



Al-Mustaqbal University / College of Technical Engineering

Department of Fuel and Energy Technical Engineering

Class (Third Year)

Subject (Heat Transfer-2) / Code (UOMU0206062)

Lecturer (Asst. Lect. Sameer Saad Raheem)

2<sup>nd</sup> term – Lecture No. 10 & Lecture Name (Design Heat Exchangers Using NTU)

## The Effectiveness - NTU Method ( $\epsilon$ -NTU)

A kind of problem encountered in heat exchanger analysis is the determination of the heat transfer rate, and the outlet temperatures of the hot and cold fluids for prescribed fluid mass flow rates and inlet temperatures when the type and size of the heat exchanger are specified. The heat transfer surface area  $A$  of the heat exchanger in this case is known, but the outlet temperatures are not. Here the task is to determine the heat transfer performance of a specified heat exchanger or to determine if a heat exchanger available in storage will do the job. This method is based on a dimensionless parameter called the heat transfer effectiveness  $\epsilon$ , defined as,

$$\epsilon = \frac{\dot{Q}}{Q_{\max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

The actual heat transfer rate in a heat exchanger can be determined from an energy balance on the hot or cold fluids and can be expressed as,

$$\dot{Q} = C_c(T_{c, \text{out}} - T_{c, \text{in}}) = C_h(T_{h, \text{in}} - T_{h, \text{out}})$$

Where  $C_h = \dot{m}_h C_{ph}$  and  $C_c = \dot{m}_c C_{pc}$  are the heat capacity rates, (  $\text{W}/^\circ\text{C}$ , or  $\text{kW}/^\circ\text{C}$  ) of the hot and the cold fluids respectively.

To determine the maximum possible heat transfer rate in a heat exchanger, we first recognize that the maximum temperature difference in a heat exchanger is the difference between the inlet temperatures of the hot and cold fluids. That is,



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$$\Delta T = T_{h,in} - T_{c,in}$$

The heat transfer in a heat exchanger will reach its maximum value when:

- (1) The cold fluid is heated to the inlet temperature of the hot fluid or
- (2) The hot fluid is cooled to the inlet temperature of the cold fluid.

These two limiting conditions will not be reached simultaneously unless the heat capacity rates of the hot and cold fluids are identical (i.e.,  $C_c = C_h$ ). When  $C_c \neq C_h$ , which is usually the case, the fluid with the smaller heat capacity rate will experience a larger temperature change, and thus it will be the first to experience the maximum temperature, at which point the heat transfer will come to a halt. Therefore, the maximum possible heat transfer rate in a heat exchanger is,

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in})$$

Where  $C_{\min}$  is the smaller of,  $C_h = \dot{m}_h C_{ph}$  and  $C_c = \dot{m}_c C_{pc}$ .

The determination of  $\dot{Q}_{\max}$  requires the availability of the inlet temperature of the hot and cold fluids and their mass flow rates, which are usually specified. Then, once the effectiveness of the heat exchanger is known, the actual heat transfer rate  $\dot{Q}$  can be determined from,

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min}(T_{h,in} - T_{c,in})$$

The effectiveness of a heat exchanger  $\varepsilon$  depends on the geometry of the heat exchanger as well as the flow arrangement. Therefore, different types of heat



exchangers have different effectiveness relations. The effectiveness of heat exchangers can be determined from effectiveness relations or charts.

The effectiveness  $E$  relation for the double-pipe parallel-flow heat exchanger is:

$$\epsilon_{\text{parallel flow}} = \frac{1 - \exp\left[-\frac{UA_s}{C_{\min}}\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]}{1 + \frac{C_{\min}}{C_{\max}}}$$

Again  $C_{\min}$  is the smaller heat capacity rate and  $C_{\max}$  is the larger one, and it makes no difference whether  $C_{\min}$  belongs to the hot or cold fluid.

Effectiveness relations of the heat exchangers typically involve the dimensionless group  $UA_s/C_{\min}$ . This quantity is called the number of transfer units NTU and is expressed as,

$$NTU = \frac{UA_s}{C_{\min}} = \frac{UA_s}{(\dot{m}C_p)_{\min}} \quad NTU = \frac{UA_s}{C_{\min}}$$

In heat exchanger analysis, it is also convenient to define another dimensionless quantity called the heat capacity ratio ( $c$ ) as:

$$c = \frac{C_{\min}}{C_{\max}}$$

Effectiveness relations have been developed for a large number of heat exchangers, and the results are given in the Table. The effectiveness of some common types of heat exchangers is also plotted in the Figure.



Heat exchanger type	Effectiveness relation
1 Double pipe: Parallel-flow	$\varepsilon = \frac{1 - \exp[-NTU(1 + c)]}{1 + c}$
Counter-flow	$\varepsilon = \frac{1 - \exp[-NTU(1 - c)]}{1 - c \exp[-NTU(1 - c)]}$
2 Shell and tube: One-shell pass 2, 4, ... tube passes	$\varepsilon = 2 \left\{ 1 + c + \sqrt{1 + c^2} \frac{1 + \exp[-NTU\sqrt{1 + c^2}]}{1 - \exp[-NTU\sqrt{1 + c^2}]} \right\}^{-1}$
3 Cross-flow (single-pass)	
Both fluids unmixed	$\varepsilon = 1 - \exp \left\{ \frac{NTU^{0.22}}{c} [\exp(-cNTU^{0.78}) - 1] \right\}$
$C_{\max}$ mixed, $C_{\min}$ unmixed	$\varepsilon = \frac{1}{c} (1 - \exp\{1 - c[1 - \exp(-NTU)]\})$
$C_{\min}$ mixed, $C_{\max}$ unmixed	$\varepsilon = 1 - \exp \left\{ -\frac{1}{c} [1 - \exp(-cNTU)] \right\}$
4 All heat exchangers with $c = 0$	$\varepsilon = 1 - \exp(-NTU)$



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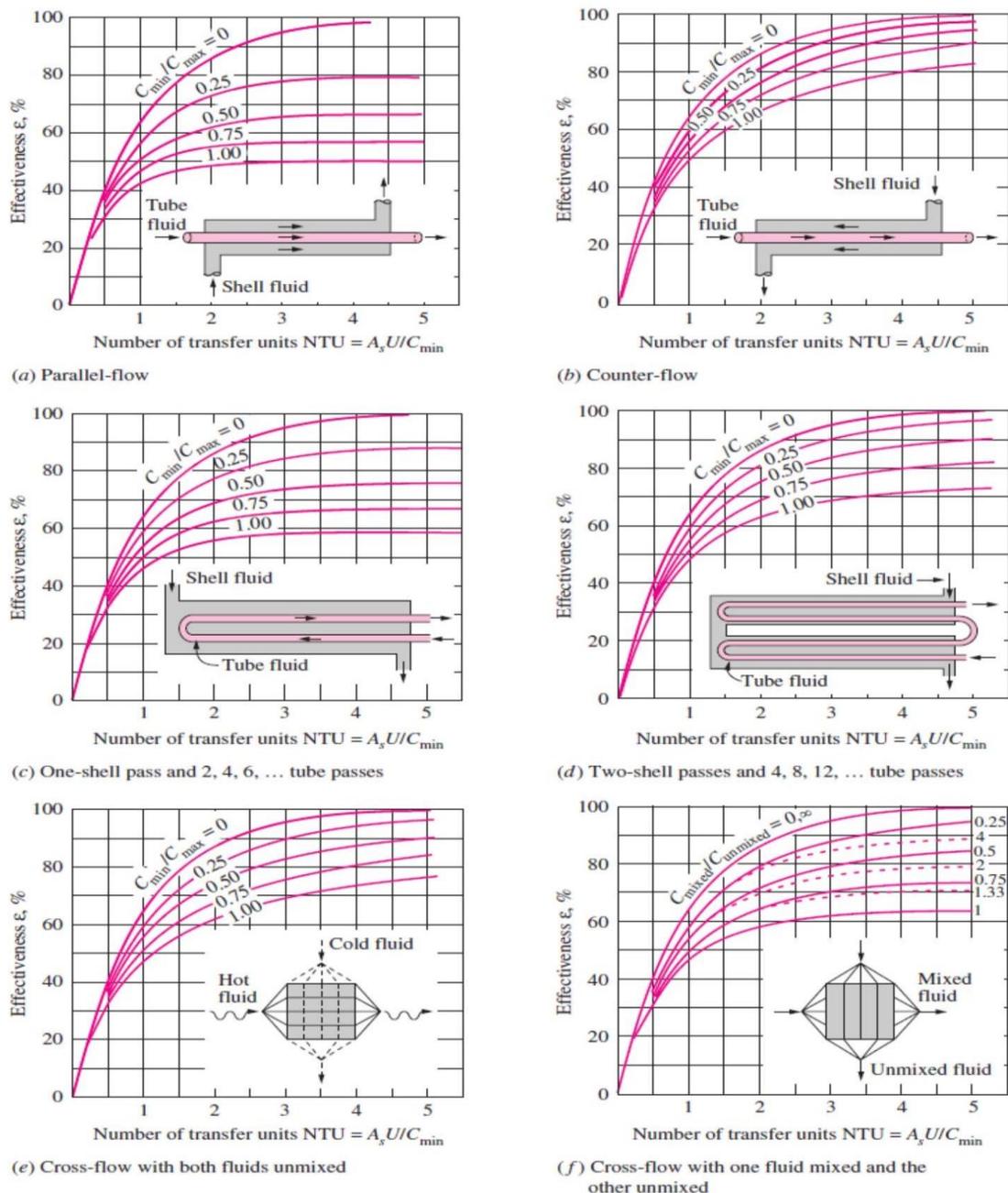


Figure (2): Effectiveness for heat exchangers



**Exercise-4: ( $\epsilon$ -NTU method)**

Hot oil is to be cooled by water in a 1-shell-pass and 8-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is  $310 \text{ W/m}^2 \cdot ^\circ\text{C}$ . Water flows through the tubes at a rate of 0.2 kg/s, and the oil through the shell at a rate of 0.3 kg/s. The water and the oil enter at temperatures of  $20^\circ\text{C}$  and  $150^\circ\text{C}$ , respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.

**Solution:**

The outlet temperatures are not specified, and they cannot be determined from an energy balance. The use of the LMTD method in this case will involve tedious iterations, and thus the  $\epsilon$ -NTU method is indicated.

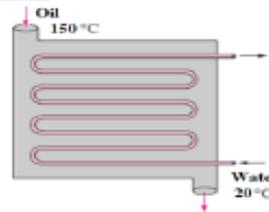
(From tables:  $C_{p,\text{oil}} = 2.13 \text{ kJ/kg} \cdot ^\circ\text{C}$ ,  $C_{p,\text{water}} = 4.18 \text{ kJ/kg} \cdot ^\circ\text{C}$ )

$$C_h = \dot{m}_h C_{ph} = (0.3 \text{ kg/s})(2.13 \text{ kJ/kg} \cdot ^\circ\text{C}) = 0.639 \text{ kW/}^\circ\text{C}$$

$$C_c = \dot{m}_c C_{pc} = (0.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot ^\circ\text{C}) = 0.836 \text{ kW/}^\circ\text{C}$$

$$C_{\min} = C_h = 0.639 \text{ kW/}^\circ\text{C}$$

$$c = \frac{C_{\min}}{C_{\max}} = \frac{0.639}{0.836} = 0.764$$



Then the maximum heat transfer rate is determined to be,

$$\begin{aligned} \dot{Q}_{\max} &= C_{\min}(T_{h,\text{in}} - T_{c,\text{in}}) \\ &= (0.639 \text{ kW/}^\circ\text{C})(150 - 20)^\circ\text{C} = 83.1 \text{ kW} \end{aligned}$$

$$A_s = n(\pi DL) = 8\pi(0.014 \text{ m})(5 \text{ m}) = 1.76 \text{ m}^2$$

Then the NTU of this heat exchanger becomes:

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{(310 \text{ W/m}^2 \cdot ^\circ\text{C})(1.76 \text{ m}^2)}{639 \text{ W/}^\circ\text{C}} = 0.853$$

The effectiveness of this heat exchanger corresponding to  $c = 0.764$  and  $\text{NTU} = 0.853$  is determined from Figure(2) to be,  $\epsilon = 0.47$ . Then the actual rate of heat transfer becomes

$$\dot{Q} = \epsilon \dot{Q}_{\max} = (0.47)(83.1 \text{ kW}) = 39.1 \text{ kW}$$

Finally, the outlet temperatures of the cold and the hot fluid streams are determined to be,

$$\dot{Q} = C_c(T_{c,\text{out}} - T_{c,\text{in}}) \longrightarrow T_{c,\text{out}} = T_{c,\text{in}} + \frac{\dot{Q}}{C_c} = 20^\circ\text{C} + \frac{39.1 \text{ kW}}{0.836 \text{ kW/}^\circ\text{C}} = 66.8^\circ\text{C}$$

$$\dot{Q} = C_h(T_{h,\text{in}} - T_{h,\text{out}}) \longrightarrow T_{h,\text{out}} = T_{h,\text{in}} - \frac{\dot{Q}}{C_h} = 150^\circ\text{C} - \frac{39.1 \text{ kW}}{0.639 \text{ kW/}^\circ\text{C}} = 88.8^\circ\text{C}$$

Therefore, the temperature of the cooling water will rise from  $20^\circ\text{C}$  to  $66.8^\circ\text{C}$  as it cools the hot oil from  $150^\circ\text{C}$  to  $88.8^\circ\text{C}$  in this heat exchanger.



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### Primary References

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- *Kern, D.Q., Process Heat Transfer, McGraw-Hill.*

### Additional Reference

- *Çengel, Y.A. & Ghajar, A.J., Heat and Mass Transfer: Fundamentals and Applications, McGraw-Hill.*