)$$

$$Ai + Bj + CK = 2i + 4j - 2K$$

$$\therefore \boxed{x = 2t - 2}$$

$$\boxed{y = 4t} \quad , \quad \boxed{z = -2t + 4}$$

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Ex)

Find parametric equations for line through the points $P(-3,2,-3)$ & $Q(1,-1,4)$?

(Solution)

$$\overrightarrow{PQ} = (1-(-3))\,i + (-1-2)\,j + (4-(-3))\,k$$

$$\boxed{\overrightarrow{PQ} = 4i - 3j + 7k} = \boxed{Ai + Bj + Ck}$$

* let $(x_0, y_0, z_0) = (-3, 2, -3)$

$$\therefore \boxed{x = 4t - 3} \; ; \; \boxed{y = -3t + 2} \; ; \; \boxed{z = 7t - 3}$$

or;

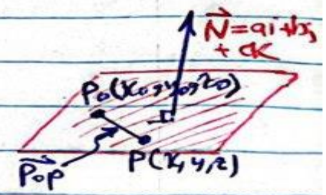
* let $(x_0, y_0, z_0) = (1, -1, 4)$

$$\therefore \boxed{x = 4t + 1} \; ; \; \boxed{y = -3t + 1} \; ; \; \boxed{z = 7t + 4}$$

Equation of the plane : معادلة المستوي

To find the eq. of the plane that passes through the point $P_0(x_0, y_0, z_0)$ & it's normal Vector is $\vec{N} = ai + bj + ck$

Let $P(x,y,z)$ be any point in the plane



$$\overrightarrow{P_0P} = (x-x_0)\,i + (y-y_0)\,j + (z-z_0)\,k$$

$$\overrightarrow{P_0P} \perp N \Longrightarrow \overrightarrow{P_0P} \cdot \vec{N} = 0$$

$$\Longrightarrow a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$\Longrightarrow ax + by + cz = ax_0 + by_0 + cz_0$$

or

$$ax + by + cz = d \longleftarrow \text{Equation of the plane}$$

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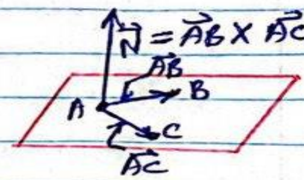
Ex1

Find the equation of the plane having the points $A(2,3,5)$, $B(7,2,1)$ & $C(1,1,1)$

Solution



في هذا المثال توجد نقاط ثلاث، من هذه النقاط الثلاث يمكننا إيجاد متجهين اثنين، وليكونا $\overrightarrow{AB}$ & $\overrightarrow{AC}$

ولكننا نحتاج $\vec{N}$ فهنا نستطيع إيجاد $\vec{N}$ من خلال عمل Cross prod. أو vector prod. بين $\overrightarrow{AB}$ و $\overrightarrow{AC}$ وكما في التالي:-

$$\overrightarrow{AB} = (7-2)i + (2-3)j + (1-5)K$$

$$\boxed{\overrightarrow{AB} = 5i - j - 4K}$$

$$\overrightarrow{AC} = (1-2)i + (1-3)j + (1-5)K$$

$$\boxed{\overrightarrow{AC} = -i - 2j - 4K}$$

$$\therefore \vec{N} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & K \\ 5 & -1 & -4 \\ -1 & -2 & -4 \end{vmatrix} = i \begin{vmatrix} -1 & -4 \\ -2 & -4 \end{vmatrix} - j \begin{vmatrix} 5 & -4 \\ -1 & -4 \end{vmatrix} + K \begin{vmatrix} 5 & -1 \\ -1 & -2 \end{vmatrix}$$

$$\vec{N} = \boxed{-4i + 24j - 11K}$$

$\therefore$ To find the value of "d", we do a substitution of any point A, B, or C as follows;

$$ax + by + cz = d \quad ; \quad d = ax_0 + by_0 + cz_0$$

let $P_0 \longrightarrow C(1,1,1) \Longrightarrow d = (-4 * 1) + (24 * 1) + (-11 * 1)$

$$\boxed{d = +9}$$

$$\therefore -4x + 24y - 11z = +9 \longleftarrow \text{Eqs of the plane}$$

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Questions for discussions :-

(1) Find the parametric eqs for the lines for

a/ The line through the point $P(3,-4,-1)$ parallel to the vector $\vec{V} = i + j + k$

b/ The line through $P(-2,0,3)$ & $Q(3,5,-2)$

c/ The line through the origin parallel to vector $\vec{V} = 2j + k$

d/ The line through the point $(3,-2,1)$ parallel to the line $x = 1+2t$, $y = 2-t$ & $z = 3t$

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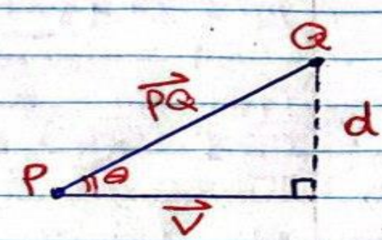
The Distance From a Point to a Line:

ايجاد المسافة من نقطة الى خط مستقيم بواسطة تطبيق المتجهات

To find a distance from a point Q to a line that passes through a point P parallel to a vector $\vec{V}$, we use the following eqs:

$$d = \frac{|\vec{PQ} \times \vec{V}|}{|\vec{V}|}$$



Ex)

Find the distance from point $p(1,1,5)$ to the line $x = 1+t$, $y = 3-t$, & $z = 2t$

Solution:

First of all, we need to find vector $\vec{V}$ from the eqs of the line

$$\left.\begin{aligned} x &= 1+t \\ y &= 3-t \\ z &= 2t \end{aligned}\right\} \xrightarrow[\text{with}]{\text{Compare}} \left.\begin{aligned} x &= x_0 + At \\ y &= y_0 + Bt \\ z &= z_0 + Ct \end{aligned}\right\} \Longrightarrow \vec{V} = Ai + Bj + Ck$$

$\therefore A = 1$, $B = -1$, $C = 2$.

$\therefore \vec{V} = i - j + 2K$

To find $Q(x_0, y_0, z_0)$, put $t = 0$ ⟶

$Q = (1, 3, 0)$

$\therefore \vec{PQ} = (1-1)i + (3-1)j + (0-5)K = 2j - 5K$

$$\vec{PQ} \times \vec{V} = \begin{vmatrix} i & j & k \\ 0 & 2 & -5 \\ 1 & -1 & 2 \end{vmatrix} = (2\times2-(-1\times5))i-(0\times2-(1\times(5)$$
$$j+(0\times(-1)-1\times2)k.$$

$$\vec{PQ}\times\vec{V} = -i+5j-2k$$

$$|\vec{PQ}\times\vec{V}| = \sqrt{(-1)^2+(-5)^2+(-2)^2} = \sqrt{30}$$

$$|\vec{V}| = \sqrt{1^2+(-1)^2+(2)^2} = \sqrt{6}$$

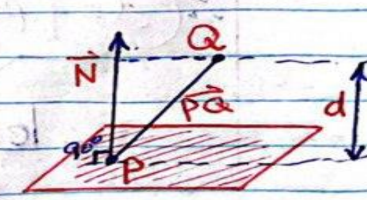
$$\therefore d = \frac{|\vec{PQ}\times\vec{V}|}{|\vec{V}|} = \frac{\sqrt{30}}{\sqrt{6}} = \sqrt{\frac{30^{5}}{6}} = \boxed{\sqrt{5}}$$

The Distance from a point to a plane:

إيجاد المسافة من نقطة الى مستوي بواسطة قوانين الجبر

If P is a point with normal $\vec{N}$, then the distance from any point (Q) to the plane is the length of the vector projection of $\vec{PQ}$ onto $\vec{N}$

$$d = |\text{Proj}_{\vec{N}}\vec{PQ}| = \left|\frac{\vec{PQ}\cdot\vec{N}}{|\vec{N}|}\right|$$



Ex1

Find the distance from the point $P(3,1,3)$ to the plane whose eq. is $3x-5y+z=4$

Solution

First we need to find a point on a plane, so, put $x=y=0$ into the given eq. of the plane yields

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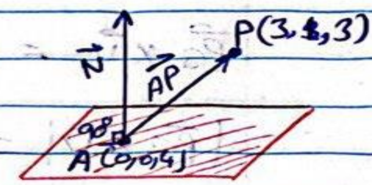
$3(0) - 5(0) + z = 4 \longrightarrow \boxed{z=4}$

$\therefore$ point $A(0,0,4)$ is satisfied the eq. of the plane.

- lets Find a vector $\vec{AP}$



$\vec{AP} = (3-0)i + (1-0)j + (3-4)k$

$\boxed{\vec{AP} = 3i + j - k}$

From the eq. of the plane (given), we can Find normal vector $\vec{N}$, as Follows

The typical eq. of the plane is

$ax + by + cz = d$ & $\vec{N} = ai + bj + ck$

$\therefore$ the eq. of the plane is $\longrightarrow 3x - 5y + z = 4$

$\therefore \vec{N} = 3i - 5j + k$

$$\therefore d = \left|\text{proj}_{\vec{N}} \vec{AP}\right| = \left|\frac{\vec{AP}\cdot\vec{N}}{|\vec{N}|}\right|$$

$$= \frac{|(3\times3) + (1\times-5) + (-1\times1)|}{\sqrt{(3)^2 + (-5)^2 + (1)^2}} = \boxed{\frac{3}{\sqrt{35}}}$$

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Vector Functions:

If $\vec{F}(x,y,z) = f_1(x,y,z)i + f_2(x,y,z)j + f_3(x,y,z)k$

Then;

$$\vec{R}(t) = x(t)\,i + y(t)\,j + z(t)\,k$$

$\vec{R}(t)$: is the position vector.

$$\vec{V}(t) = \frac{d\vec{R}(t)}{dt} = \frac{dx}{dt}i + \frac{dy}{dt}j + \frac{dz}{dt}k$$

$\vec{V}(t)$: is the velocity vector.

$$\vec{a}(t) = \frac{d^2\vec{R}}{dt^2} = \frac{d^2x}{dt^2}i + \frac{d^2y}{dt^2}j + \frac{d^2z}{dt^2}k$$

$\vec{a}(t)$: is the acceleration vector

$$|\vec{V}| = \sqrt{\left(\frac{dR}{dt}\right)^2} = \text{speed}$$

$$\frac{V}{|V|} = \frac{dR/dt}{\sqrt{(dR/dt)^2}} = \text{Direction}$$

Ex) The vector $\vec{R}(t) = (3\cos t)i + (3\sin t)j + t^2 k$ gives the position of a moving body at time. Find the body's speed & direction when $t = 2$.

Solution)

$$\vec{V}(t) = \frac{d\vec{R}(t)}{dt} = -(3\sin t)i + (3\cos t)j + 2t\,k$$

$$\text{speed} = |\vec{V}(t)| = \sqrt{(-3\sin t)^2 + (3\cos t)^2 + (2t)^2}$$

$$|\vec{V}(t)|_{t=2} = \sqrt{(-3\sin 2)^2 + (3\cos 2)^2 + (2\times 2)^2} = \boxed{5} \text{ unit of speed}$$

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Department of Fuel and Energy Techniques
Class (2nd)
Subject (Math-3)
Lecturer (Dr Hussein K. Halwas)
1st term – Lect. (Vectors)

$$\text{Direction}\Big|_{t=2} = \frac{\vec{v}(2)}{|\vec{v}(2)|} = \frac{(-3\sin 2)i + (3\cos 2)j + (2*2)k}{5}$$

$$\boxed{\text{Direction} = \frac{1}{5}(-0.105i + 3j + 4k)} \quad \underline{\text{Ans}}$$

H.Ws :-

غير داخل-1 ① Find the acute angles between the lines

a- $3x + y = 5$; $2x - y = 4$

b- $12x + 5y = 1$; $2x - 2y = 3$

② a- Find the area of the triangle determined by $P(1,1,1)$, $Q(2,1,3)$, & $R(3,-1,1)$?

b- Find a unit vector perpendicular to plane PQR ?

③ Let $u = 5i - j + k$, $v = j - 5k$ & $w = -15i + 3j - 3k$ which vectors are (a) perpendicular (b) parallel ?

④ Find the point where the line $x = \frac{8}{3} + 2t$, $y = -2t$ & $z = 1 + t$ intersects the plane $3x + 2y + 6z = 6$?

⑤ Find the distance from $S(1,1,3)$ to the plane $3x + 2y + 6z = 6$

" نهاية محاضرة --Vectors, Vectors in Space, Unit Vector, Scalar Product, Vector Product, line & plane eqs, plane-tangent-perpendicular line-vector function المتجهات، المتجهات فى الفضاء، وحدة العددية، ضرب المتجه، معادلة الخط والمستوى، الخط المماس المتجه، ضرب القيمة والعمودى على المستوى، دالة المتجه"--