Al-Mustaqbal University / College of Technical Engineering
Department of Fuel and Energy Technical Engineering
Class (Third Year)
Subject (Heat Transfer-2) / Code (UOMU0206062)
Lecturer (Asst. Lect. Sameer Saad Raheem)
2nd term – Lecture No. 9 & Lecture Name (Heat Exchangers Effectiveness)

Heat Exchangers Effectiveness

1:-Log Mean Temperature Difference Method (LMTD)

The temperature difference between the hot and cold fluids varies along the heat exchanger, and it is convenient to have a mean temperature difference ΔTm for use in the relation $Q = UA_s\Delta T_m$.

In the parallel-flow double-pipe heat exchanger shown in the Figure, the temperature difference ΔT between the hot and cold fluids is large at the inlet of the heat exchanger but decreases exponentially toward the outlet. The temperature of the hot fluid decreases and the temperature of the cold fluid increases along the heat exchanger, but the temperature of the cold fluid can never exceed that of the hot fluid no matter how long the heat exchanger is. An energy balance on each fluid in a differential section of the heat exchanger can be expressed as,

$$\delta\dot{Q} = -\dot{m}_h C_{ph} dT_h \, , \delta\dot{Q} = \dot{m}_c C_{pc} dT_c$$

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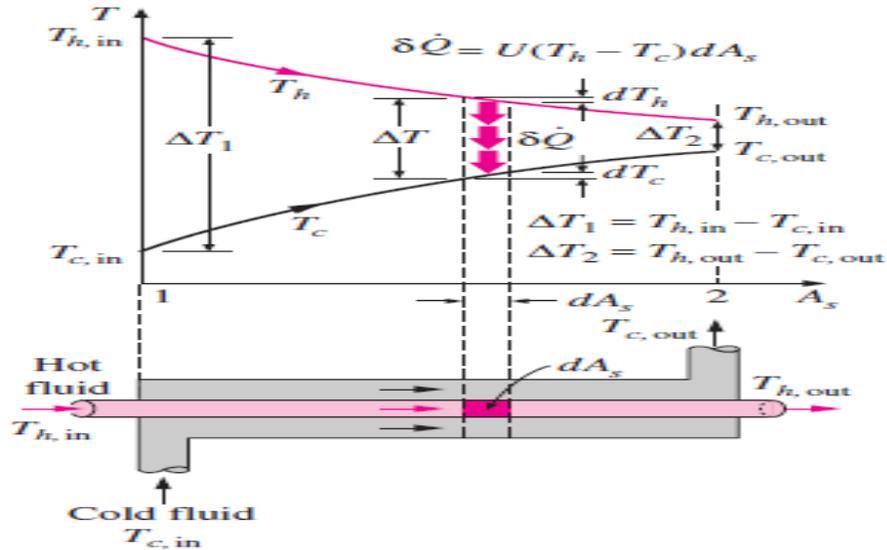
That is, the rate of heat loss from the hot fluid at any section of a heat exchanger is equal to the rate of heat gain by the cold fluid in that section.

Solving the equations above for dT_h and dT_c gives,

$$dT_h = -\frac{\delta \dot{Q}}{\dot{m}_h C_{ph}} \;,\; dT_c = \frac{\delta \dot{Q}}{\dot{m}_c C_{pc}}$$

Taking their difference, we get,

$$dT_h - dT_c = d(T_h - T_c) = -\delta \dot{Q} \left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}} \right)$$

The rate of heat transfer in the differential section of the heat exchanger can also be expressed as,

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$$\delta\dot{Q} = U(T_h - T_c)dA_s$$

Substituting this equation into the previous equation and rearranging gives,

$$\frac{d(T_h - T_c)}{T_h - T_c} = -UdA_s\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right)$$

Integrating from the inlet of the heat exchanger to its outlet, we obtain,

$$\ln\frac{T_{h,\,out} - T_{c,\,out}}{T_{h,\,in} - T_{c,\,in}} = -UA_s\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right)$$

Finally, solving the two equations,

$$\left[\dot{Q} = \dot{m}_c C_{pc}\left(T_{c,\,out} - T_{c,\,in}\right) \text{ and } \dot{Q} = \dot{m}_h C_{ph}\left(T_{h,\,in} - T_{h,\,out}\right)\right]$$

Substituting for ($\dot{m}_c C_{pc}$) and, ($\dot{m}_h C_{ph}$) into the above equation gives, after some rearrangement:

$$\dot{Q} = UA_s\Delta T_{\mathrm{lm}}, \boldsymbol{U} = \text{ Overall heat transfer coefficient}, As = \text{ Surface area } (\pi\mathrm{DL})$$

Where:

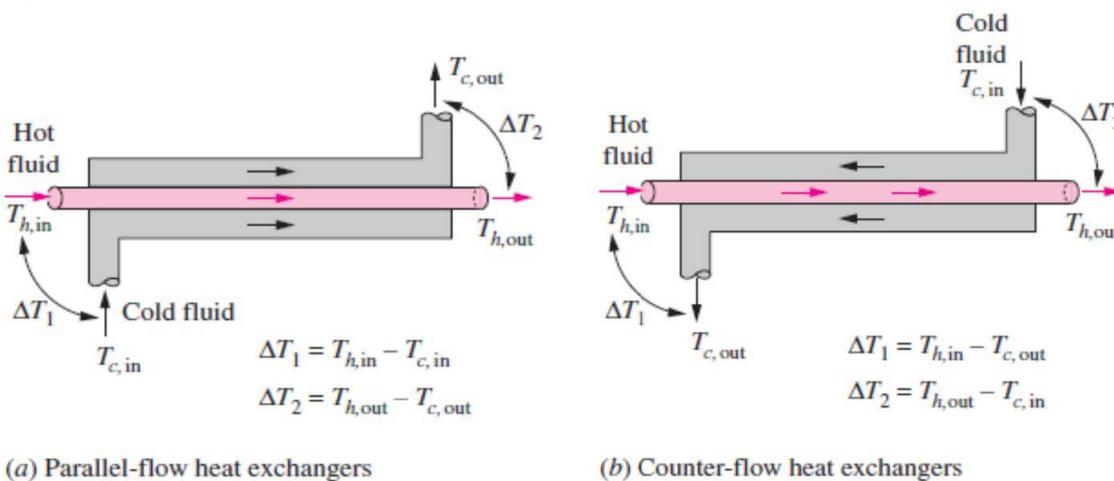$$\Delta T_{\mathrm{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

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is the **Log** mean temperature difference (**LMT**). Here ΔT_1 and ΔT_2 represent the temperature difference between the two fluids at the two ends (inlet and outlet) of the heat exchanger.

$\Delta T_1 = T_{h,in} - T_{c,in}$
$\Delta T_2 = T_{h,out} - T_{c,out}$

(*a*) Parallel-flow heat exchangers

$\Delta T_1 = T_{h,in} - T_{c,out}$
$\Delta T_2 = T_{h,out} - T_{c,in}$

(*b*) Counter-flow heat exchangers

The temperature difference between the two fluids decreases from ΔT_1 at the inlet to ΔT_2 at the outlet. Thus, it is tempting to use the arithmetic mean temperature, $\Delta T_{am} = \frac{1}{2}(\Delta T_1 + \Delta T_2)$ as the average temperature difference. The logarithmic mean temperature difference ΔT_{lm} is obtained by tracing the actual temperature profile of the fluids along the heat exchanger and is an exact representation of the average temperature difference between the hot and cold fluids. Therefore, we should always use the logarithmic mean temperature difference LMTD when determining the rate of heat transfer in a heat exchanger.

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Counter-Flow Heat Exchangers

The variation of temperatures of hot and cold fluids in a counter-flow heat exchanger is given in the Figure. Note that the hot and cold fluids enter the heat exchanger from opposite ends, and the outlet temperature of the cold fluid in this case may exceed the outlet temperature of the hot fluid.

For specified inlet and outlet temperatures, the **log** mean temperature difference for a counter-flow heat exchanger is always greater than that for a parallel-flow heat exchanger.
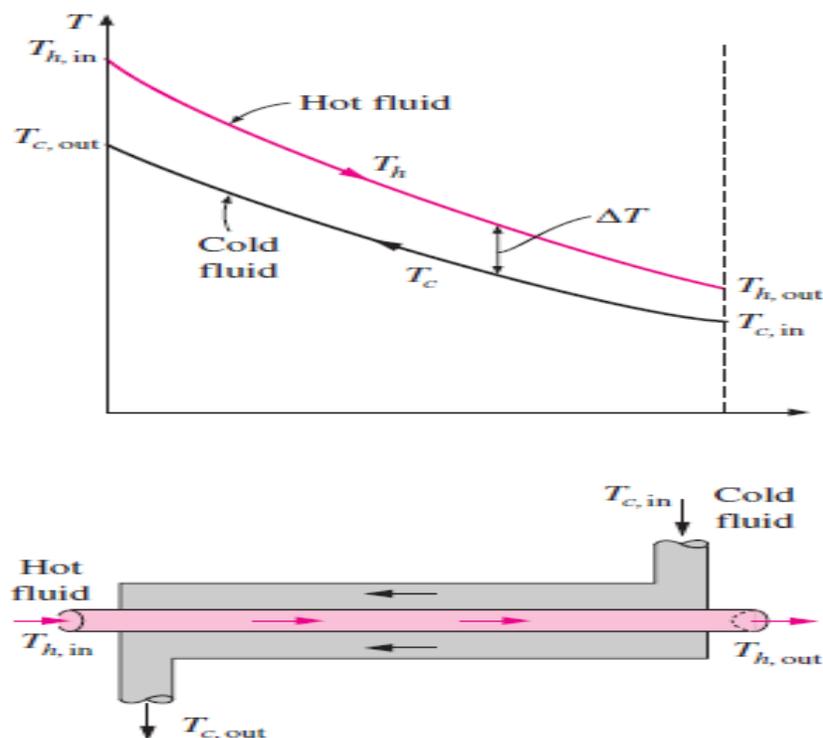
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That   is,

$$\Delta T_{\text{lm},CF} > \Delta T_{\text{lm},PF}$$

$$\dot{Q} = UA_s\Delta T_{\text{lm}}$$

And thus a smaller surface area (and thus a smaller heat exchanger) is needed to achieve a specified heat transfer rate in a counter flow heat exchanger.

Therefore, it is common practice to use counter-flow arrangem nts in heat exchangers.

Primary References

• *Holman, J.P., Heat Transfer, 10th Edition, McGraw-Hill.*

• *Kern, D.Q., Process Heat Transfer, McGraw-Hill.*

Additional Reference

• *Çengel, Y.A. & Ghajar, A.J., Heat and Mass Transfer: Fundamentals and Applications, McGraw-Hill.*