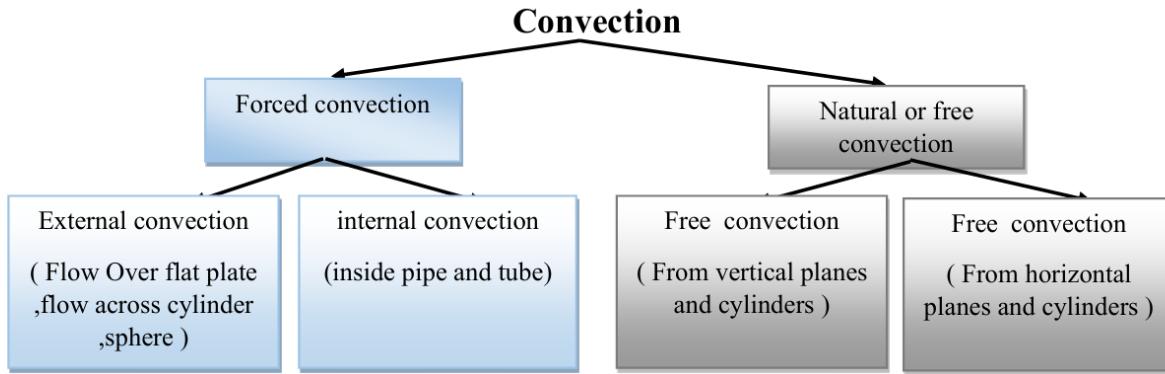




CHAPTER FOUR

CONVECTION HEAT TRANSFER



6-1 ▪ PHYSICAL MECHANISM OF CONVECTION

We mentioned earlier that there are three basic mechanisms of heat transfer: conduction, convection, and radiation. Conduction and convection are similar in that both mechanisms require the presence of a material medium. But they are different in that convection requires the presence of fluid motion.

Heat transfer through a solid is always by conduction, since the molecules of a solid remain at relatively fixed positions. Heat transfer through a liquid or gas, however, can be by conduction or convection, depending on the presence of any bulk fluid motion. Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it. Therefore, conduction in a fluid can be viewed as the limiting case of convection, corresponding to the case of quiescent fluid (Fig. 6-1).

Convection heat transfer is complicated by the fact that it involves fluid motion as well as heat conduction. The fluid motion enhances heat transfer, since it brings hotter and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in a fluid. Therefore, the rate of heat transfer through a fluid is much higher by convection than it is by conduction. In fact, the higher the fluid velocity, the higher the rate of heat transfer.



$$\dot{q}_{\text{conv}} = h(T_s - T_\infty) \quad (\text{W/m}^2)$$

(6-1)

or

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \quad (\text{W})$$

(6-2)

where

h = convection heat transfer coefficient, $\text{W/m}^2 \cdot ^\circ\text{C}$

A_s = heat transfer surface area, m^2

T_s = temperature of the surface, $^\circ\text{C}$

T_∞ = temperature of the fluid sufficiently far from the surface, $^\circ\text{C}$

Judging from its units, the **convection heat transfer coefficient h** can be defined as *the rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference*.

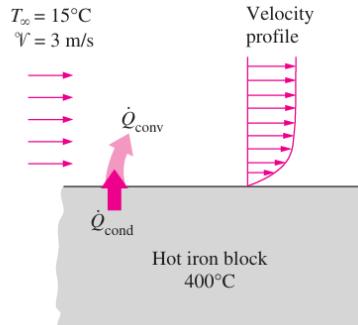
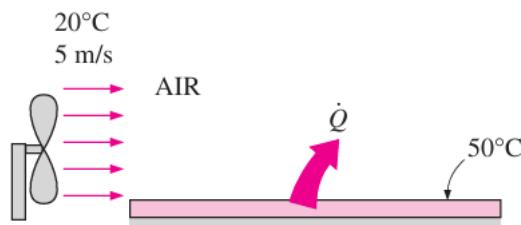
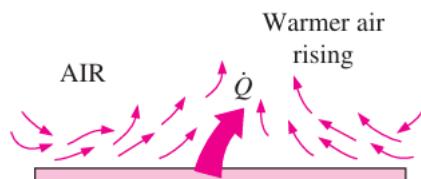


FIGURE 6-3

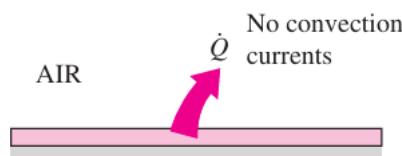
The cooling of a hot block by forced convection.



(a) Forced convection



(b) Free convection



(c) Conduction

FIGURE 6-1

Heat transfer from a hot surface to the surrounding fluid by convection and conduction.



Nusselt Number

In convection studies, it is common practice to nondimensionalize the governing equations and combine the variables, which group together into *dimensionless numbers* in order to reduce the number of total variables. It is also common practice to nondimensionalize the heat transfer coefficient h with the Nusselt number, defined as

$$\text{Nu} = \frac{hL_c}{k} \quad (6-5)$$

where k is the thermal conductivity of the fluid and L_c is the *characteristic length*. The Nusselt number is named after Wilhelm Nusselt, who made significant contributions to convective heat transfer in the first half of the twentieth century, and it is viewed as the *dimensionless convection heat transfer coefficient*.

$$\dot{q}_{\text{conv}} = h\Delta T$$

and

$$\dot{q}_{\text{cond}} = k \frac{\Delta T}{L}$$

Taking their ratio gives

$$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu}$$



CLASSIFICATION OF FLUID FLOWS

Convection heat transfer is closely tied with fluid mechanics, which is the science that deals with the behavior of fluids at rest or in motion, and the interaction of fluids with solids or other fluids at the boundaries. There are a wide variety of fluid flow problems encountered in practice, and it is usually convenient to classify them on the basis of some common characteristics to make it feasible to study them in groups. There are many ways to classify the fluid flow problems, and below we present some general categories.

Viscous versus Inviscid Flow

When two fluid layers move relative to each other, a friction force develops between them and the slower layer tries to slow down the faster layer. This internal resistance to flow is called the **viscosity**, which is a measure of internal stickiness of the fluid. Viscosity is caused by cohesive forces between the molecules in liquids, and by the molecular collisions in gases. There is no fluid with zero viscosity, and thus all fluid flows involve viscous effects to some degree. Flows in which the effects of viscosity are significant are called **viscous flows**. The effects of viscosity are very small in some flows, and neglecting those effects greatly simplifies the analysis without much loss in accuracy. Such idealized flows of zero-viscosity fluids are called frictionless or **inviscid flows**.

Internal versus External Flow

A fluid flow is classified as being internal and external, depending on whether the fluid is forced to flow in a confined channel or over a surface. The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is **external flow**. The flow in a pipe or duct is **internal flow** if the fluid is completely bounded by solid surfaces. Water flow in a pipe, for example, is internal flow, and air flow over an exposed pipe during a windy day is external flow (Fig. 6-7). The flow of liquids in a pipe is called *open-channel flow* if the pipe is partially filled with the liquid and there is a free surface. The flow of water in rivers and irrigation ditches are examples of such flows.

Compressible versus Incompressible Flow

A fluid flow is classified as being *compressible* or *incompressible*, depending on the density variation of the fluid during flow. The densities of liquids are essentially constant, and thus the flow of liquids is typically incompressible. Therefore, liquids are usually classified as *incompressible substances*. A pressure of 210 atm, for example, will cause the density of liquid water at 1 atm to change by just 1 percent. Gases, on the other hand, are highly compressible. A

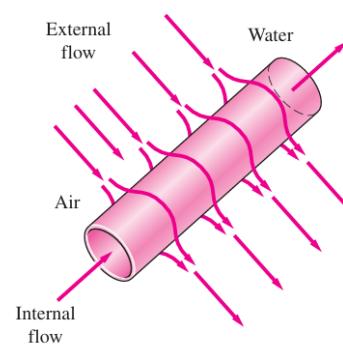


FIGURE 6-7
Internal flow of water in a pipe and the external flow of air over the same pipe.

pressure change of just 0.01 atm, for example, will cause a change of 1 percent in the density of atmospheric air. However, gas flows can be treated as incompressible if the density changes are under about 5 percent, which is usually the case when the flow velocity is less than 30 percent of the velocity of sound in that gas (i.e., the Mach number of flow is less than 0.3). The velocity of sound in air at room temperature is 346 m/s. Therefore, the compressibility effects of air can be neglected at speeds under 100 m/s. Note that the flow of a gas is not necessarily a compressible flow.

Laminar versus Turbulent Flow

Some flows are smooth and orderly while others are rather chaotic. The highly ordered fluid motion characterized by smooth streamlines is called **laminar**. The flow of high-viscosity fluids such as oils at low velocities is typically laminar. The highly disordered fluid motion that typically occurs at high velocities characterized by velocity fluctuations is called **turbulent**. The flow of low-viscosity fluids such as air at high velocities is typically turbulent. The flow regime greatly influences the heat transfer rates and the required power for pumping.

Natural (or Unforced) versus Forced Flow

A fluid flow is said to be natural or forced, depending on how the fluid motion is initiated. In **forced flow**, a fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In **natural flows**, any fluid motion is due to a natural means such as the buoyancy effect, which manifests itself as the rise of the warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid. This thermosiphoning effect is commonly used to replace pumps in solar water heating systems by placing the water tank sufficiently above the solar collectors (Fig. 6-8).

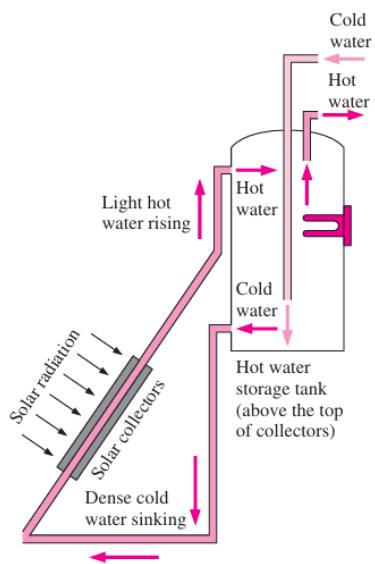


FIGURE 6-8

Natural circulation of water in a solar water heater by thermosiphoning.

Steady versus Unsteady (Transient) Flow

The terms *steady* and *uniform* are used frequently in engineering, and thus it is important to have a clear understanding of their meanings. The term **steady** implies *no change with time*. The opposite of steady is **unsteady**, or **transient**. The term *uniform*, however, implies *no change with location* over a specified region.

Many devices such as turbines, compressors, boilers, condensers, and heat exchangers operate for long periods of time under the same conditions, and they are classified as *steady-flow devices*. During steady flow, the fluid properties can change from point to point within a device, but at any fixed point they remain constant.

One-, Two-, and Three-Dimensional Flows

A flow field is best characterized by the velocity distribution, and thus a flow is said to be one-, two-, or three-dimensional if the flow velocity \mathcal{V} varies in one, two, or three primary dimensions, respectively. A typical fluid flow involves a three-dimensional geometry and the velocity may vary in all three dimensions rendering the flow three-dimensional [$\mathcal{V}(x, y, z)$ in rectangular or $\mathcal{V}(r, \theta, z)$ in cylindrical coordinates]. However, the variation of velocity in

certain direction can be small relative to the variation in other directions, and can be ignored with negligible error. In such cases, the flow can be modeled conveniently as being one- or two-dimensional, which is easier to analyze.

When the entrance effects are disregarded, fluid flow in a circular pipe is *one-dimensional* since the velocity varies in the radial r direction but not in the angular θ - or axial z -directions (Fig. 6-9). That is, the velocity profile is the same at any axial z -location, and it is symmetric about the axis of the pipe. Note that even in this simplest flow, the velocity cannot be uniform across the cross section of the pipe because of the no-slip condition. However, for convenience in calculations, the velocity can be assumed to be constant and thus *uniform* at a cross section. Fluid flow in a pipe usually approximated as *one-dimensional uniform flow*.

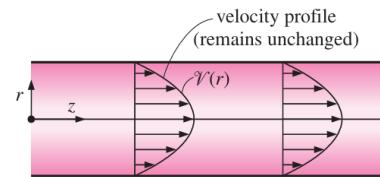


FIGURE 6-9

One-dimensional flow in a circular pipe.

VELOCITY BOUNDARY LAYER

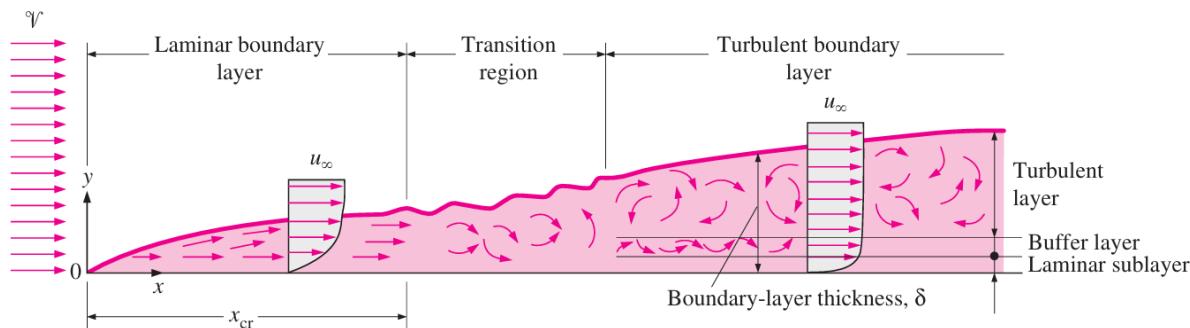


FIGURE 6-10

The development of the boundary layer for flow over a flat plate, and the different flow regimes.

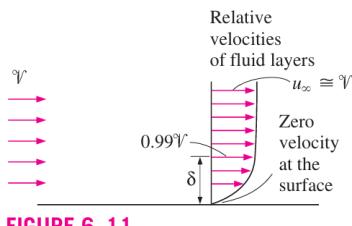


FIGURE 6-11

The region of the flow above the plate bounded by δ in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the **velocity boundary layer**. The *boundary layer thickness*, δ , is typically defined as the distance y from the surface at which $u = 0.99u_\infty$.

The hypothetical line of $u = 0.99u_\infty$ divides the flow over a plate into two regions: the **boundary layer region**, in which the viscous effects and the velocity changes are significant, and the **inviscid flow region**, in which the frictional effects are negligible and the velocity remains essentially constant.

The determination of the surface shear stress τ_s from Eq. 6-9 is not practical since it requires a knowledge of the flow velocity profile. A more practical approach in external flow is to relate τ_s to the upstream velocity V as

$$\tau_s = C_f \frac{\rho V^2}{2} \quad (\text{N/m}^2) \quad (6-10)$$

where C_f is the dimensionless **friction coefficient**, whose value in most cases is determined experimentally, and ρ is the density of the fluid. Note that the friction coefficient, in general, will vary with location along the surface. Once the average friction coefficient over a given surface is available, the friction force over the entire surface is determined from

$$F_f = C_f A_s \frac{\rho V^2}{2} \quad (N) \quad (6-11)$$

where A_s is the surface area.

The friction coefficient is an important parameter in heat transfer studies since it is directly related to the heat transfer coefficient and the power requirements of the pump or fan.

Prandtl Number

The relative thickness of the velocity and the thermal boundary layers is best described by the *dimensionless* parameter **Prandtl number**, defined as

$$Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k} \quad (6-12)$$

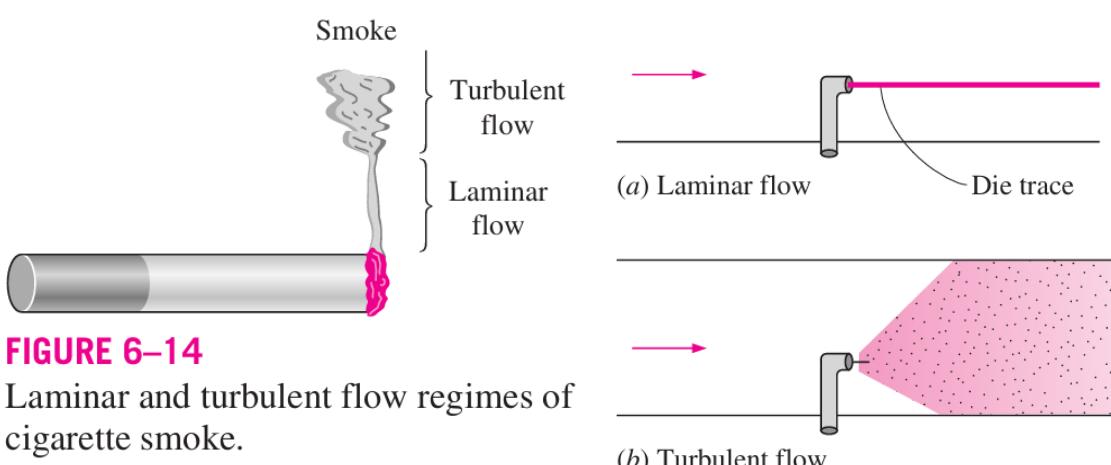


FIGURE 6-14

Laminar and turbulent flow regimes of cigarette smoke.

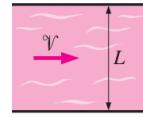


Reynolds Number

The transition from laminar to turbulent flow depends on the *surface geometry*, *surface roughness*, *free-stream velocity*, *surface temperature*, and *type of fluid*, among other things. After exhaustive experiments in the 1880s, Osborn Reynolds discovered that the flow regime depends mainly on the ratio of the *inertia forces* to *viscous forces* in the fluid. This ratio is called the **Reynolds number**, which is a *dimensionless* quantity, and is expressed for external flow as (Fig. 6-16)

$$Re = \frac{\text{Inertia forces}}{\text{Viscous}} = \frac{\mathcal{V}L_c}{\nu} = \frac{\rho\mathcal{V}L_c}{\mu} \quad (6-13)$$

where \mathcal{V} is the upstream velocity (equivalent to the free-stream velocity u_∞ for a flat plate), L_c is the characteristic length of the geometry, and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid. For a flat plate, the characteristic length is the distance x from the leading edge. Note that kinematic viscosity has the unit m^2/s , which is identical to the unit of thermal diffusivity, and can be viewed as *viscous diffusivity* or *diffusivity for momentum*.



$$\begin{aligned} Re &= \frac{\text{Inertia forces}}{\text{Viscous forces}} \\ &= \frac{\rho\mathcal{V}^2/L}{\mu\mathcal{V}L^2} \\ &= \frac{\rho\mathcal{V}L}{\mu} \\ &= \frac{\mathcal{V}L}{\nu} \end{aligned}$$

FIGURE 6-16

The Reynolds number can be viewed as the ratio of the inertia forces to viscous forces acting on a fluid volume element.

External Flow

Surface Shear Stress

$$\tau_s = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} \quad (\text{N/m}^2) \quad 1$$

where the constant of proportionality μ is called the **dynamic viscosity** of the fluid, whose unit is $\text{kg}/\text{m} \cdot \text{s}$ (or equivalently, $\text{N} \cdot \text{s}/\text{m}^2$, or $\text{Pa} \cdot \text{s}$, or poise $= 0.1 \text{ Pa} \cdot \text{s}$).

The determination of the surface shear stress τ_s from Eq. 6-9 is not practical since it requires a knowledge of the flow velocity profile. A more practical approach in external flow is to relate τ_s to the upstream velocity \mathcal{V} as

$$\tau_s = C_f \frac{\rho\mathcal{V}^2}{2} \quad (\text{N/m}^2) \quad (6-10)$$

where C_f is the dimensionless **friction coefficient**, whose value in most cases is determined experimentally, and ρ is the density of the fluid. Note that the friction coefficient, in general, will vary with location along the surface. Once the average friction coefficient over a given surface is available, the friction force over the entire surface is determined from

$$F_f = C_f A_s \frac{\rho\mathcal{V}^2}{2} \quad (\text{N}) \quad (6-11)$$

where A_s is the surface area.

The friction coefficient is an important parameter in heat transfer studies since it is directly related to the heat transfer coefficient and the power requirements of the pump or fan.

$$\frac{u}{u_\infty} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^3 \text{ for laminar flow}$$

4



For turbulent flow

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

The experimental data for heat transfer is often represented conveniently with reasonable accuracy by a simple power-law relation of the form

$$Nu = C \operatorname{Re}_L^m \operatorname{Pr}^n$$

where m and n are constant exponents, and the value of the constant C depends on geometry and flow.

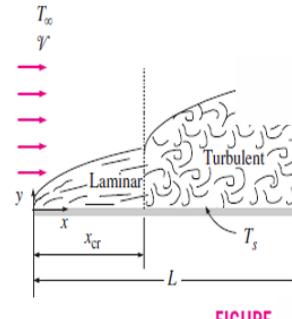
The fluid temperature in the thermal boundary layer varies from T_s at the surface to about T_∞ at the outer edge of the boundary. The fluid properties also vary with temperature, and thus with position across the boundary layer. In order to account for the variation of the properties with temperature, the fluid properties are usually evaluated at the so-called **film temperature**, defined as

$$T_f = \frac{T_s + T_\infty}{2}$$

Parallel Flow over flat plates

Consider the parallel flow of a fluid over a flat plate of length L in the flow direction, as shown in Figure . The x -coordinate is measured along the plate surface from the leading edge in the direction of the flow. The fluid approaches the plate in the x -direction with uniform upstream velocity \mathcal{V} and temperature T_∞ . The flow in the velocity boundary layer starts out as laminar, but if the plate is sufficiently long, the flow will become turbulent at a distance x_{cr} from the leading edge where the Reynolds number reaches its critical value for transition.

The transition from laminar to turbulent flow depends on the *surface geometry*, *surface roughness*, *upstream velocity*, *surface temperature*, and the *type of fluid*, among other things, and is best characterized by the Reynolds number. The Reynolds number at a distance x from the leading edge of a flat plate is expressed as



Laminar and turbulent flow over a flat plate.

$$\operatorname{Re}_x = \frac{\rho \mathcal{V} x}{\mu} = \frac{\mathcal{V} x}{\nu} \quad (7-10)$$

Note that the value of the Reynolds number varies for a flat plate along the flow, reaching $\operatorname{Re}_L = \mathcal{V} L / \nu$ at the end of the plate.

For flow over a flat plate, transition from laminar to turbulent is usually taken to occur at the *critical Reynolds number* of

$$\operatorname{Re}_{cr} = \frac{\rho \mathcal{V} x_{cr}}{\mu} = 5 \times 10^5 \quad (7-11)$$



Friction Coefficient

Based on analysis, the boundary layer thickness and the local friction coefficient at location x for laminar flow over a flat plate were determined in Chapter 6 to be

$$\text{Laminar: } \delta_{v,x} = \frac{5x}{\text{Re}_x^{1/2}} \quad \text{and} \quad C_{f,x} = \frac{0.664}{\text{Re}_x^{1/2}}, \quad \text{Re}_x < 5 \times 10^5 \quad 6-7$$

The corresponding relations for turbulent flow are

$$\text{Turbulent: } \delta_{v,x} = \frac{0.382x}{\text{Re}_x^{1/5}} \quad \text{and} \quad C_{f,x} = \frac{0.0592}{\text{Re}_x^{1/5}}, \quad 5 \times 10^5 \leq \text{Re}_x \leq 10^7 \quad 8-9$$

where x is the distance from the leading edge of the plate and $\text{Re}_x = \mathcal{V}x/\nu$ is the Reynolds number at location x . Note that $C_{f,x}$ is proportional to $\text{Re}_x^{-1/2}$ and thus to $x^{-1/2}$ for laminar flow. Therefore, $C_{f,x}$ is supposedly *infinite* at the leading edge ($x = 0$) and decreases by a factor of $x^{-1/2}$ in the flow direction. The local friction coefficients are higher in turbulent flow than they are in laminar flow because of the intense mixing that occurs in the turbulent boundary layer. Note that $C_{f,x}$ reaches its highest values when the flow becomes fully turbulent, and then decreases by a factor of $x^{-1/5}$ in the flow direction.

The *average* friction coefficient over the entire plate is determined by substituting the relations above into Eq. 6-7 and performing the integrations. We get

$$\text{Laminar: } C_f = \frac{1.328}{\text{Re}_L^{1/2}} \quad \text{Re}_L < 5 \times 10^5 \quad 10$$

$$\text{Turbulent: } C_f = \frac{0.074}{\text{Re}_L^{1/5}} \quad 5 \times 10^5 \leq \text{Re}_L \leq 10^7 \quad 11$$

The first relation gives the average friction coefficient for the entire plate when the flow is *laminar* over the *entire* plate. The second relation gives the average friction coefficient for the entire plate only when the flow is *turbulent* over the *entire* plate, or when the laminar flow region of the plate is too small relative to the turbulent flow region (that is, $x_{cr} \ll L$ where the length of the plate x_{cr} over which the flow is laminar can be determined from $\text{Re}_{cr} = 5 \times 10^5 = \mathcal{V}x_{cr}/\nu$).

For combined flow (laminar & turbulent)

$$C_f = \frac{0.074}{\text{Re}_L^{1/5}} - \frac{1742}{\text{Re}_L} \quad 5 \times 10^5 \leq \text{Re}_L \leq 10^7$$



Heat Transfer Coefficient

The local Nusselt number at a location x for laminar flow over a flat plate was determined by solving the differential energy equation to be

$$\text{Laminar: } \text{Nu}_x = \frac{h_x x}{k} = 0.332 \text{ Re}_x^{0.5} \text{ Pr}^{1/3} \quad \text{Pr} > 0.60 \quad 15$$

16

The corresponding relation for turbulent flow is

$$\text{Turbulent: } \text{Nu}_x = \frac{h_x x}{k} = 0.0296 \text{ Re}_x^{0.8} \text{ Pr}^{1/3} \quad 0.6 \leq \text{Pr} \leq 60$$
$$5 \times 10^5 \leq \text{Re}_x \leq 10^7 \quad 14$$

Note that h_x is proportional to $\text{Re}_x^{0.5}$ and thus to $x^{-0.5}$ for laminar flow. Therefore, h_x is *infinite* at the leading edge ($x = 0$) and decreases by a factor of $x^{-0.5}$ in the flow direction.

The *average* Nusselt number over the entire plate is determined by substituting the relations above into Eq. 7-8 and performing the integrations. We get

$$\text{Laminar: } \text{Nu} = \frac{hL}{k} = 0.664 \text{ Re}_L^{0.5} \text{ Pr}^{1/3} \quad \text{Re}_L < 5 \times 10^5 \quad (7-21)$$

$$\text{Turbulent: } \text{Nu} = \frac{hL}{k} = 0.037 \text{ Re}_L^{0.8} \text{ Pr}^{1/3} \quad 0.6 \leq \text{Pr} \leq 60$$
$$5 \times 10^5 \leq \text{Re}_L \leq 10^7 \quad (7-22)$$

The first relation gives the average heat transfer coefficient for the entire plate when the flow is *laminar* over the *entire* plate. The second relation gives the average heat transfer coefficient for the entire plate only when the flow is *turbulent* over the *entire* plate, or when the laminar flow region of the plate is too small relative to the turbulent flow region.

For combined flow (laminar & turbulent)

$$\text{Nu} = \frac{hL}{k} = (0.037 \text{ Re}_L^{0.8} - 871) \text{Pr}^{1/3} \quad 0.6 \leq \text{Pr} \leq 60$$
$$5 \times 10^5 \leq \text{Re}_L \leq 10^7 \quad 17$$

Uniform Heat Flux

When a flat plate is subjected to *uniform heat flux* instead of uniform temperature, the local Nusselt number is given by

$$\text{Laminar: } \text{Nu}_x = 0.453 \text{ Re}_x^{0.5} \text{ Pr}^{1/3} \quad 18$$

$$\text{Turbulent: } \text{Nu}_x = 0.0308 \text{ Re}_x^{0.8} \text{ Pr}^{1/3} \quad 19$$



Ex1: Engine oil at 60°C flows over the upper surface of a 5-m-long flat plate whose temperature is 20°C with a velocity of 2 m/s. Determine the total friction force and the rate of heat transfer per unit width of the entire plate

Properties The properties of engine oil at the film temperature of $T_f = (T_s + T_\infty)/2 = (20 + 60)/2 = 40^\circ\text{C}$ are (Table A-14).

$$\rho = 876 \text{ kg/m}^3$$

$$k = 0.144 \text{ W/m} \cdot ^\circ\text{C}$$

$$\text{Pr} = 2870$$

$$\nu = 242 \times 10^{-6} \text{ m}^2/\text{s}$$

Analysis Noting that $L = 5 \text{ m}$, the Reynolds number at the end of the plate is

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(2 \text{ m/s})(5 \text{ m})}{0.242 \times 10^{-5} \text{ m}^2/\text{s}} = 4.13 \times 10^4$$

which is less than the critical Reynolds number. Thus we have *laminar flow* over the entire plate, and the average friction coefficient is

it from moving will feel like he or she is using as much force as is necessary to hold a 18-kg mass from dropping.

Similarly, the Nusselt number is determined using the laminar flow relations for a flat plate,

$$\text{Nu} = \frac{hL}{k} = 0.664 \text{ Re}_L^{0.5} \text{ Pr}^{1/3} = 0.664 \times (4.13 \times 10^4)^{0.5} \times 2870^{1/3} = 1918$$

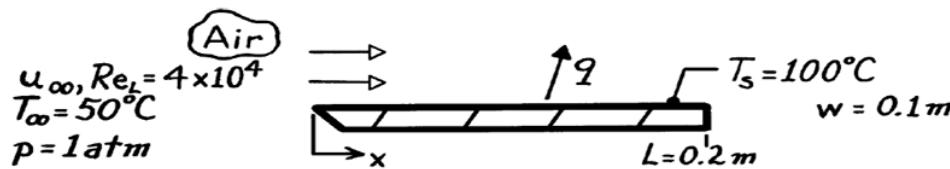
Then,

$$h = \frac{k}{L} \text{ Nu} = \frac{0.144 \text{ W/m} \cdot ^\circ\text{C}}{5 \text{ m}} (1918) = 55.2 \text{ W/m}^2 \cdot ^\circ\text{C}$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (55.2 \text{ W/m}^2 \cdot ^\circ\text{C})(5 \times 1 \text{ m}^2)(60 - 20)^\circ\text{C} = \mathbf{11,040 \text{ W}}$$

Ex3: Air at a pressure of 1 atm and temperature of 50 °C is parallel flow over the top surface of a flat plate that is heated to uniform temperature of 100 °C. The plate has a length of 0.2 m (in the flow direction) and a width of 0.1 m. The Reynolds number based on the plate length is 40000. What is the rate of heat transfer from the plate to the air? If the free stream velocity of the air is doubled and the pressure is increased to 10 atm. What is the rate of heat transfer?



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature, (3) Negligible radiation, (4) $Re_{X_c} = 5 \times 10^5$.

PROPERTIES: Table A-4, Air ($T_f = 348\text{K}$, 1 atm): $k = 0.0299 \text{ W/mK}$, $Pr = 0.70$.

ANALYSIS: (a) The heat rate is

$$q = \bar{h}_L (w \times L) (T_s - T_{\infty})$$

Since the flow is laminar over the entire plate for $Re_L = 4 \times 10^4$, it follows that

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = 0.664 Re_L^{1/2} Pr^{1/3} = 0.664 (40,000)^{1/2} (0.70)^{1/3} = 117.9.$$

$$\text{Hence } \bar{h}_L = 117.9 \frac{k}{L} = 117.9 \frac{0.0299 \text{ W/m}\cdot\text{K}}{0.2\text{m}} = 17.6 \text{ W/m}^2\cdot\text{K}$$

$$\text{and } q = 17.6 \frac{\text{W}}{\text{m}^2\cdot\text{K}} (0.1\text{m} \times 0.2\text{m}) (100 - 50)^{\circ}\text{C} = 17.6 \text{ W.} \quad <$$

(b) With $p_2 = 10 p_1$, it follows that $\rho_2 = 10 \rho_1$ and $v_2 = v_1/10$. Hence

$$Re_{L,2} = \left(\frac{u_{\infty} L}{v} \right)_2 = 2 \times 10 \left(\frac{u_{\infty} L}{v} \right)_1 = 20 Re_{L,1} = 8 \times 10^5$$

and mixed boundary layer conditions exist on the plate. Hence

$$\frac{\overline{Nu}_L}{\overline{Nu}_L} = \frac{\bar{h}_L L}{k} = \left(0.037 Re_L^{4/5} - 871 \right) Pr^{1/3} = \left[0.037 \times (8 \times 10^5)^{4/5} - 871 \right] (0.70)^{1/3}$$

$$\overline{Nu}_L = 961.$$

$$\text{Hence, } \bar{h}_L = 961 \frac{0.0299 \text{ W/m}\cdot\text{K}}{0.2\text{m}} = 143.6 \text{ W/m}^2\cdot\text{K}$$

$$q = 143.6 \frac{\text{W}}{\text{m}^2\cdot\text{K}} (0.1\text{m} \times 0.2\text{m}) (100 - 50)^{\circ}\text{C} = 143.6 \text{ W.} \quad <$$



Properties of air at 1 atm pressure

Temp. $T, ^\circ\text{C}$	Density $\rho, \text{kg/m}^3$	Specific Heat $c_p, \text{J/kg}\cdot\text{K}$	Thermal Conductivity $k, \text{W/m}\cdot\text{K}$	Thermal Diffusivity $\alpha, \text{m}^2/\text{s}$	Dynamic Viscosity $\mu, \text{kg/m}\cdot\text{s}$	Kinematic Viscosity $\nu, \text{m}^2/\text{s}$	Prandtl Number Pr
-150	2.866	983	0.01171	4.158×10^{-6}	8.636×10^{-6}	3.013×10^{-6}	0.7246
-100	2.038	966	0.01582	8.036×10^{-6}	1.189×10^{-5}	5.837×10^{-6}	0.7263
-50	1.582	999	0.01979	1.252×10^{-5}	1.474×10^{-5}	9.319×10^{-6}	0.7440
-40	1.514	1002	0.02057	1.356×10^{-5}	1.527×10^{-5}	1.008×10^{-5}	0.7436
-30	1.451	1004	0.02134	1.465×10^{-5}	1.579×10^{-5}	1.087×10^{-5}	0.7425
-20	1.394	1005	0.02211	1.578×10^{-5}	1.630×10^{-5}	1.169×10^{-5}	0.7408
-10	1.341	1006	0.02288	1.696×10^{-5}	1.680×10^{-5}	1.252×10^{-5}	0.7387
0	1.292	1006	0.02364	1.818×10^{-5}	1.729×10^{-5}	1.338×10^{-5}	0.7362
5	1.269	1006	0.02401	1.880×10^{-5}	1.754×10^{-5}	1.382×10^{-5}	0.7350
10	1.246	1006	0.02439	1.944×10^{-5}	1.778×10^{-5}	1.426×10^{-5}	0.7336
15	1.225	1007	0.02476	2.009×10^{-5}	1.802×10^{-5}	1.470×10^{-5}	0.7323
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255
45	1.109	1007	0.02699	2.416×10^{-5}	1.941×10^{-5}	1.750×10^{-5}	0.7241
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202
70	1.028	1007	0.02881	2.780×10^{-5}	2.052×10^{-5}	1.995×10^{-5}	0.7177
80	0.9994	1008	0.02953	2.931×10^{-5}	2.096×10^{-5}	2.097×10^{-5}	0.7154
90	0.9718	1008	0.03024	3.086×10^{-5}	2.139×10^{-5}	2.201×10^{-5}	0.7132
100	0.9458	1009	0.03095	3.243×10^{-5}	2.181×10^{-5}	2.306×10^{-5}	0.7111
120	0.8977	1011	0.03235	3.565×10^{-5}	2.264×10^{-5}	2.522×10^{-5}	0.7073
140	0.8542	1013	0.03374	3.898×10^{-5}	2.345×10^{-5}	2.745×10^{-5}	0.7041
160	0.8148	1016	0.03511	4.241×10^{-5}	2.420×10^{-5}	2.975×10^{-5}	0.7014
180	0.7788	1019	0.03646	4.593×10^{-5}	2.504×10^{-5}	3.212×10^{-5}	0.6992
200	0.7459	1023	0.03779	4.954×10^{-5}	2.577×10^{-5}	3.455×10^{-5}	0.6974
250	0.6746	1033	0.04104	5.890×10^{-5}	2.760×10^{-5}	4.091×10^{-5}	0.6946
300	0.6158	1044	0.04418	6.871×10^{-5}	2.934×10^{-5}	4.765×10^{-5}	0.6935
350	0.5664	1056	0.04721	7.892×10^{-5}	3.101×10^{-5}	5.475×10^{-5}	0.6937
400	0.5243	1069	0.05015	8.951×10^{-5}	3.261×10^{-5}	6.219×10^{-5}	0.6948
450	0.4880	1081	0.05298	1.004×10^{-4}	3.415×10^{-5}	6.997×10^{-5}	0.6965
500	0.4565	1093	0.05572	1.117×10^{-4}	3.563×10^{-5}	7.806×10^{-5}	0.6986
600	0.4042	1115	0.06093	1.352×10^{-4}	3.846×10^{-5}	9.515×10^{-5}	0.7037
700	0.3627	1135	0.06581	1.598×10^{-4}	4.111×10^{-5}	1.133×10^{-4}	0.7092
800	0.3289	1153	0.07037	1.855×10^{-4}	4.362×10^{-5}	1.326×10^{-4}	0.7149
900	0.3008	1169	0.07465	2.122×10^{-4}	4.600×10^{-5}	1.529×10^{-4}	0.7206
1000	0.2772	1184	0.07868	2.398×10^{-4}	4.826×10^{-5}	1.741×10^{-4}	0.7260
1500	0.1990	1234	0.09599	3.908×10^{-4}	5.817×10^{-5}	2.922×10^{-4}	0.7478
2000	0.1553	1264	0.11113	5.664×10^{-4}	6.630×10^{-5}	4.270×10^{-4}	0.7539